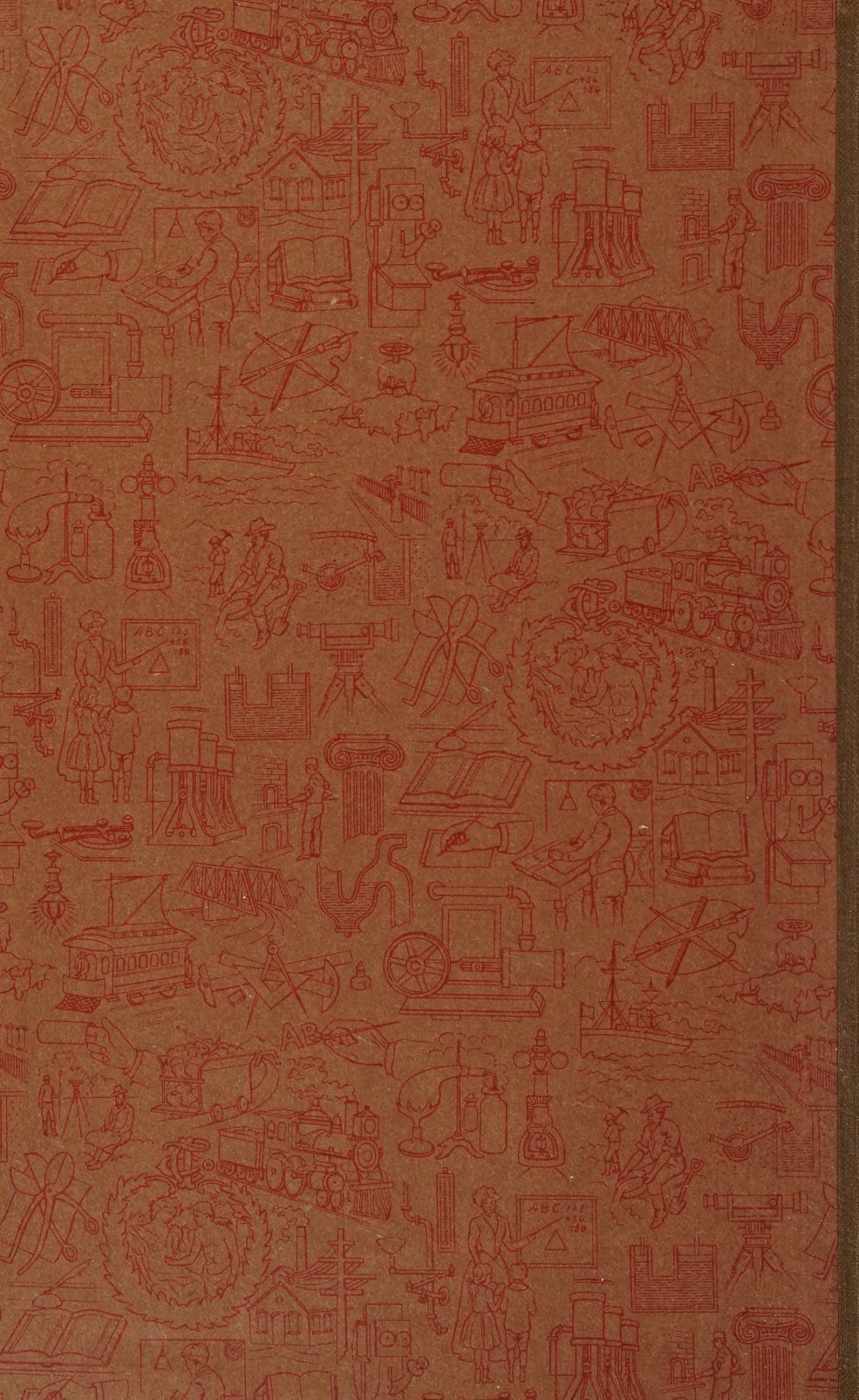

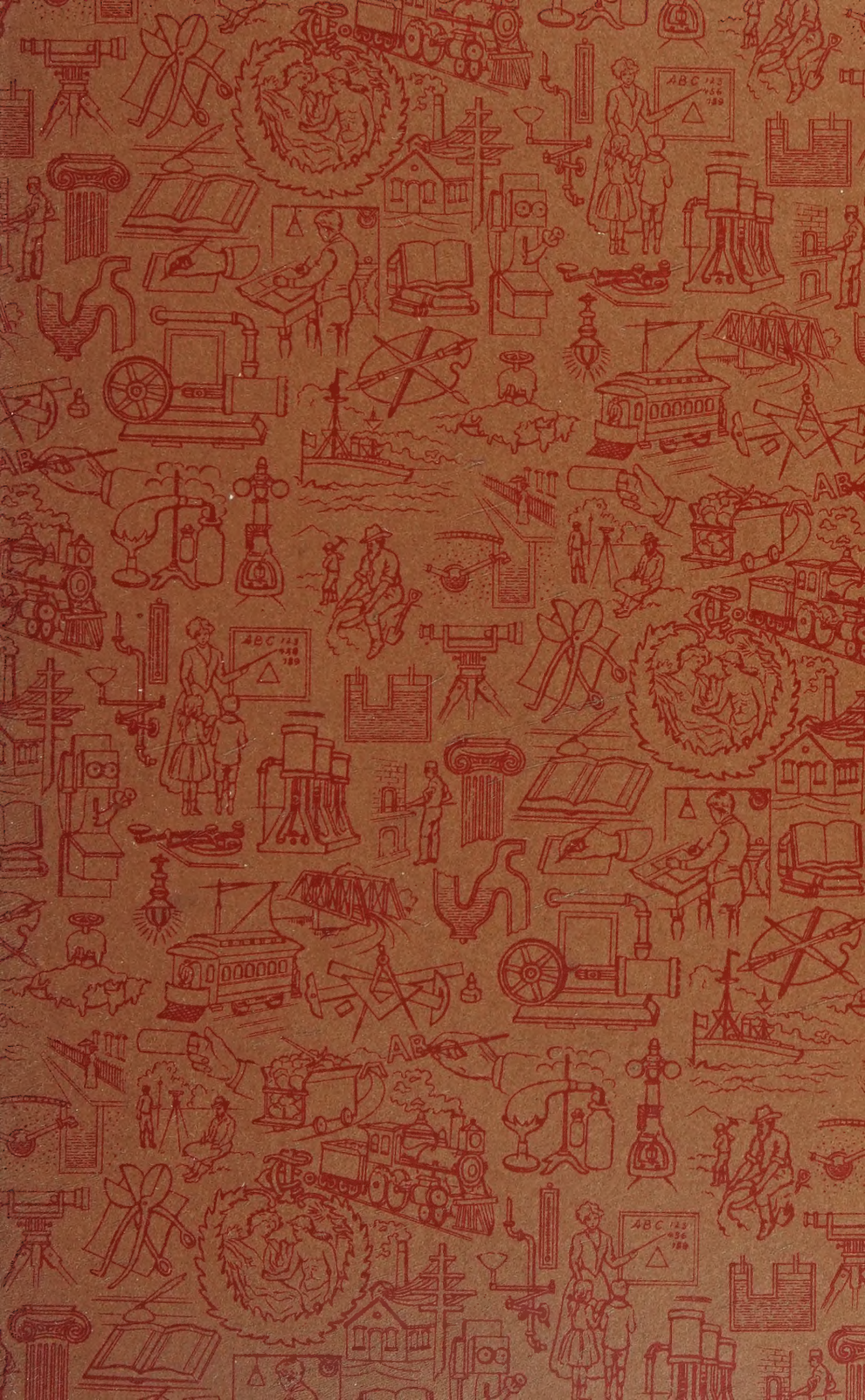

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PREFACE

Formerly it was our practice to send to each student entitled to receive them a set of volumes printed and bound especially for the Course for which the student enrolled. In consequence of the vast increase in the enrolment, this plan became no longer practicable and we therefore concluded to issue a single set of volumes, comprising all our textbooks, under the general title of I. C. S. Reference Library. The students receive such volumes of this Library as contain the instruction to which they are entitled. Under this plan some volumes contain one or more Papers not included in the particular Course for which the student enrolled, but in no case are any subjects omitted that form a part of such Course. This plan is particularly advantageous to those students who enroll for more than one Course, since they no longer receive volumes that are, in some cases, practically duplicates of those they already have. This arrangement also renders it much easier to revise a volume and keep each subject up to date.

Each volume in the Library contains, in addition to the text proper, the Examination Questions and (for those subjects in which they are issued) the Answers to the Examination Questions.

In preparing these textbooks, it has been our constant endeavor to view the matter from the student's standpoint, and try to anticipate everything that would cause him trouble. The utmost pains have been taken to avoid and correct any and all ambiguous expressions—both those due to faulty rhetoric and those due to insufficiency of statement or explanation. As the best way to make a statement, explanation, or description clear is to give a picture or a

diagram in connection with it, illustrations have been used almost without limit. The illustrations have in all cases been adapted to the requirements of the text, and projections and sections or outline, partially shaded, or full-shaded perspectives have been used, according to which will best produce the desired results.

The method of numbering pages and articles is such that each part is complete in itself; hence, in order to make the indexes intelligible, it was necessary to give each part a number. This number is placed at the top of each page, on the headline, opposite the page number; and to distinguish it from the page number, it is preceded by a section mark (§). Consequently, a reference, such as § 3, page 10, can be readily found by looking along the inside edges of the headlines until § 3 is found, and then through § 3 until page 10 is found.

INTERNATIONAL CORRESPONDENCE SCHOOLS

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ARITHMETIC.

(PART 1.)

DEFINITIONS.

1. **Arithmetic** is the science of numbers and the art of computation.

2. A **unit** is *one*, or a single thing, as *one* boy, *one* horse, *one*, *one* dozen.

3. A **number** is a unit or a collection of units, as *three* apples, *five* boys, *seven*.

4. The **unit of a number** is one of the units included in the collection of units forming the number. Thus, the unit of *twelve* is *one*, of *twenty* dollars is *one* dollar.

5. A **concrete number** is a number applied to some particular kind of object or quantity, as three *horses*, five *dollars*, ten *pounds*.

6. An **abstract number** is a number not applied to any object or quantity, as *three*, *five*, *ten*.

7. **Like numbers** are numbers that express units of the *same kind*, as 6 *days* and 10 *days*, 2 *feet* and 5 *feet*.

8. **Unlike numbers** are numbers that express units of *different kinds*, as ten *months* and eight *miles*, seven *dollars* and five *feet*.

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NOTATION AND NUMERATION.

9. Numbers are expressed in three ways: (1) by words; (2) by figures; (3) by letters.

10. **Notation** is the art of expressing numbers by figures or letters.

11. **Numeration** is the art of reading numbers expressed by figures or letters.

ARABIC NOTATION.

12. The **Arabic notation** is the method of expressing numbers by figures. This method employs ten characters, called **figures,** to represent numbers, viz.:

Figures	0	1	2	3	4	5	6	7	8	9
Names	*naught, cipher, or zero*	*one*	*two*	*three*	*four*	*five*	*six*	*seven*	*eight*	*nine*

The first figure (0) is called **naught, cipher,** or **zero,** and, when standing alone, has no value.

The other nine figures are called **digits,** and each one has a value of its own.

An **integer** is any whole number.

13. Since there are only ten *figures* used in expressing numbers, each *figure* must have different *values*, determined by the way in which it is used.

14. The value of a figure depends upon its *position* in relation to others.

15. Figures have **simple** values, and **local,** or **place, values.**

16. The **simple** value of a figure is the value it expresses when standing alone.

17. The **local**, or **place**, value of a figure is its value as determined by its position in a number.

Thus, 6 standing alone means *six ones*	6
In the second place it denotes *six tens*	60
In the third place it denotes *six hundreds*	600
In the fourth place it denotes *six thousands* . . .	6,000
In the fifth place it denotes *six ten-thousands* . . .	60,000
In the sixth place it denotes *six hundred-thousands*	600,000
In the seventh place it denotes *six millions* . . .	6,000,000

18. The **value** of a figure increases tenfold with each remove to the left.

19. The **cipher** has no value in itself, but it is useful in fixing the place of other figures. To represent the number *four hundred five*, only two significant figures are necessary, one to denote *four hundred*, and the other to denote *five;* but if these two figures are placed together, as 45, the 4, being in the second place, will mean 4 *tens*. To denote 4 *hundreds* it should be in the third place. A cipher, therefore, must be inserted in the *tens* place to show that the number is composed of *hundreds* and *units* only, and that there are no *tens*. *Four hundred five* is therefore written 405. If the number were *four thousand five*, two ciphers would be inserted; thus, 4,005. If it were *four hundred fifty*, the cipher would be in the units place to show that there are no *units*, but only *hundreds* and *tens;* thus, 450. *Four thousand fifty* is written 4,050, the ciphers indicating that there are no hundreds and no units.

20. In reading numbers, it is usual to divide them by commas into groups of three figures each, called **periods**, beginning at the right. The first figure is said to belong to the *first order*, the second to the *second order*, etc. Each **period** contains three orders, named as shown in the following table:

TABLE.

Name of period.	*Trillions.*			*Billions.*			*Millions.*			*Thousands.*			*Units.*		
Name of order.	hundred-trillions.	ten-trillions.	trillions.	hundred-billions.	ten-billions.	billions.	hundred-millions.	ten-millions.	millions.	hundred-thousands.	ten-thousands.	thousands.	hundreds.	tens.	units.
Number.	9	8	7,	4	3	2,	1	9	8,	7	6	5,	4	3	2

The first period, beginning at the right, contains *units*, *tens*, *hundreds;* the second, *thousands*, *ten-thousands*, *hundred-thousands;* the third, *millions*, *ten-millions*, *hundred-millions;* etc.

The number in the table is read, *nine hundred eighty-seven trillion, four hundred thirty-two billion, one hundred ninety-eight million, seven hundred sixty-five thousand, four hundred thirty-two.*

21. The writing of numbers is called **notation,** and the reading of numbers is called **numeration.** It will be noticed that in reading and writing numbers the *s* at the end of thousands, millions, etc. is omitted.

ROMAN NOTATION.

22. **Roman notation** is a method of expressing numbers by means of seven capital letters: These letters are I, V, X, L, C, D, and M. Their values, when standing alone, are as follows: I = 1, V = 5, X = 10, L = 50, C = 100, D = 500, M = 1,000. By combinations of these letters all other numbers are expressed. Their combinations are in accordance with the following.

PRINCIPLES.

1. *Repeating a letter repeats its value.*

Thus, II = 2, XX = 20, XXX = 30, CC = 200, CCC = 300.

V, D, and L are never repeated, and only I, X, C, and M are ever used more than once in any combination.

2. *If a letter precedes one of greater value, their difference is denoted; if it follows, their sum is denoted.*

Thus, IV = 4, VI = 6, IX = 9, XI = 11, XL = 40, LX = 60.

3. *A bar placed over a letter multiplies its value by one thousand.*

Thus, $\bar{X}$ = 10,000, $\bar{L}$ = 50,000, $\overline{XC}$DXVII = 90,517.

23. The following table illustrates more fully the foregoing principles:

VII	= 7	LX	= 60	CLXIX	= 169	$\overline{IX}$LX	= 9060
XIII	= 13	LXIX	= 69	CLXXX	= 180	$\bar{L}$CXC	= 50190
XIV	= 14	LXX	= 70	CCXL	= 240	$\overline{XIX}$	= 19000
XV	= 15	LXXX	= 80	CCCLIX	= 359	$\bar{X}$LDX	= 40510
XX	= 20	XC	= 90	CCCCL	= 450	$\bar{X}$CVII	= 97000
XXV	= 25	XCIX	= 99	CCCCXC	= 490	$\overline{DC}$	= 700000
XXVIII	= 28	CIX	= 109	DCCXL	= 740	$\bar{X}\overline{CIX}$	= 99000
XXIX	= 29	CXI	= 111	DCCCXI	= 811	$\bar{M}$	= 1000000
XXX	= 30	CXIX	= 119	DCCCC	= 900	$\overline{MM}$	= 2000000
XL	= 40	CLVI	= 156	DCCCCXL	= 940	$\overline{MDC}$	= 1700000

24. The four fundamental processes of arithmetic are **addition, subtraction, multiplication,** and **division.** They are called fundamental processes because all operations in arithmetic are based upon them.

ADDITION.

25. **Addition** is the *process* of *finding* a number that is equal to two or more numbers taken together. The sign of addition is +. It is read *plus*, and means *more*. Thus, 5 + 6 is read 5 *plus* 6, and means that 5 and 6 are to be added.

26. The sign of equality is =. It is read *equals*, or *is equal to*. Thus, $5 + 6 = 11$ may be read *5 plus 6 equals 11*.

27. Only like numbers can be added. Thus, 6 dollars can be added to 7 dollars, and the sum will be 13 dollars, but 6 *dollars* cannot be added to 7 *feet*.

28. The following table gives the sum of any two numbers from 1 to 12:

1 and 1 is 2	2 and 1 is 3	3 and 1 is 4	4 and 1 is 5
1 and 2 is 3	2 and 2 is 4	3 and 2 is 5	4 and 2 is 6
1 and 3 is 4	2 and 3 is 5	3 and 3 is 6	4 and 3 is 7
1 and 4 is 5	2 and 4 is 6	3 and 4 is 7	4 and 4 is 8
1 and 5 is 6	2 and 5 is 7	3 and 5 is 8	4 and 5 is 9
1 and 6 is 7	2 and 6 is 8	3 and 6 is 9	4 and 6 is 10
1 and 7 is 8	2 and 7 is 9	3 and 7 is 10	4 and 7 is 11
1 and 8 is 9	2 and 8 is 10	3 and 8 is 11	4 and 8 is 12
1 and 9 is 10	2 and 9 is 11	3 and 9 is 12	4 and 9 is 13
1 and 10 is 11	2 and 10 is 12	3 and 10 is 13	4 and 10 is 14
1 and 11 is 12	2 and 11 is 13	3 and 11 is 14	4 and 11 is 15
1 and 12 is 13	2 and 12 is 14	3 and 12 is 15	4 and 12 is 16
5 and 1 is 6	6 and 1 is 7	7 and 1 is 8	8 and 1 is 9
5 and 2 is 7	6 and 2 is 8	7 and 2 is 9	8 and 2 is 10
5 and 3 is 8	6 and 3 is 9	7 and 3 is 10	8 and 3 is 11
5 and 4 is 9	6 and 4 is 10	7 and 4 is 11	8 and 4 is 12
5 and 5 is 10	6 and 5 is 11	7 and 5 is 12	8 and 5 is 13
5 and 6 is 11	6 and 6 is 12	7 and 6 is 13	8 and 6 is 14
5 and 7 is 12	6 and 7 is 13	7 and 7 is 14	8 and 7 is 15
5 and 8 is 13	6 and 8 is 14	7 and 8 is 15	8 and 8 is 16
5 and 9 is 14	6 and 9 is 15	7 and 9 is 16	8 and 9 is 17
5 and 10 is 15	6 and 10 is 16	7 and 10 is 17	8 and 10 is 18
5 and 11 is 16	6 and 11 is 17	7 and 11 is 18	8 and 11 is 19
5 and 12 is 17	6 and 12 is 18	7 and 12 is 19	8 and 12 is 20
9 and 1 is 10	10 and 1 is 11	11 and 1 is 12	12 and 1 is 13
9 and 2 is 11	10 and 2 is 12	11 and 2 is 13	12 and 2 is 14
9 and 3 is 12	10 and 3 is 13	11 and 3 is 14	12 and 3 is 15
9 and 4 is 13	10 and 4 is 14	11 and 4 is 15	12 and 4 is 16
9 and 5 is 14	10 and 5 is 15	11 and 5 is 16	12 and 5 is 17
9 and 6 is 15	10 and 6 is 16	11 and 6 is 17	12 and 6 is 18
9 and 7 is 16	10 and 7 is 17	11 and 7 is 18	12 and 7 is 19
9 and 8 is 17	10 and 8 is 18	11 and 8 is 19	12 and 8 is 20
9 and 9 is 18	10 and 9 is 19	11 and 9 is 20	12 and 9 is 21
9 and 10 is 19	10 and 10 is 20	11 and 10 is 21	12 and 10 is 22
9 and 11 is 20	10 and 11 is 21	11 and 11 is 22	12 and 11 is 23
9 and 12 is 21	10 and 12 is 22	11 and 12 is 23	12 and 12 is 24

This table should be carefully committed to memory. Since 0 has no value, the sum of any number and 0 is the number itself; thus, 17 and 0 is 17.

29. For *addition*, place the numbers to be added directly under one another, taking care to place *units* under *units*, *tens* under *tens*, *hundreds* under *hundreds*, and so on.

EXAMPLE 1.—What is the sum of 131, 222, 21, 2, and 413?

SOLUTION.—

$$\begin{array}{r} 131 \\ 222 \\ 21 \\ 2 \\ 413 \\ \hline \textit{sum}\ 789 \end{array}$$ Ans.

EXPLANATION.—After placing the numbers in proper order, begin at the bottom of the units column and add, mentally repeating the different sums. Thus, three, five, six, eight, nine, the sum of the numbers in the units column. Place the 9 directly beneath, as the units figure in the sum.

The sum of the numbers in the tens column is 8, which is the tens figure in the sum.

The sum of the numbers in the hundreds column is 7, which is the hundreds figure in the sum.

EXAMPLE 2.—What is the sum of 61,803 + 43,429 + 47,712 + 62,138?

SOLUTION.—

$$\begin{array}{r} 61803 \\ 43429 \\ 47712 \\ 62138 \\ \hline 22 \\ 60 \\ 2000 \\ 13000 \\ 200000 \\ \hline \textit{sum}\ 215082 \end{array}$$ Ans.

EXPLANATION.—The sum of the units column is 22; of the tens column it is 6 tens, or 60; of the hundreds column it is 20 hundreds, or 2,000; of the thousands column it is 13 thousands, or 13,000; and of the ten-thousands column the sum is 20 ten-thousands, or 200,000. The sum of these numbers

is 215,082. Ordinarily, the work would be performed as follows, the unnecessary ciphers being omitted:

$$
\begin{array}{r}
61803 \\
43429 \\
47712 \\
62138 \\
\hline
\quad\;\; 22 \\
\quad\;\;\; 6 \\
\quad 20 \\
\;\; 13 \\
20 \\
\hline
\textit{sum}\;\; 215082
\end{array}
$$

This method is very convenient to use when adding a long column of figures.

30. In practice, addition is performed as follows:

$$
\begin{array}{r}
425 \\
\;\; 36 \\
9215 \\
\quad\;\; 4 \\
\;\; 907 \\
\hline
\textit{sum}\;\; 10587
\end{array}
$$

Explanation.—The sum of the numbers in the units column = 27 units, or 2 tens and 7 units. Write the 7 units as the first, or right-hand, figure in the sum. Reserve the 2 tens and add them to the tens column. The sum of the figures in the tens column plus the 2 tens reserved from the units column = 8, which is written as the second figure in the sum. There is nothing to carry to the next column. The sum of the numbers in the next column is 15, or 1 thousand and 5 hundreds. Write the 5 as the third, or hundreds, figure in the sum and carry the 1 to the next column: $1 + 9 = 10$, which is written at the left of the other figures.

The second method saves space and figures, but the first is to be preferred when adding a long column.

EXAMPLE.—Add the following numbers:

SOLUTION.—

$$\begin{array}{r} 890 \\ 82 \\ 90 \\ 393 \\ 281 \\ 80 \\ 770 \\ 83 \\ 492 \\ 80 \\ 383 \\ 84 \\ 191 \\ \hline \textit{sum}\ 3899 \end{array}$$ Ans.

EXPLANATION.—The sum of the first column is 19, or 1 ten and 9 units. Write 9 and carry 1 to the next column. The sum of the second column + 1 = 109 tens, or 10 hundreds and 9 tens. Write 9 and carry 10 to the next column. The sum of this column plus the 10 reserved is 38. Write 38. The total sum is 3,899.

31. Rule.—I. *Begin at the right, add each column separately, and write the sum, if it be only one figure, under the column added.*

II. *If the sum of any column consists of two or more figures, put the right-hand figure of the sum under that column, and add the remaining figure or figures to the next column.*

32. Proof.—*To prove addition, add each column from top to bottom. If the same result is obtained as by adding from bottom to top, the work is probably correct.*

EXAMPLES FOR PRACTICE.

Find the the sum of:

(*a*) 104 + 203 + 613 + 214.
(*b*) 1,875 + 3,143 + 5,826 + 10,832.
(*c*) 4,865 + 2,145 + 8,173 + 40,084.
(*d*) 14,204 + 8,173 + 1,065 + 10,042.
(*e*) 10,832 + 4,145 + 3,133 + 5,872.
(*f*) 214 + 1,231 + 141 + 5,000.
(*g*) 123 + 104 + 425 + 126 + 327.
(*h*) 6,354 + 2,145 + 2,042 + 1,111 + 3,333.

Ans. (*a*) 1,134. (*b*) 21,676. (*c*) 55,267. (*d*) 33,484. (*e*) 23,982. (*f*) 6,586. (*g*) 1,105. (*h*) 14,985.

RAPID ADDITION.

33. There is nothing more useful to the bookkeeper and business man than the ability to add rapidly and correctly; but this can be acquired only by persevering practice. Any time employed in practicing addition will be well spent. If the student will practice addition ten or fifteen minutes daily for a month or so, he will be greatly benefited.

In order to become expert in adding, it is absolutely essential that, when two figures are seen or heard pronounced, the student can instantly give their sum. Thus, 15 should suggest itself as soon as 6 and 9, or 8 and 7, are seen, or heard pronounced. It should not be necessary to say, mentally, 6 and 9 are 15, but 6, 15, the 9 not being pronounced either mentally or orally. (In no case should the student contract the habit of adding aloud; it is not necessary, and it is a very difficult habit to break.)

In adding 5, 6, 1, 9, 7, 5, 2, 4, 8, 9, do not say 5 and 6 is 11 and 1 is 12, and 9 is 21, etc., but *think* 5, 11, 12, 21, 28, 33, 35, 39, 47, 56, repeating the sums about as fast as they can be pronounced.

While rapidity is of great importance, accuracy is more important, and accuracy is attained only by practice. Below will be found some examples on which the student can practice. He should construct similar ones for himself.

(1)	(2)	(3)	(4)	(5)
4175	9286	3582	48275	2681753
5698	5735	6896	27368	5176382
8527	8267	5275	59487	9628625
3489	3489	9182	61735	8763897
7678	9261	7364	98275	6374859
2537	7082	5385	63842	6283926

The answers to the above are: (1) 32,104; (2) 43,120; (3) 37,684; (4) 358,982; (5) 38,909,442.

34. To add rapidly, it is necessary for the student to become accustomed to group the figures of a column and add

the sums of the groups. It is most convenient to choose the groups, as far as possible, so that the sum of each group shall be either 10 or 20. An example will show how this is accomplished:

3
9
2
5
8
2
6
4
3
5
0
9
1
2
59

Commencing at the bottom of the column, we see at once that 1 and 9 form a group whose sum is 10; hence we mentally say 2, 12 instead of 2, 3, 12. Now the next two figures, 5 and 3, we group together, and, instead of adding 5 and 3 separately, we add the sum 8. The next two figures, 6 and 4, form a group whose sum is 10, and so do the next two figures, 8 and 2. Looking now at the four figures at the top of the column, we readily see that 5, 2, and 3 form a last group whose sum is 10, leaving only the 9 outside of a group. In adding the column, we would repeat mentally 2, 12, 20, 30, 40, 50, 59. Another grouping would readily appear to the skilful accountant; the 2 at the bottom and the 5 and 3 above the 0 form a group whose sum is 10. Recognizing this group, the mental addition would be: 10, 20, 30, 40, 50, 59.

This process of forming groups may be extended to include those whose sums are 15 or 20. By a judicious selection of the figures composing a group, its sum may usually be made either 10, 15, or 20, and these sums should always be sought in preference to others, since two 15's make 30, and the numbers 10, 20, and 30 are added with little mental labor. Thus, in the following example, the grouping is shown by the braces. The mental addition is 15, 30, 37, 47, 67, 72. After some practice, the student will be able to recognize a group whose sum is 10 or 20 almost instantly; he should persevere in the solution of examples until he is able to form the groups rapidly and with ease. It is not always possible to get consecutive numbers which will form groups whose sums are 10, 15, or 20. Such groups, however, can often be found by skipping one or more figures in the column. For instance, in the following example, the first two figures, 7 and 3, make a ten, and skipping the fourth

5
9
3 } 20
8
6
4 } 10
7
2
9 } 15
4
8
7 } 15
72

3 figure, 9, the third and fifth figures, 4 and 6, make another
4 10. Skipping the 8, the 3, 4, and 3 at the top of the
8 column make another 10, thus making three groups of
3 10 and the figures 9 and 8. In adding, however, we
0 should not leave the figures skipped to be added at the
6 end of the operation, but should add them as they occur.
9 In this example we would say 10, 20, 29, 39, 47. The
4 order of adding may be shown by the following arrange-
3 ment:

7

47

$$\overbrace{7+3}^{10}+\overbrace{6+4}^{10}+9+\overbrace{3+4+3}^{10}+8.$$

The following examples may be used for practice:

(1)	(2)	(3)	(4)	(5)
3	13	317	1127	13103
5	27	226	2632	61706
2	92	232	1946	43285
6	35	518	3217	39134
3	64	396	1184	96286
4	22	435	1936	45173
0	30	707	2003	78135
9	97	992	5114	26232
3	18	311	976	19375
1	33	417	5634	8428

The answers are: (1) 36; (2) 431; (3) 4,551; (4) 25,769; (5) 430,857.

35. It is frequently advantageous to be able to add *horizontally*, as it is termed; by this is meant the adding of several numbers as they stand in a horizontal row, without arranging the numbers in vertical columns. Thus,

$$123 + 567 + 792 + 221 + 546 = 2{,}249.$$

When adding in this manner straight addition must be employed; i. e., the method of Art. **34** cannot be used. The process would be 6, 7, 9, 16, 19; 5, 7, 16, 22, 24; 7, 9, 16, 21, 22. As an example of this method, consider the following table, which is supposed to give, by days, the grain export, in bushels, of a certain city for one week. It is required to find the

amount of grain exported each day, the total amount of each kind of grain exported during the week, and, finally, the total amount of grain exported during the week.

GRAIN EXPORT OF A CITY FOR ONE WEEK (in bushels).

	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.	Total
Corn	28325	15236	35715	29128	75183	46217	*****
Wheat . . .	35719	41719	50108	32546	59275	81126	*****
Oats	12136	9237	18265	7268	6950	17230	*****
Barley . . .	18230	15738	21375	15928	19263	13637	*****
Rye	5275	6829	7201	11325	7825	13261	*****
Totals . . .	*****	*****	*****	*****	*****	*****	*****

The student should find the totals, and prove that the results are correct by adding the totals in the right-hand column, and then adding the totals in the bottom row; the two results should be the same, viz., 757,270 bushels. The other results are: corn, 229,804; wheat, 300,493; oats, 71,086; barley, 104,171; rye, 51,716; Mon., 99,685; Tues., 88,759; Wed., 132,664; Thurs., 96,195; Fri., 168,496; Sat., 171,471.

EXAMPLES FOR PRACTICE.

36. Find the sums of the following:

(1)	(2)	(3)	(4)	(5)	(6)
4568	15431	7386	49850	6542	62165
7391	29685	45371	17370	63834	16732
7854	73648	13764	68429	76343	85696
53469	34519	9887	23156	80931	71883
13470	78234	64348	21017	79883	50149
58143	7843	14627	67154	83578	31572
			64353	35647	76844

EXPLANATION.—Having found the sums of the above, add the sums horizontally, and thus obtain the total of all the numbers. Ans.: (1) 144,895; (2) 239,360; (3) 155,383; (4) 311,329; (5) 426,758; (6) 395,041; total, 1,672,766.

Add all of the above groups horizontally, beginning with the numbers in the top row. Ans.: Top row, 145,942; second row, 180,383; third row, 325,734; fourth row, 273,845; fifth row, 307,101; sixth row, 262,917; seventh row, 176,844.

SUBTRACTION.

37. In Arithmetic, **subtraction** is the process of finding how much greater one number is than another.

The greater of the two numbers is the **minuend.**

The lesser of the two numbers is the **subtrahend.**

The number left after subtracting the subtrahend from the minuend is the **difference,** or **remainder.**

38. The sign of subtraction is −. It is read **minus,** and means *less.* Thus, 12 − 7 is read 12 *minus* 7, and means that 7 is to be taken from 12.

EXAMPLE.—From 7,568 take 3,425.

SOLUTION.—

minuend	7568	
subtrahend	3425	
remainder	4143	Ans.

EXPLANATION.—Begin at the units column and subtract in succession each figure in the subtrahend from the one directly above it in the minuend, and write the remainders below the line. The result is the remainder.

39. When there are more figures in the minuend than in the subtrahend, and when some figures in the minuend are less than the figures directly under them in the subtrahend, proceed as in the following example:

First Method.—EXAMPLE 1.—From 8,453 take 844.

SOLUTION.—

minuend	8453	
subtrahend	844	
remainder	7609	Ans.

EXPLANATION.—Begin at the units column to subtract. We cannot take 4 from 3, and must, therefore, borrow 1 *ten* from 5 in *tens* column, and add, or prefix, it to the 3 units, making 13 units. Then, 4 from 13 equals 9, which is written as the units figure of the remainder.

Since 1 *ten* was borrowed from 5 *tens*, only 4 *tens* remain; 4 from 4 equals 0, the *tens* figure of the remainder.

We cannot take 8 *hundreds* from 4 *hundreds*, and must borrow 1 *thousand* from 8 in the *thousands* column. This equals 10 *hundreds;* and 10 *hundreds* plus 4 *hundreds* equals 14 hundreds; 8 from 14 is 6, the *hundreds* figure of the remainder.

Since 1 *thousand* was borrowed from 8 *thousands*, only 7 thousands remain. There being nothing to subtract from this, the *thousands* figure of the remainder is 7.

Second Method.—EXAMPLE 2.—From 84,532 take 8,447.

SOLUTION.—

minuend	8 4 5 3 2
subtrahend	8 4 4 7
remainder	7 6 0 8 5 Ans.

EXPLANATION.—As in the preceding example, 7 is taken from 12, leaving 5. Now, instead of subtracting 4 from 12, as explained above, 1 is added to 4, and the sum is subtracted from 13, leaving 8. That is, when 1 is borrowed from the minuend it is added to that figure of the subtrahend under the figure of the minuend from which it was borrowed. Continuing, 5 from 5 is 0, 8 from 14 is 6, and 1 from 8 is 7.

The method here described should be thoroughly learned, since it is easier than the first to apply in practice, and has other advantages that the student will doubtless notice.

40. It often happens that there are ciphers in the minuend, from which, of course, nothing can be borrowed. The following example will explain the difficulty:

EXAMPLE.—From 20,000 take 8,763.

SOLUTION.—

minuend	2 0 0 0 0
subtrahend	8 7 6 3
remainder	1 1 2 3 7 Ans.

EXPLANATION.—Although 1 cannot be borrowed from tens column, it is customary to regard the ciphers as tens, and increase each figure in the subtrahend by 1, explained in Art. **39.** Then, in performing the above subtraction, the process would be as follows: 3 from 10 is 7; 7 from 10 is 3; 8 from 10 is 2; 9 from 10 is 1; and 1 from 2 is 1.

41. Rule.—*Write the subtrahend under the minuend in the same manner as for addition, with the units of the same order standing in the same column.*

Commencing at the right, subtract each figure in the lower number from the one above it, and write the difference in the line below.

If any figure in the lower number is greater than the one above it, add 10 to the upper figure, perform the subtraction, and then add 1 to the next figure on the left, in the lower number. So proceed with the remaining figures.

42. Proof.—*Add the remainder to the subtrahend. The sum should equal the minuend. If not, the work is wrong.*

Proof of the last example:

subtrahend	8 7 6 3
remainder	1 1 2 3 7
minuend	2 0 0 0 0

43. There is still another method of subtraction better than either of the two previously described. It may be called the addition method. It is as follows:

Taking example 1 of Art. **39,**

minuend	8 4 5 3
subtrahend	8 4 4
remainder	7 6 0 9

Instead of subtracting 4 from 13, we add to 4 a number that will make 13; this number is 9, since 4 and 9 is 13. Write the 9 and carry the 1, as in addition; then $4 + 1 = 5$. Since

$5 + 0 = 5$, for the next figure in the subtrahend we write 0, as shown. There is nothing to carry; hence, since 8 is greater than 4, we consider 4 to be 14, and find what number added to 8 will make 14; this is 6, which write below the line, as shown. We have 1 to carry, but, as we have no figure in the subtrahend to add it to, we say 1 and 7 is 8, and write the 7 below the line, as shown.

Again, consider the following example:

minuend	10000
subtrahend	8763
remainder	1237

Here we say 3 and 7 is 10, and write the 7; then, 7 and 3 is 10 (carrying the 1 and adding it to the 6), and write the 3; next, 8 and 2 is 10, and write the 2; finally, 9 and 1 is 10, and write the 1. The student may use whichever method he prefers, but we strongly recommend the addition method as being less productive of errors.

EXAMPLES FOR PRACTICE.

44. From:

(*a*) 94,278 take 62,574.
(*b*) 53,714 take 25,824.
(*c*) 71,832 take 58,109.
(*d*) 20,804 take 10,408.
(*e*) 310,465 take 102,141.
(*f*) (81,043 + 1,041) take 14,831.
(*g*) (20,482 + 18,216) take 21,214.
(*h*) (2,040 + 1,213 + 542) take 3,791.

Ans. (*a*) 31,704. (*b*) 27,890. (*c*) 13,723. (*d*) 10,396. (*e*) 208,324. (*f*) 67,253. (*g*) 17,484. (*h*) 4.

45. In many cases it is inconvenient to write the subtrahend *under* the minuend. For example, it might be required to find the sum of a series of numbers, and to subtract this sum from some other number. In such cases, both time and space may be saved by writing the minuend under the subtrahend, drawing the line, and then subtracting downwards.

The rule given in Art. **41** may be applied to this case by changing the words *under* to read *above; above* to read *below; lower* to read *upper;* and *upper* to read *lower.*

EXAMPLE.—Writing the subtrahend above the minuend, subtract 5,267,148 from 10,342,927.

SOLUTION.—

subtrahend	5 2 6 7 1 4 8
minuend	1 0 3 4 2 9 2 7
remainder	5 0 7 5 7 7 9 Ans.

EXAMPLES FOR PRACTICE.

46. In the following examples, subtract the upper number from the lower:

(1)	(2)	(3)	(4)
2 3 4 5 6 7 8 9	3 7 5 2 9 5 1 0	9 8 9 0 9 7 8	1 9 8 2 7
8 0 7 0 6 0 4 0	4 3 1 8 4 2 9 6	1 0 0 0 1 0 0 0 1	8 4 3 6 2

(5)	(6)	(7)	(8)
7 0 9 0 5 0 8	9 8 9 7 9 6 0	8 1 9 0 7	3 1 6 1 7 4 6 7 8 9
8 0 9 0 4 0 3	1 2 3 4 5 6 7 8 9	9 4 3 7 1	4 2 1 3 1 5 0 0 0 0

ANS.—(1) 57,249,251; (2) 5,654,786; (3) 90,119,023; (4) 64,535; (5) 999,895; (6) 113,558,829; (7) 12,464; (8) 1,051,403,211.

GENERAL REMARKS ON SUBTRACTION.

47. Subtraction is a much simpler and easier operation than addition; but the student should, nevertheless, study the subject thoroughly. Its very simplicity is deceiving, and it is probable that more mistakes are made in subtraction than in addition. The student should practice subtraction until he feels that he has thoroughly mastered the subject. He should not try to subtract rapidly at first, but endeavor to make as few mistakes as possible. When he has attained such proficiency that he can solve, say, ten examples similar to those given in

Art. **46,** a part of them having the subtrahend above the minuend, without making a mistake, he may then strive for rapidity. A rapid calculator will subtract as fast as he can write the figures of the result.

It will be a great help to the student if he will reverse the addition table. At odd moments, when walking or working, let him say to himself, 6 from 11 is how much? 9 from 17 is what? etc., and in a short time the right answer will present itself without any mental effort whatever.

48. Addition and subtraction form an extremely important part of a bookkeeper's work. The books must *balance*, as it is termed; i. e., the sum of the columns on the debit side must *exactly* equal the sum of the columns on the credit side. If an error of even one cent is made it will manifest itself in the trial balance, and a day or more may be spent in finding the error. Hence, the importance of accuracy. It is also important that no time be wasted in adding the columns, and in subtracting the sums of the two columns to find the balance. To illustrate, we give part of a page of a ledger. The double vertical lines separate the debit side on the left from the credit side on the right.

Dr.			Price & Howard					Cr.	
1888					1888				
June	4	Merchandise,	$1893	42	June	14	Merchandise,	$1538	38
"	20	"	149	37	"	30	Cash,	500	
July	3	Sundries,	115	26	July	17	Bills Receivable,	900	
Aug.	16	Merchandise,	1326	97	Aug.	1	Cash,	375	
"	30	Sundries,	490	63	"	24	Merchandise,	984	88
Sept.	19	Merchandise,	1085	75	Oct.	1	Balance,	763	14
			$5061	40				$5061	40
Oct.	1	Balance,	$763	14					

Now, what the bookkeeper has to do is to add the two columns and subtract the less sum from the greater; then he must write the difference in the column that contains the less sum, and also write it under the total of the column containing the greater sum. This difference is indicated by the word *balance*. In order not to mar the appearance of the ledger, no figures except those written above should be used.

If the student were obliged to write the two sums on a piece of waste paper in order to subtract them, considerable time would be lost, and confusion would result. There is a much shorter and easier method, when it can be readily seen which column contains the greater sum. It is as follows:

In the above account it is readily seen that the debit, or left-hand, column is the greater. Hence, add this column, and write the result, $5,061.40, underneath (the period separates the dollars from the cents). Now add the extreme right-hand column—8, 16—and subtract 16 from the first figure (right-hand figure) of the total in the debit column. This figure is a cipher; hence, we prefix 2 to the cipher, making 20; whence, $20 - 16 = 4$, which write in the credit column, as shown. Now add the second column on the credit side, carrying the 2—thus, 2, 10, 13—obtaining 13 as the sum. Now subtract 13 from 4, the second figure of the total on the debit side. But 13 cannot be taken from 4; hence, we prefix 1 to the 4, and 13 from 14 leaves 1, which write in the second column on the credit side, as shown. Then add the third column of the credit side, first adding the 1 that was prefixed—thus, 1, 5, 10, 18—obtaining 18 as the sum; subtract 18 from 21 (prefixing 2) and get 3, which write in the third column on the credit side. The sum of the fourth column, with the 2 that was prefixed, is 20. Subtracting 20 from 6, prefixing 2 to the 6, getting 26, leaves 6, which write in the fourth column on the credit side. The sum of the fifth column, with the 2 that was prefixed, is 33. Subtracting 33 from 40, formed by prefixing 4 to 0, leaves 7, which write in the fifth column on the credit side. Carrying 4 to the sixth column and adding it to the 1 gives 5, and 5 from 5 equals 0. Hence, the balance is $763.14. If the work has been done correctly, the debit and credit sides, when added, should give the same total. Adding the credit side, the total is $5,061.40, the same as the debit side; hence, the work is correct.

49. The student will find the addition method of subtraction the best to use in this case. Thus, adding the first

column on the right, the sum is 16. The right figure of 5,061.40 is a cipher; hence, we write 2 before the cipher, getting 20, and say 16 and 4 is 20, and write the 4 on the credit side, as shown. Now, carrying the 2, the sum of the second column is 13, and 13 and 1 is 14, writing the 1 as before. Carrying the 1, the sum of the numbers in the third column is 18, and 18 and 3 is 21; write the 3 and carry the 2. The sum of the third column plus the 2 carried is 20, and 20 and 6 is 26; write the 6 and carry the 2. The sum of the fourth column plus the 2 carried is 33, and 33 and 7 is 40; write the 7 and carry the 4. The sum of the fifth column plus the 4 carried is 5, and 5 and 0 is 5.

The student should apply both methods in solving the following examples.

EXAMPLES FOR PRACTICE.

50. Let the student practice this method on the following examples, one of which is worked out. If it is not apparent which set of numbers is the greater, he should add up the two left-hand columns and be guided by the sums so obtained.

(1)

12394		33604	
47826		9775	
52482		11628	
26103		84721	
40079		5109	
178884	*Balance*	34047	
Balance 34047		178884	

(2)

81720	81220	
22222	21413	
42730	37500	
81075	41200	
41630	71700	
22500		
71946		

(3)

23725	11227
90000	21836
80000	71749
71824	64800
21875	11875
	53898
	20313

(4)

107500	37560
231842	21824
81210	71737
93840	24445
431200	94633
	22248
	10875

By the same method perform the following:

	(5)	(6)	(7)
minuend	$301.23	$4214.60	9234621
subtrahend	24.99	875.95	897742
	63.75	469.98	3456987
	9.87	93.49	747678
	48.84	721.74	498765
	59.28	1803.42	1768496
remainder			

Ans. (1) 34,047; (2) 110,790; (3) 31,726; (4) 662,270; (5) $94.50; (6) $250.02; (7) 1,864,953.

MULTIPLICATION.

51. To **multiply** a number is to *add* the number to itself a certain number of times.

52. **Multiplication** is the process of multiplying one number by another.

The *number* thus added to itself, or the number to be multiplied, is called the **multiplicand.**

The *number* that shows how many times the *multiplicand* is to be taken, or the *number* by which we *multiply*, is called the **multiplier.**

The result obtained by multiplying is called the **product.**

53. The sign of multiplication is $\times$. It is read *times*, or *multiplied by*. Thus, 9×6 is read 9 *times* 6, or 9 *multiplied by* 6.

54. It matters not in what order the numbers to be multiplied together are placed. Thus, 6×9 is the same as 9×6.

MULTIPLICATION TABLE.

55. In the following table, the product of any two numbers (neither of which exceeds twelve) may be found.

1 times 1 is 1	2 times 1 is 2	3 times 1 is 3
1 times 2 is 2	2 times 2 is 4	3 times 2 is 6
1 times 3 is 3	2 times 3 is 6	3 times 3 is 9
1 times 4 is 4	2 times 4 is 8	3 times 4 is 12
1 times 5 is 5	2 times 5 is 10	3 times 5 is 15
1 times 6 is 6	2 times 6 is 12	3 times 6 is 18
1 times 7 is 7	2 times 7 is 14	3 times 7 is 21
1 times 8 is 8	2 times 8 is 16	3 times 8 is 24
1 times 9 is 9	2 times 9 is 18	3 times 9 is 27
1 times 10 is 10	2 times 10 is 20	3 times 10 is 30
1 times 11 is 11	2 times 11 is 22	3 times 11 is 33
1 times 12 is 12	2 times 12 is 24	3 times 12 is 36
4 times 1 is 4	5 times 1 is 5	6 times 1 is 6
4 times 2 is 8	5 times 2 is 10	6 times 2 is 12
4 times 3 is 12	5 times 3 is 15	6 times 3 is 18
4 times 4 is 16	5 times 4 is 20	6 times 4 is 24
4 times 5 is 20	5 times 5 is 25	6 times 5 is 30
4 times 6 is 24	5 times 6 is 30	6 times 6 is 36
4 times 7 is 28	5 times 7 is 35	6 times 7 is 42
4 times 8 is 32	5 times 8 is 40	6 times 8 is 48
4 times 9 is 36	5 times 9 is 45	6 times 9 is 54
4 times 10 is 40	5 times 10 is 50	6 times 10 is 60
4 times 11 is 44	5 times 11 is 55	6 times 11 is 66
4 times 12 is 48	5 times 12 is 60	6 times 12 is 72
7 times 1 is 7	8 times 1 is 8	9 times 1 is 9
7 times 2 is 14	8 times 2 is 16	9 times 2 is 18
7 times 3 is 21	8 times 3 is 24	9 times 3 is 27
7 times 4 is 28	8 times 4 is 32	9 times 4 is 36
7 times 5 is 35	8 times 5 is 40	9 times 5 is 45
7 times 6 is 42	8 times 6 is 48	9 times 6 is 54
7 times 7 is 49	8 times 7 is 56	9 times 7 is 63
7 times 8 is 56	8 times 8 is 64	9 times 8 is 72
7 times 9 is 63	8 times 9 is 72	9 times 9 is 81
7 times 10 is 70	8 times 10 is 80	9 times 10 is 90
7 times 11 is 77	8 times 11 is 88	9 times 11 is 99
7 times 12 is 84	8 times 12 is 96	9 times 12 is 108
10 times 1 is 10	11 times 1 is 11	12 times 1 is 12
10 times 2 is 20	11 times 2 is 22	12 times 2 is 24
10 times 3 is 30	11 times 3 is 33	12 times 3 is 36
10 times 4 is 40	11 times 4 is 44	12 times 4 is 48
10 times 5 is 50	11 times 5 is 55	12 times 5 is 60
10 times 6 is 60	11 times 6 is 66	12 times 6 is 72
10 times 7 is 70	11 times 7 is 77	12 times 7 is 84
10 times 8 is 80	11 times 8 is 88	12 times 8 is 96
10 times 9 is 90	11 times 9 is 99	12 times 9 is 108
10 times 10 is 100	11 times 10 is 110	12 times 10 is 120
10 times 11 is 110	11 times 11 is 121	12 times 11 is 132
10 times 12 is 120	11 times 12 is 132	12 times 12 is 144

This table should be carefully committed to memory.
Since 0 has no value, the product of 0 and any number is **0.**

56. Practice the following until you can give them all readily without referring to the table.

3 × 7	12 × 9	9 × 2	4 × 4	6 × 7
4 × 8	7 × 8	5 × 9	6 × 3	12 × 8
5 × 12	9 × 6	7 × 4	10 × 11	10 × 4
6 × 11	10 × 10	9 × 11	8 × 11	8 × 10
7 × 10	11 × 11	10 × 7	7 × 12	4 × 7
8 × 9	12 × 12	8 × 8	9 × 10	10 × 3
9 × 5	7 × 7	4 × 5	10 × 6	12 × 2
5 × 8	8 × 5	5 × 10	11 × 9	4 × 12
3 × 10	2 × 8	6 × 12	12 × 11	3 × 2
4 × 3	2 × 12	7 × 9	2 × 5	11 × 3
3 × 11	9 × 3	6 × 10	3 × 8	11 × 2
2 × 10	3 × 12	6 × 6	6 × 2	11 × 6
5 × 4	8 × 3	12 × 5	10 × 5	12 × 10
10 × 9	3 × 6	8 × 2	11 × 12	10 × 2
11 × 8	6 × 4	9 × 12	12 × 4	4 × 11
8 × 4	5 × 11	12 × 6	7 × 5	2 × 7
5 × 3	12 × 7	11 × 5	2 × 6	3 × 4
6 × 8	9 × 9	4 × 9	3 × 5	2 × 4
7 × 2	8 × 12	5 × 5	4 × 2	12 × 3
9 × 8	3 × 3	7 × 6	5 × 2	9 × 4
8 × 7	4 × 10	9 × 7	6 × 9	10 × 12
3 × 9	5 × 7	10 × 8	7 × 11	11 × 4
4 × 6	6 × 5	11 × 10	7 × 3	2 × 3
5 × 6	11 × 7	2 × 11	2 × 9	8 × 6

57. To multiply any number by a number of one figure.

EXAMPLE.—Multiply 425 by 5.

SOLUTION.—

multiplicand	4 2 5
multiplier	5
product	2 1 2 5 Ans.

EXPLANATION.—For convenience, the multiplier is written under the right-hand figure of the multiplicand. Multiplying the first figure at the right of the multiplicand, or 5, by the multiplier 5, the result is 5 times 5 units are 25 units, 2 tens and 5 units. Write the five units in units place in the product, and reserve the 2 tens to add to the product of tens. Multiplying the second figure of the multiplicand by the multiplier 5, the result is 10 tens, which, plus the 2 tens reserved, is 12 tens, or 1 hundred plus 2 tens. Write the 2 tens in tens place and reserve the 1 hundred to add to the product of hundreds. Multiplying the third, or last, figure of the multiplicand by the

multiplier 5, the result is 20 hundreds, which, plus the 1 hundred reserved, is 21 hundreds, or 2 thousands 1 hundred, which we write in the thousands and hundreds places, respectively.

Hence, the product is 2,125.

This result is the same as the sum of five 425's. Thus,

$$\begin{array}{r} 425 \\ 425 \\ 425 \\ 425 \\ 425 \\ \hline \textit{sum}\ \ 2125 \end{array}$$

EXAMPLES FOR PRACTICE.

58. Find the product of:

(*a*) $61{,}483 \times 6$.
(*b*) $12{,}375 \times 5$.
(*c*) $10{,}426 \times 7$.
(*d*) $10{,}835 \times 3$.
(*e*) $98{,}376 \times 4$.
(*f*) $10{,}873 \times 8$.
(*g*) $71{,}543 \times 9$.
(*h*) $218{,}734 \times 2$.

Ans. (*a*) 368,898. (*b*) 61,875. (*c*) 72,982. (*d*) 32,505. (*e*) 393,504. (*f*) 86,984. (*g*) 643,887. (*h*) 437,468.

59. To multiply a number by a number of two or more figures.

EXAMPLE.—Multiply 475 by 234.

SOLUTION.—

$$\begin{array}{rr} \textit{multiplicand} & 475 \\ \textit{multiplier} & 234 \\ \hline & 1900 \\ & 1425 \\ & 950 \\ \hline \textit{product} & 111150 \end{array}$$ Ans.

EXPLANATION.—For convenience, the multiplier is generally written under the multiplicand, placing units under units, tens under tens, etc.

We cannot multiply by 234 at one operation; we must, therefore, multiply by the parts and then add the **partial products.**

The parts by which we are to multiply are 4 units, 3 tens and 2 hundreds. 4 times 475 = 1,900, the first partial product;

3 times 475 = 1,425, the second partial product, the right-hand figure of which is written directly under the figure multiplied by, or 3; 2 times 475 = 950, the third partial product, the right-hand figure of which is written directly under the figure multiplied by, or 2.

The sum of these three partial products is 111,150, which is the entire product.

60. **Rule.—I.** *Write the multiplier under the multiplicand, so that units are under units, tens under tens, etc.*

II. *Begin at the right, and multiply each figure of the multiplicand by each successive figure of the multiplier, placing the right-hand figure of each partial product directly under the figure used as a multiplier.*

III. *The sum of the partial products will be the required product.*

61. **Proof.**—*Review the work carefully, or multiply the multiplier by the multiplicand; if the results agree, the work is correct.*

62. The student will find the following test useful in determining whether the answer in multiplication is correct:

Find the sum of the digits in the multiplicand. If the sum consists of more than one figure, add the digits of the sum, and so continue until the sum is one figure. Do the same with the multiplier. Multiply together the final sums thus obtained, and if the result consists of more than one figure, add its digits until one figure is obtained. If this result is the same as is obtained by adding the digits of the product until one figure is obtained, the work is probably correct.

To illustrate, multiply 837,295 by 4,631.

SOLUTION.— *multiplicand* 837295
multiplier 4631
product 3877513145

PROOF.— 8+3+7+2+9+5 = 34; 3+4 = 7
4+6+3+1 = 14; 1+4 = 5
35; 3+5 = 8.
3+8+7+7+5+1+3+1+4+5 = 44; 4+4 = 8.

The reason for this proof will be explained later.

The proof given in this article is not absolute, because two or more errors might cause the product to fulfil the conditions of the test. But if, upon trial, the final sum of the digits of the product does not agree with that of the product of the final sums of the multiplicand and the multiplier, it is certain that the work is wrong.

63. There are many short methods of multiplication, and some of these will be given under the heading "Aliquot Parts." It is important that the student should notice the abbreviation that is possible when a cipher occurs in the multiplier, and when the multiplicand or the multiplier ends with one or more ciphers.

EXAMPLE 1.—Multiply 49,076 by 40,807.

SOLUTION.—

multiplicand		49076
multiplier		40807
		343532
partial products		392608
		196304
product		2002644332 Ans.

EXPLANATION.—The process is exactly the same as the preceding, except that when a cipher occurs in the multiplier it is not used to multiply by, the next digit of the multiplier being used instead. The first figure of the partial product is always written directly under the figure by which we multiply, as stated in the rule for multiplication, Art. **60.**

EXAMPLE 2.—Multiply 49,076 by 48,700.

SOLUTION.—

multiplicand		49076
multiplier		48700
		343532
partial products		392608
		196304
product		2390001200 Ans.

EXPLANATION.—Here the multiplier consists of three digits and two ciphers, as in the preceding examples, but

in this case the two ciphers occupy the units and tens places in the multiplier. Write the multiplier as shown, so that the ciphers lie to the right of the right-hand figure of the multiplicand, or, in other words, so that the right-hand *digit* of the multiplier lies under the right-hand digit of the multiplicand. Then, without paying attention to the ciphers on the right of the multiplier, multiply in the usual manner, annexing the two ciphers (the number of ciphers to the right of the right-hand digit of the multiplier) to the product, as shown. If the multiplicand ends in ciphers, the process is exactly the same; thus, multiplying 4,907,600 by 487:

multiplicand	4907600
multiplier	487
partial products	343532
	392608
	196304
product	2390001200

If both multiplicand and multiplier end in ciphers, place the right-hand digits under each other, as above, and add to the product as many ciphers as are contained in both multiplicand and multiplier on the right of their right-hand digits.

EXAMPLE 3.—Multiply 590,000 by 420.

SOLUTION.—

multiplicand	590000
multiplier	420
	118
	236
product	247800000 Ans.

64. It would be well to apply the principle given in Art. **62** to all cases of multiplication, until the student has attained confidence. Thus, in example 1 of Art. **63,** the number obtained by adding the digits of the multiplicand is $4 + 9 + 7 + 6 = 26$, and $2 + 6 = 8$; by adding the digits of the multiplier, $4 + 8 + 7 = 19$, $1 + 9 = 10$, and $1 + 0 = 1$. Whence, $8 \times 1 = 8$. Adding the digits of the product, $2 + 2 + 6 + 4 + 4 + 3 + 3 + 2 = 26$, or $2 + 6 = 8$; hence, the work is probably correct.

65. To be able to multiply rapidly is almost as valuable to a bookkeeper or business man as to be able to add rapidly. The only way that any one can become expert in multiplying is by practice. To give the student a good idea of how a rapid multiplier would proceed, we will now give the entire process pursued, choosing two numbers whose digits consist of 7's, 8's, and 9's only, as these are the hardest to use.

EXAMPLE.— $987789 \times 897 = ?$

SOLUTION.—

987789	sum of digits = 48 = 12, or	3
897	sum of digits = 24, or	6
6914523		18, or 9
8890101		
7902312		
886046733	sum of digits = 45, or 9.	

EXPLANATION.—Say *7 times 9 is 63;* write the 3 and carry 6. Say *7 times 8 is 56 and 6 is 62;* write the 2 and carry 6. Say *7 times 7 is 49 and 6 is 55;* write the 5 and carry 5. Say *7 times 7 is 49 and 5 is 54;* write the 4 and carry 5. Say *7 times 8 is 56 and 5 is 61;* write the 1 and carry 6. Say *7 times 9 is 63 and 6 is 69;* write the 69. We multiply by 9 and 8 in the same way, and then prove the work by the test given in Art. **62.**

66. After the student has attained considerable proficiency in multiplying as described in Art. **65** he may shorten his work considerably by merely repeating the digit by which he multiplies, the product of that digit and the desired digit in the multiplicand, and the sum of this product and the number carried. Thus, instead of saying *7 times 8 is 56 and 6 is 62*, think *7, 56, 62.* In other words, in multiplying 987,789 by 9, think *9, 81* (write the 1); *9, 72, 80* (write the 0); *9, 63, 71; 9, 63, 70; 9, 72, 79;* and, finally, *9, 81, 88.* By practicing this method for some time, he should be able to multiply nearly as fast as he can write the results.

67. Besides working the examples for practice which follow, the student should make up many others and work them out. He should continue to do this until he can mul-

tiply rapidly, and with ease and certainty. This remark applies, also, to addition and subtraction, and to division, which follows. He should study the multiplication table until he can name the product of any two numbers between 1 and 12 instantly, without any hesitation whatever. He can learn to do this only by repeating the table over and over again. The result attained will be well worth the time and labor spent.

EXAMPLES FOR PRACTICE.

68. Find the product of:

(*a*)	$3,842 \times 26$.	Ans.	(*a*) 99,892.
(*b*)	$3,716 \times 45$.		(*b*) 167,220.
(*c*)	$1,817 \times 124$.		(*c*) 225,308.
(*d*)	675×38.		(*d*) 25,650.
(*e*)	$1,875 \times 33$.		(*e*) 61,875.
(*f*)	$4,836 \times 47$.		(*f*) 227,292.
(*g*)	$5,682 \times 543$.		(*g*) 3,085,326.
(*h*)	$3,257 \times 246$.		(*h*) 801,222.
(*i*)	$2,875 \times 302$.		(*i*) 868,250.
(*j*)	$17,819 \times 1,004$.		(*j*) 17,890,276.
(*k*)	$38,674 \times 205$.		(*k*) 7,928,170.
(*l*)	$18,304 \times 100$.		(*l*) 1,830,400.
(*m*)	$7,834 \times 10$.		(*m*) 78,340.
(*n*)	$87,543 \times 1,000$.		(*n*) 87,543,000.
(*o*)	$48,763 \times 100$.		(*o*) 4,876,300.

DIVISION.

69. **Division** is the process of finding how many times one number is contained in another of the same kind, or it is the process of separating a number into a given number of equal parts. Thus, to separate 48 dollars into four equal amounts is division.

This form of division is generally called **partition.**

70. The **dividend** is the number to be divided, or to be separated into equal parts.

71. The **divisor** is the number by which the dividend is divided.

72. The **quotient** is the number showing how many times the dividend contains the divisor. In *partition* the quotient shows one of the equal parts of the dividend.

73. The **sign of division** is ÷. It is read *divided by*. Thus, 54 ÷ 9 denotes that 54 is to be divided by 9. In this case 54 is the *dividend* and 9 is the *divisor*.

74. To divide when the divisor consists of but one figure.

EXAMPLE.—What is the quotient of 875 ÷ 7?

SOLUTION.—

divisor dividend quotient

```
7 ) 8 7 5 ( 1 2 5   Ans.
    7
    ---
    1 7
    1 4
    ---
      3 5
      3 5
      ---
```

EXPLANATION.—7 is contained in 8 hundreds 1 hundred times. Place the 1 as left-hand figure of the quotient. Multiply the divisor 7 by the 1 hundred of the quotient, and place the product, 7 hundreds, under the 8 hundreds in the dividend, and subtract. On the right of the remainder, 1, bring down the next, or *tens*, figure of the dividend, in this case 7, making 17 tens; 7 is contained in 17, 2 times. Write the 2 as the second figure of the quotient. Multiply the divisor 7 by the 2 in the quotient, and subtract the product from 17. To the remainder, 3, annex the next, or *units*, figure of the dividend, in this case 5, making 35 units. 7 is contained in 35, 5 times, which is placed in the quotient. Multiplying the divisor by the last figure of the quotient, 5 times 7 = 35, which subtracted from 35, under which it is placed, leaves 0. Therefore, the quotient is 125. This method is called **long division.**

75. In **short division**, only the divisor, dividend, and quotient are written.

dividend

divisor 7) $8^{1}7^{3}5$

quotient 1 2 5 Ans.

The operation is as follows: 7 is contained in 8 once and 1 remainder; 1 placed before 7 makes 17; 7 is contained in 17 2 times and 3 over; the 3 placed before 5 makes 35; 7 is contained in 35, 5 times. These partial quotients placed in order as they are found, make the entire quotient, 125.

76. If the divisor consists of *2 or more* figures, proceed as in the following example:

EXAMPLE.—Divide 2,702,826 by 63.

divisor *dividend* *quotient*

SOLUTION.— 63) 2702826 (42902 Ans.

252

182

126

568

567

126

126

EXPLANATION.—As 63 is not contained in the first two figures, 27, we must use the first three figures, 270. Now, by trial we must find how many times 63 is contained in 270; 6 is contained in the first two figures of 270, 4 times. Place the 4 as the first figure in the quotient. Multiply the divisor, 63, by 4, and subtract the product 252 from 270. The remainder is 18, to which we annex the next figure of the dividend, 2, making 182. Now, 6 is contained in the first two figures of 182 3 times, but on multiplying 63 by 3, we see that the product 189 is too great, so we try 2 as the second figure of the quotient. Multiplying the divisor 63 by 2, and subtracting the product 126 from 182, the remainder is 56, to which we annex the next figure of the dividend, making 568; 6 is contained in 56 about 9 times. Multiply the divisor, 63, by 9, and subtract

the product 567 from 568. The remainder is 1, and bringing down the next figure of the dividend, 2, gives 12. As 12 is less than 63, we write 0 in the quotient and bring down the next figure, 6, making 126; 63 is contained in 126, 2 times, without a remainder. Therefore, 42,902 is the quotient.

77. Rule.—I. *Write the divisor at the left of the dividend with a curved line between them.*

II. *Find how many times the divisor is contained in the least number of the left-hand figures of the dividend that will contain it, and write the result at the right of the dividend, with a line between, as the first figure of the quotient.*

III. *Multiply the divisor by this quotient; write the product under the partial dividend used, and subtract, annexing to the remainder the next figure of the dividend. Divide as before, and continue thus until all the figures of the dividend have been used.*

IV. *If any partial dividend will not contain the divisor, write a cipher in the quotient, annex the next figure of the dividend and proceed as before.*

V. *If there is at last a remainder, write it after the quotient, with the divisor underneath.*

78. Proof.—*Multiply the quotient by the divisor, and add the remainder, if there be any, to the product. The result will be the dividend.*

divisor dividend quotient

Thus, 63) 4235 (67 $\frac{14}{63}$ Ans.

378

455

441

remainder 14

Proof, *quotient* 67

divisor 63

201

402

4221

remainder 14

dividend 4235

SHORT METHOD OF DIVISION.

79. The following method saves about half the figures required by the method just given, and we think that there will be fewer mistakes made when using it.

EXAMPLE.—Divide 39,913,910 by 5,494.

SOLUTION.—

dividend *divisor*

3 9 9 1 3 9 1 0) 5 4 9 4

1 4 5 5 9 7 2 6 5 *quotient.* Ans.

3 5 7 1 1

2 7 4 7 0

EXPLANATION.—The addition method of subtraction (see Art. **43**) is used in this case; the divisor is written on the right of the dividend, and the quotient underneath the divisor. The different figures of the quotient are obtained in exactly the same manner as by the preceding method. Thus, the divisor is contained in the first five figures of the dividend 7 times, and 7 is written for the first figure of the quotient. Now, instead of multiplying the divisor by 7, writing the product under the first five figures of the dividend, and then subtracting, we multiply each figure of the divisor by 7, and by the addition method, subtract from the dividend, writing only the remainder. Thus, 7 times 4 is 28, and 5 is 33; write the 5 under the 3 in the dividend, and carry 3. Then, 7 times 9 is 63 and 3 is 66, and 66 and 5 is 71; write the 5 and carry 7. 7 times 4 is 28 and 7 is 35, 35 and 4 is 39; write the 4 and carry 3. 7 times 5 is 35 and 3 is 38, and 38 and 1 is 39; write the 1. Now bring down the next figure of the dividend, 9, and annex it to the remainder. $14{,}559 \div 5{,}494 = 2$; write 2 as the second figure of the quotient. Then, as above, $2 \times 4 = 8$, and $8 + 1 = 9$; write the 1 under the 9, as shown. $2 \times 9 = 18$, and $18 + 7 = 25$; write the 7 and carry the 2. $2 \times 4 = 8$, $8 + 2 = 10$, and $10 + 5 = 15$; write the 5 and carry the 1. $2 \times 5 = 10$, $10 + 1 = 11$, and $11 + 3 = 14$; write the 3. Bringing down 1, the next figure of the dividend, $35{,}711 \div 5{,}494 = 6$, the third figure of the quotient. Proceed as above with the remaining figures.

A fast computer would work as follows: In multiplying by 6, he would repeat to himself 6, 24, and 7 is 31 (writing the 7 and carrying the 3). 6, 54, 57, and 4 is 61. 6, 24, 30, and 7 is 37. 6, 30, 33, and 2 is 35.

The object of writing the divisor on the right is to make it easier to multiply by the figures of the quotient; it also saves space, as may readily be seen. The student is strongly advised to learn this method thoroughly, and always to use it. The best way to attain facility in division, is first to practice dividing by small numbers, from 2 to 12, and using the method of short division. After he has become proficient in this, he should practice long division by the method just described. Some special methods which may be used when dividing by certain numbers will be mentioned farther on.

80. The principle given in Art. **62** may be used to test the work of division, when the principle has been slightly modified. Add the digits of the divisor, the dividend, the quotient, and the remainder, if any, as described in Art. **62**, obtaining a single figure for the sum of each. Multiply the number thus obtained for the divisor by the number obtained for the quotient, and add to the product the number obtained for the remainder, if any. If the work has been done correctly, the result must equal the number obtained for the dividend. Thus, in the last example, the sum of the digits in the divisor (reduced to a single figure) is 4, of those in the quotient, 2, and in the remainder, 0. Hence, $4 \times 2 = 8$; $8 + 0 = 8$. Adding the digits in the dividend, the result (reduced to a single figure) is also 8; hence, the work is very probably correct.

Applying this method to the example in Art. **78**, we have, for the divisor 9, for the quotient 4, for the remainder 5. Hence, $9 \times 4 + 5 = 41$, and $4 + 1 = 5$. For the dividend, $4 + 2 + 3 + 5 = 14$, and $1 + 4 = 5$, also. The student will find this principle very useful.

Addition, subtraction, multiplication, and divison are the four corner stones of arithmetic; everything else in arithmetic depends upon them.

EXAMPLES FOR PRACTICE.

81. Divide the following:

(*a*) 126,498 by 58.
(*b*) 3,207,594 by 767.
(*c*) 11,408,202 by 234.
(*d*) 2,100,315 by 581.
(*e*) 969,936 by 4,008.
(*f*) 7,481,888 by 1,021.
(*g*) 1,525,915 by 5,003.
(*h*) 1,646,301 by 381.

Ans. (*a*) 2,181. (*b*) 4,182. (*c*) 48,753. (*d*) 3,615. (*e*) 242. (*f*) 7,328. (*g*) 305. (*h*) 4,321.

CANCELATION.

82. **Cancelation** is the process of shortening operations in division by casting out equal factors from both dividend and divisor.

83. The **factors** of a number are those numbers which, when multiplied together, will equal that number. Thus, 5 and 3 are the factors of 15, since $5 \times 3 = 15$. Likewise, 8 and 7 are the factors of 56, since $8 \times 7 = 56$.

84. A **prime number** is a number that cannot be divided by any number except itself and 1; 1 is not considered a factor. Thus, 2, 3, 11, 29, etc. are prime numbers.

85. A **prime factor** of a number is any factor that is a prime number.

Any number that is not a prime is called a **composite** number, and may be produced by multiplying together its prime factors. Thus, 60 is a composite number, and is equal to the product of its prime factors, $2 \times 2 \times 3 \times 5$.

Two numbers are said to be **prime to each other** when they have no common factor, as, for example, 15 and 28; there is no number, except 1, that will divide *both* 15 and 28 without a remainder.

86. Canceling equal factors from both dividend and divisor does *not* change the quotient.

The canceling of a factor in both dividend and divisor is the *same as dividing them both by the same number*, which, by a principle of division, does not change the quotient.

Write the numbers forming the dividend above the line, and those forming the divisor below it.

EXAMPLE.—Divide $4 \times 45 \times 60$ by 9×24.

SOLUTION.—Placing the dividend over the divisor, and canceling,

$$\frac{\overset{5}{\cancel{4}} \times \overset{10}{\cancel{45}} \times \cancel{60}}{\underset{6}{\cancel{9}} \times \cancel{24}} = 50. \quad \text{Ans.}$$

EXPLANATION.—The 4 in the dividend and 24 in the divisor are both divisible by 4, since 4 divided by 4 equals 1, and 24 divided by 4 equals 6. Cancel the 4, and the 24, and write the 6 under 24. Thus,

$$\frac{\cancel{4} \times 45 \times 60}{9 \times \underset{6}{\cancel{24}}} =$$

60 in the dividend and 6 in the divisor are divisible by 6, since 60 divided by 6 equals 10, and 6 divided by 6 equals 1. Cancel the 60 and write 10 over it; also, cancel the 6. Thus,

$$\frac{\cancel{4} \times 45 \times \overset{10}{\cancel{60}}}{9 \times \underset{6}{\cancel{24}}} =$$

Again, 45 in the dividend and 9 in the divisor are each divisible by 9, since 45 divided by 9 equals 5, and 9 divided by 9 equals 1. Cancel the 45 and write the 5 over it; also, cancel the 9. Thus,

$$\frac{\overset{5}{\cancel{4}} \times \overset{10}{\cancel{45}} \times \cancel{60}}{\underset{6}{\cancel{9}} \times \cancel{24}} =$$

Since there are no two remaining numbers (one in the dividend and one in the divisor) divisible by any number greater than 1 without a remainder, it is impossible to cancel further.

Multiply together all the uncanceled numbers in the dividend and divide their product by the product of all the

uncanceled numbers in the divisor. The result will be the quotient. The product of all the uncanceled numbers in the dividend is $5 \times 10 = 50$, and there are no uncanceled numbers in the divisor.

$$\text{Hence, } \frac{\overset{1}{\cancel{4}} \times \overset{5}{\cancel{15}} \times \overset{10}{\cancel{60}}}{\underset{1}{\cancel{9}} \times \underset{6}{\cancel{24}}} = 5 \times 10 = 50.$$

87. **Rule.—I.** *Cancel the common factors from both the dividend and divisor.*

II. *Then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor, and the result will be the quotient.*

EXAMPLES FOR PRACTICE

88. Divide:

(*a*)	$14 \times 18 \times 16 \times 40$ by $7 \times 8 \times 6 \times 5 \times 3$.	Ans.	(*a*) 32.
(*b*)	$3 \times 65 \times 50 \times 100 \times 60$ by $30 \times 60 \times 13 \times 10$.		(*b*) 250.
(*c*)	$8 \times 4 \times 3 \times 9 \times 11$ by $11 \times 9 \times 4 \times 3 \times 8$.		(*c*) 1.
(*d*)	$164 \times 321 \times 6 \times 7 \times 4$ by $82 \times 321 \times 7$.		(*d*) 48.
(*e*)	$50 \times 100 \times 200 \times 72$ by $1{,}000 \times 144 \times 100$.		(*e*) 5.
(*f*)	$48 \times 63 \times 55 \times 49$ by $7 \times 21 \times 11 \times 48$.		(*f*) 105.
(*g*)	$110 \times 150 \times 84 \times 32$ by $11 \times 15 \times 100 \times 64$.		(*g*) 42.
(*h*)	$115 \times 120 \times 400 \times 1{,}000$ by $23 \times 1{,}000 \times 60 \times 800$.		(*h*) 5.

ARITHMETIC.

(PART 2.)

FRACTIONS.

1. A **fraction** is one or more of the equal parts of a unit.

2. Two numbers are required to express a fraction, one called the **numerator,** and the other, the **denominator.**

3. The numerator is placed above the denominator, with a *line* between them, as $\frac{2}{3}$. Here 3 is the *denominator*, and shows into how many equal parts the unit is divided. The *numerator* 2 shows how many of these equal parts are taken or considered. The denominator also indicates the name of the parts.

$\frac{1}{2}$ is read one-half.
$\frac{3}{4}$ is read three-fourths.
$\frac{3}{8}$ is read three-eighths.
$\frac{5}{16}$ is read five-sixteenths.
$\frac{29}{47}$ is read twenty-nine forty-sevenths.

4. In the expression "$\frac{3}{4}$ of an apple," the denominator 4 shows that the apple is divided into four *equal* parts, and the numerator 3 shows that three of these parts, or fourths, are taken or considered.

If each of the parts, or fourths, of the apple were cut into two equal pieces, there would then be twice as many pieces as before, or $4 \times 2 = 8$ pieces in all; one of these pieces would be called one-eighth, and would be expressed in figures as $\frac{1}{8}$. Three of these pieces would be called three-eighths, written $\frac{3}{8}$. The words three-fourths, three-eighths, five-sixteenths, etc. are abbreviations of three one-fourths, three

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one-eighths, five one-sixteenths, etc. It is evident that, the greater the denominator, the greater is the number of parts into which the unit is divided; consequently, the parts themselves are smaller, and the value of the fraction is less for the same number of parts taken. In other words, $\frac{7}{9}$, for example, is less than $\frac{7}{8}$, because, if a unit is divided into 9 parts, the parts are less than if the same unit had been divided into 8 parts; and, since $\frac{1}{9}$ is less than $\frac{1}{8}$, it is clear that 7 one-ninths is less than 7 one-eighths. Hence, also, $\frac{3}{8}$ is less than $\frac{3}{4}$.

5. The **value** of a fraction is the *numerator* divided by the *denominator;* as, $\frac{4}{2} = 2$, $\frac{6}{2} = 3$.

6. The line between the *numerator* and the *denominator* means *divided by*, or $\div$.

$\frac{3}{4}$ is equivalent to $3 \div 4$.
$\frac{5}{8}$ is equivalent to $5 \div 8$.

7. The *numerator* and *denominator* of a fraction are called the **terms** of a fraction.

8. The *value* of a fraction when its terms are equal is **1.**

$\frac{4}{4}$, or four-fourths $= 1$.
$\frac{8}{8}$, or eight-eighths $= 1$.
$\frac{64}{64}$, or sixty-four sixty-fourths $= 1$.

9. A **proper fraction** is a fraction whose numerator is *less* than its denominator. Its value is *less* than 1, as $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{16}$.

10. An **improper fraction** is a fraction whose numerator *equals or is greater than the denominator.* Its value is 1 or *more* than 1, as $\frac{4}{4}$, $\frac{9}{8}$, $\frac{42}{32}$.

11. A **mixed number** is a *whole number and a fraction united.* $4\frac{2}{3}$ is a mixed number, and is equivalent to $4 + \frac{2}{3}$. It is read *four and two-thirds.*

REDUCTION OF FRACTIONS.

12. **Reduction of fractions** is the process of changing the form of fractions without changing their *value.*

13. A fraction is reduced to *higher terms* by *multiplying both terms of the fraction by the same number.* Thus,

$\frac{3}{4}$ is reduced to $\frac{6}{8}$ by multiplying both terms of the fraction by 2.

$$\frac{3\times 2}{4\times 2} = \frac{6}{8}.$$

The *value* is not changed. For, suppose that a unit, say an apple, is divided into 8 equal parts. If these parts be arranged in 4 piles, each containing 2 parts, it is evident that each pile will be composed of the same part of the apple as would have been the case had the apple been originally cut into 4 equal parts. Now, if one of these piles (containing 2 parts) be removed, there will be 3 piles left, each containing 2 equal parts, or 6 equal parts in all, i. e., six-eighths. But, since one pile, or one-quarter, was removed, there are three-quarters left. Hence, $\frac{3}{4} = \frac{6}{8}$. The same reasoning may be applied to any similar case. Therefore, multiplying both terms of a fraction by the same number does not alter its value.

14. To reduce a fraction to an equivalent fraction having a given denominator.

EXAMPLE.—Reduce $\frac{7}{8}$ to an equivalent fraction having 96 for a denominator.

SOLUTION.—Both the numerator and the denominator must be multiplied by the same number in order not to change the value of the fraction. The denominator must be multiplied by some number which will make the product 96; this number is evidently $96 \div 8 = 12$, since $8\times 12 = 96$. Hence, $\frac{7\times 12}{8\times 12} = \frac{84}{96}$. Ans.

15. Rule.—*Divide the given denominator by the denominator of the given fraction, and multiply both terms of the fraction by the quotient.*

EXAMPLE.—Reduce $\frac{3}{4}$ to 100ths.

SOLUTION.— $100 \div 4 = 25$; hence, $\frac{3\times 25}{4\times 25} = \frac{75}{100}$. Ans.

16. *A fraction is reduced to lower terms by dividing both terms by the same number.* Thus, $\frac{8}{10}$ is reduced to $\frac{4}{5}$ by dividing both terms by 2.

$$\frac{8\div 2}{10\div 2} = \frac{4}{5}.$$

That $\frac{8}{10} = \frac{4}{5}$ is readily seen from the explanation given in

Art. **13;** for, multiplying both terms of the fraction $\frac{4}{5}$ by 2, $\frac{4\times2}{5\times2} = \frac{8}{10}$, and, if $\frac{4}{5} = \frac{8}{10}$, $\frac{8}{10}$ must equal $\frac{4}{5}$. Hence, dividing both terms of a fraction by the same number does not alter its value.

17. *A fraction is reduced to lowest terms, or simplest form, when its numerator and denominator cannot both be divided by the same number without a remainder.* As, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{11}{24}$, $\frac{8}{15}$.

EXAMPLES FOR PRACTICE.

18. Reduce the following:

(*a*) $\frac{7}{16}$ to 128ths.
(*b*) $\frac{24}{182}$ to its lowest terms.
(*c*) $\frac{64}{1000}$ to its lowest terms.
(*d*) $\frac{5}{7}$ to 49ths.
(*e*) $\frac{13}{16}$ to 10,000ths.

Ans. (*a*) $\frac{56}{128}$. (*b*) $\frac{2}{11}$. (*c*) $\frac{8}{125}$. (*d*) $\frac{35}{49}$. (*e*) $\frac{8125}{10000}$.

---

19. To reduce a whole number or a mixed number to an improper fraction.

EXAMPLE 1.—How many *fourths* in 5?

SOLUTION.—Since there are 4 *fourths* in 1, in 5 there will be 5×4 fourths, or 20 fourths; i.e., $5\times\frac{4}{4} = \frac{20}{4}$. Ans.

EXAMPLE 2.—Reduce $8\frac{3}{4}$ to an improper fraction.

SOLUTION.— $8\times\frac{4}{4} = \frac{32}{4}$. $\frac{32}{4}+\frac{3}{4} = \frac{35}{4}$. Ans.

20. Rule.—*Multiply the whole number by the denominator of the fraction, add the numerator to the product and place the denominator under the result. If it is desired to reduce a whole number to a fraction, multiply the whole number by the given denominator and the result will be the numerator of the required fraction.*

EXAMPLES FOR PRACTICE.

21. Reduce to improper fractions:

(*a*) $4\frac{1}{8}$.
(*b*) $5\frac{1}{9}$.
(*c*) $10\frac{2}{10}$.
(*d*) $37\frac{3}{4}$.
(*e*) $50\frac{4}{5}$.
(*f*) Reduce 7 to a fraction whose denominator is 16.

Ans. (*a*) $\frac{33}{8}$. (*b*) $\frac{46}{9}$. (*c*) $\frac{102}{10}$. (*d*) $\frac{151}{4}$. (*e*) $\frac{254}{5}$. (*f*) $\frac{112}{16}$.

22. To reduce an improper fraction to a whole or a mixed number.

EXAMPLE.—Reduce $\frac{21}{4}$ to a mixed number.

SOLUTION.— 4 is contained in 21, 5 times and 1 remaining (see Art. 5); as this remainder is also divided by 4, its value is $\frac{1}{4}$. Therefore, $5+\frac{1}{4}$, or $5\frac{1}{4}$, is the number.

23. Rule.—*Divide the numerator by the denominator, and write the result as in ordinary division.* (See part V of Rule, Art. **77**, § 1.)

EXAMPLES FOR PRACTICE.

24. Reduce to whole or mixed numbers:

(*a*) $\frac{145}{6}$.
(*b*) $\frac{185}{3}$.
(*c*) $\frac{701}{8}$.
(*d*) $\frac{149}{3}$.
(*e*) $\frac{76}{19}$.
(*f*) $\frac{125}{25}$.

Ans. (*a*) $24\frac{1}{6}$. (*b*) $61\frac{2}{3}$. (*c*) $116\frac{5}{8}$. (*d*) $49\frac{2}{3}$. (*e*) 4. (*f*) 5.

---

25. A **common denominator** of *two or more fractions* is a number that will contain (i. e., which may be divided by) all of the *denominators* of the fractions without a remainder. The **least common denominator** is the least number that will contain all the denominators of the fractions without a remainder.

26. To find the least common denominator.

EXAMPLE.—Find the least common denominator of $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{9}$, and $\frac{1}{16}$.

SOLUTION.—We first place the denominators in a row, separated by commas.

$$\begin{array}{rl} 2\,) & 4,\ 3,\ 9,\ 16 \\ \hline 2\,) & 2,\ 3,\ 9,\ 8 \\ \hline 3\,) & \quad 3,\ 9,\ 4 \\ \hline & \quad\quad 3,\ 4 \end{array}$$

$2\times2\times3\times3\times4 = 144$, the least common denominator. Ans.

EXPLANATION.—Divide each of the denominators by some prime number that will divide at least two of them without a remainder if (possible), bringing down to the row below those

denominators which will not contain the divisor without a remainder. Dividing each of the numbers by 2, the second row becomes 2, 3, 9, 8, since 2 will not divide 3 and 9 without a remainder. Dividing again by 2, the result is 3, 9, 4. Dividing the third row by 3, the result is 3, 4. The numbers in the fourth row are now prime to each other (see Art. **85,** § 1), and the product of these numbers multiplied by the divisors will be the least common denominator. Thus, $2\times2\times3\times3\times4 = 144$, the least common denominator.

27. Example.—Find the least common denominator of $\frac{4}{9}$, $\frac{5}{12}$, and $\frac{7}{18}$.

Solution.—

$$\begin{array}{r|l} 3 & 9,\ 12,\ 18 \\ \hline 3 & 3,\ 4,\ 6 \\ \hline 2 & 4,\ 2 \\ \hline & 2 \end{array}$$

$3\times3\times2\times2 = 36$. Ans.

28. If one (or more) of the denominators is a factor of some other denominator, it need not be considered in the process of finding the least common denominator, for if the least common denominator will contain the larger denominator it will also contain any factor of it. Thus, in the last example, since 9 is a factor of 18, it need not be considered, and all that is necessary is to find the least common denominator of 12 and 18. Also, if *all* of the denominators have a common factor, whether prime or composite, that factor may be used as a divisor. For example, since 12 and 18 have the common factor 6, 6 may be used as a divisor instead of its prime factors 2 and 3. Hence, the entire operation of finding the least common denominator of $\frac{4}{9}$, $\frac{5}{12}$, $\frac{7}{18}$ reduces to $\begin{array}{r|l} 6 & 12,\ 18, \\ \hline & 2,\ 3, \end{array}$ or least common denominator $= 6\times2\times3 = 36$, the same result as before.

29. Any number that will exactly contain another number is called a **multiple** of that number. Thus, 48 is a multiple of 6; also, of 8, of 12, etc. Any number that will exactly contain two or more numbers is called a **common multiple** of those numbers; and the least number that will

exactly contain two or more numbers is called the **least common multiple** of those numbers. Hence, the least common denominator of two or more fractions is the least common multiple of the denominators of the fractions.

30. The least common multiple of several numbers may often be determined by inspection. For example, if it is desired to find the least common multiple of 3, 5, and 10, simple inspection will show that the least common multiple is 30, the mental process being as follows: Since 5 is a factor of 10, 5 need not be considered; and since 3 and 10 are prime to each other, their least common multiple is their product, i. e., 3×10, or 30. Therefore, 30 is the least common multiple of 3, 5, and 10.

Again, consider the example of Art. **27.** Here, it is required to find the least common multiple of 9, 12, and 18. Since 9 is a factor of 18, it need not be considered. The least common multiple of 12 and 18 must, of course, contain 18 a certain number of times. To ascertain how many times 18 the least common multiple must be, divide 12 (the other number) by the greatest factor common to both 12 and 18; the quotient multiplied by 18 will be the required least common multiple. In the present case, the greatest factor is 6, and $12 \div 6 = 2$; $2 \times 18 = 36$, the least common multiple.

31. To reduce two or more fractions to equivalent fractions having the least common denominator.

EXAMPLE.—Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{8}$ to equivalent fractions having the least common denominator.

SOLUTION.—The least common denominator is the least number that can be exactly divided by 3, 4, and 8. This number is readily seen by inspection to be 24. Now, reducing each fraction to a fraction having a denominator of 24 (Art. **14**), we obtain

$$\frac{2\times 8}{3\times 8} = \frac{16}{24}, \quad \frac{3\times 6}{4\times 6} = \frac{18}{24}, \quad \frac{7\times 3}{8\times 3} = \frac{21}{24}. \quad \text{Ans.}$$

32. Rule.—*Divide the least common denominator by the denominator of the given fraction, and multiply both terms of the fraction by the quotient.*

EXAMPLES FOR PRACTICE.

33. Reduce to fractions having a common denominator:

(*a*) $\frac{3}{4}, \frac{5}{8}, \frac{7}{8}$.
(*b*) $\frac{3}{16}, \frac{3}{4}, \frac{7}{32}$.
(*c*) $\frac{7}{8}, \frac{7}{88}, \frac{10}{11}$.
(*d*) $\frac{3}{5}, \frac{5}{8}, \frac{11}{40}$.
(*e*) $\frac{4}{10}, \frac{6}{40}, \frac{9}{20}$.
(*f*) $\frac{7}{15}, \frac{17}{30}, \frac{21}{80}$.

Ans.
(*a*) $\frac{6}{8}, \frac{5}{8}, \frac{7}{8}$.
(*b*) $\frac{6}{32}, \frac{24}{32}, \frac{7}{32}$.
(*c*) $\frac{77}{88}, \frac{7}{88}, \frac{80}{88}$.
(*d*) $\frac{24}{40}, \frac{25}{40}, \frac{11}{40}$.
(*e*) $\frac{16}{40}, \frac{6}{40}, \frac{18}{40}$.
(*f*) $\frac{14}{30}, \frac{17}{30}, \frac{21}{80}$.

ADDITION OF FRACTIONS.

34. *Fractions cannot be added unless they have a common denominator.* We cannot add $\frac{3}{4}$ to $\frac{7}{8}$ as they now stand, since the denominators represent different parts of a unit. Fourths can be added to fourths, but not to eighths.

Suppose we divide an apple into 4 equal parts, and then divide 2 of these parts into 2 equal parts. It is evident that we shall have 2 one-fourths and 4 one-eighths. Now if we add these parts the result is $2+4=6$ something. But what is this something? It is not fourths, for six fourths are $1\frac{1}{2}$, and we had only one apple to begin with; neither is it eighths, for six eighths are $\frac{3}{4}$, which is less than 1 apple. By reducing the fourths to eighths, we have $\frac{2}{4}=\frac{4}{8}$; and, adding the other 4 eighths, $4+4=8$ eighths. The result is correct, since $\frac{8}{8}=1$. Or we can, in this case, reduce the eighths to fourths. Thus, $\frac{4}{8}=\frac{2}{4}$; whence, adding $2+2=4$ quarters or fourths, a correct result, since $\frac{4}{4}=1$.

Before adding, fractions should be reduced to a common denominator, preferably the *least* common denominator.

35. EXAMPLE.—Find the sum of $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$.

SOLUTION.—The *least common denominator*, or the *least number* that will exactly contain all the *denominators*, is 8.

$$\frac{1}{2}=\frac{4}{8}, \frac{3}{4}=\frac{6}{8}, \text{ and } \frac{4}{8}+\frac{6}{8}+\frac{5}{8}=\frac{4+6+5}{8}=\frac{15}{8}=1\frac{7}{8}. \quad \text{Ans.}$$

EXPLANATION.—As the *denominator* indicates the names of the *parts*, only the *numerators* are added to obtain the total number of *parts* indicated by the *denominator*. Thus, 4 one-eighths plus 6 one-eighths plus 5 one-eighths = 15 one-eighths.

36. EXAMPLE 1.—What is the sum of $12\frac{3}{4}$, $14\frac{5}{8}$, and $7\frac{5}{16}$?

SOLUTION.—The least common denominator in this case is 16.

$$\begin{array}{rl} 12\frac{3}{4} = & 12\frac{12}{16} \\ 14\frac{5}{8} = & 14\frac{10}{16} \\ 7\frac{5}{16} = & 7\frac{5}{16} \\ \hline \textit{sum} & 33+\frac{27}{16} = 33+1\frac{11}{16} = 34\frac{11}{16}. \end{array}$$ Ans.

The sum of the fractions $= \frac{27}{16}$, or $1\frac{11}{16}$, which added to the sum of the whole numbers $= 34\frac{11}{16}$.

EXAMPLE 2.—What is the sum of 17, $13\frac{3}{16}$, $\frac{9}{32}$, and $3\frac{1}{4}$?

SOLUTION.—The least common denominator is 32. $13\frac{3}{16} = 13\frac{6}{32}$, $3\frac{1}{4} = 3\frac{8}{32}$.

$$\begin{array}{rl} & 17 \\ & 13\frac{6}{32} \\ & \frac{9}{32} \\ & 3\frac{8}{32} \\ \hline \textit{sum} & 33\frac{23}{32} \end{array}$$ Ans.

37. **Rule.—I.** *Make the fractions similar; write the sum of the numerators over the least common denominator.*

II. *When there are integers or mixed numbers, add them separately and then add the results.*

EXAMPLES FOR PRACTICE.

38. Find the sum of:

(*a*) $\frac{4}{6}$, $\frac{7}{24}$, $\frac{5}{8}$.
(*b*) $\frac{2}{3}$, $\frac{5}{15}$, $\frac{24}{45}$.
(*c*) $\frac{1}{2}$, $\frac{3}{8}$, $\frac{5}{16}$.
(*d*) $\frac{5}{6}$, $\frac{11}{72}$, $\frac{13}{18}$.
(*e*) $\frac{10}{11}$, $\frac{6}{33}$, $\frac{23}{66}$.
(*f*) $\frac{23}{45}$, $\frac{11}{15}$, $\frac{14}{45}$.
(*g*) $\frac{4}{11}$, $\frac{7}{22}$, $\frac{14}{22}$.
(*h*) $\frac{3}{7}$, $\frac{14}{49}$, $\frac{2}{7}$.

Ans. (*a*) $1\frac{7}{12}$. (*b*) $1\frac{8}{15}$. (*c*) $1\frac{3}{16}$. (*d*) $1\frac{17}{24}$. (*e*) $1\frac{23}{66}$. (*f*) $1\frac{5}{9}$. (*g*) $1\frac{7}{22}$. (*h*) 1.

SUBTRACTION OF FRACTIONS.

39. *Fractions cannot be subtracted without first reducing them to a common denominator.* This can be shown in the same manner as in the case of addition of fractions. (Art. **34.**)

EXAMPLE.—Subtract $\frac{3}{8}$ from $\frac{13}{16}$.

SOLUTION.—The least common denominator is 16.

$$\frac{3}{8} = \frac{6}{16}. \quad \frac{13}{16} - \frac{6}{16} = \frac{13-6}{16} = \frac{7}{16}. \quad \text{Ans.}$$

40. EXAMPLE.—From 7 take $\frac{5}{8}$.

SOLUTION.— $1 = \frac{8}{8}$; therefore, since $7 = 6+1$, $7 = 6+\frac{8}{8} = 6\frac{8}{8}$, and $6\frac{8}{8} - \frac{5}{8} = 6\frac{3}{8}$. Ans.

41. EXAMPLE.—What is the difference between $17\frac{9}{16}$ and $9\frac{15}{32}$?

SOLUTION.—The least common denominator of the fraction is 32. $17\frac{9}{16} = 17\frac{18}{32}$.

minuend	$17\frac{18}{32}$
subtrahend	$9\frac{15}{32}$
difference	$8\frac{3}{32}$ Ans.

42. EXAMPLE.—From $9\frac{1}{4}$ take $4\frac{7}{16}$.

SOLUTION.—The least common denominator of the fractions is 16. $9\frac{1}{4} = 9\frac{4}{16}$.

minuend	$9\frac{4}{16}$ or	$8\frac{20}{16}$
subtrahend	$4\frac{7}{16}$	$4\frac{7}{16}$
remainder	$4\frac{13}{16}$	$4\frac{13}{16}$ Ans.

EXPLANATION.—As the fraction in the subtrahend is greater than the fraction in the minuend, it cannot be subtracted; therefore, borrow 1, or $\frac{16}{16}$, from the 9 in the minuend and add it to the $\frac{4}{16}$; $\frac{4}{16} + \frac{16}{16} = \frac{20}{16}$. $\frac{7}{16}$ from $\frac{20}{16} = \frac{13}{16}$. Since 1 was borrowed from 9, 8 remains; 4 from $8 = 4$; $4 + \frac{13}{16} = 4\frac{13}{16}$.

43. EXAMPLE.—From 9 take $8\frac{3}{16}$.

SOLUTION.—

minuend	9	or $8\frac{16}{16}$
subtrahend	$8\frac{3}{16}$	$8\frac{3}{16}$
difference	$\frac{13}{16}$	$\frac{13}{16}$ Ans.

EXPLANATION.—As there is no fraction in the minuend from which to take the fraction in the subtrahend, borrow 1, or $\frac{16}{16}$, from 9. $\frac{3}{16}$ from $\frac{16}{16} = \frac{13}{16}$. Since 1 was borrowed from 9, only 8 is left. 8 from $8 = 0$.

44. **Rule.—I.** *Reduce the given fractions to fractions having the least common denominator. Subtract one numerator from the other and place the remainder over the common denominator.*

II. *When there are mixed numbers, subtract the fractions and whole numbers separately.*

III. *When the fraction in the subtrahend is greater than the fraction in the minuend, borrow 1 from the whole number in the minuend, and add it to the fraction in the minuend, from which subtract the fraction in the subtrahend.*

IV. *When the minuend is a whole number, borrow 1 from it; reduce the 1 to a fraction whose denominator is the same as the denominator of the fraction in the subtrahend, and then subtract.*

EXAMPLES FOR PRACTICE.

45. Subtract:

(*a*) $\frac{10}{24}$ from $\frac{11}{12}$.
(*b*) $\frac{7}{14}$ from $\frac{17}{28}$.
(*c*) $\frac{4}{30}$ from $\frac{5}{10}$.
(*d*) $\frac{15}{35}$ from $\frac{45}{70}$.
(*e*) $\frac{15}{16}$ from $\frac{57}{48}$.
(*f*) $13\frac{1}{4}$ from $30\frac{1}{2}$.
(*g*) $12\frac{1}{8}$ from 27.
(*h*) $5\frac{1}{4}$ from 30.

Ans. (*a*) $\frac{1}{2}$. (*b*) $\frac{3}{28}$. (*c*) $\frac{11}{30}$. (*d*) $\frac{3}{14}$. (*e*) $\frac{1}{4}$. (*f*) $17\frac{1}{4}$. (*g*) $14\frac{7}{8}$. (*h*) $24\frac{3}{4}$.

MULTIPLICATION OF FRACTIONS.

46. In multiplication of fractions it is not necessary to reduce the given fractions to fractions having a common denominator.

47. *Multiplying the numerator or dividing the denominator multiplies the fraction.*

EXAMPLE.—Multiply $\frac{3}{4}$ by 4.

SOLUTION.— $\frac{3}{4} \times 4 = \frac{3 \times 4}{4} = \frac{12}{4} = 3$. Ans.

Or, $\frac{3}{4} \times 4 = \frac{3}{4 \div 4} = \frac{3}{1} = 3$. Ans.

The word "of" in multiplication of fractions means the same as ×, or times. Thus,

$\frac{3}{4}$ of $4 = \frac{3}{4} \times 4 = 3$.

$\frac{1}{8}$ of $\frac{5}{16} = \frac{1}{8} \times \frac{5}{16} = \frac{5}{128}$.

EXAMPLE.—Multiply 2 by $\frac{3}{8}$.

SOLUTION.— $2 \times \frac{3}{8} = \frac{3}{8} \times 2 = \frac{6}{8} = \frac{3}{4}$. **Ans.**

Or $2 \times \frac{3}{8} = \frac{3}{8 \div 2} = \frac{3}{4}$. Ans.

48. EXAMPLE.—What is the product of $\frac{4}{16}$ and $\frac{7}{8}$?

SOLUTION.— $\frac{4}{16} \times \frac{7}{8} = \frac{4 \times 7}{16 \times 8} = \frac{28}{128} = \frac{7}{32}$. Ans.

Or, by cancelation, $\frac{\cancel{4} \times 7}{\cancel{16} \times 8} = \frac{7}{4 \times 8} = \frac{7}{32}$. Ans.

49. EXAMPLE.—What is $\frac{4}{8}$ of $\frac{3}{4}$ of $\frac{16}{32}$?

SOLUTION.— $\frac{\cancel{4} \times 3 \times \cancel{16}}{8 \times \cancel{4} \times \cancel{32}} = \frac{3}{8 \times 2} = \frac{3}{16}$. Ans.

50. EXAMPLE.—What is the product of $9\frac{3}{4}$ and $5\frac{5}{8}$?

SOLUTION.— $9\frac{3}{4} = \frac{39}{4}$; $5\frac{5}{8} = \frac{45}{8}$.

$$\frac{39}{4} \times \frac{45}{8} = \frac{39 \times 45}{4 \times 8} = \frac{1755}{32} = 54\frac{27}{32}. \quad \text{Ans.}$$

51. EXAMPLE.—Multiply $15\frac{7}{8}$ by 3.

SOLUTION.—

$15\frac{7}{8}$	$15\frac{7}{8}$
3	3
$47\frac{5}{8}$	$45 + \frac{21}{8} = 45 + 2\frac{5}{8} = 47\frac{5}{8}$. Ans.

52. Rule.—I. *Divide the product of the numerators by the product of the denominators. All factors common to the numerators and denominators should first be cast out by cancelation.*

II. *To multiply one mixed number by another, reduce them both to improper fractions.*

III. *To multiply a mixed number by a whole number, first multiply the fractional part by the multiplier, and if the product is an improper fraction, reduce it to a mixed number, and add the whole-number part to the product of the multiplier and the whole number.*

EXAMPLES FOR PRACTICE.

53. Find the product of:

(*a*) $7 \times \frac{3}{19}$.
(*b*) $14 \times \frac{5}{18}$.
(*c*) $\frac{21}{32} \times \frac{5}{14}$.
(*d*) $\frac{16}{27} \times 4$.
(*e*) $\frac{19}{18} \times 7$.
(*f*) $17\frac{18}{21} \times 7$.
(*g*) $\frac{105}{224} \times 32$.
(*h*) $\frac{15}{28} \times 14$.

Ans. (*a*) $1\frac{2}{19}$. (*b*) $4\frac{8}{9}$. (*c*) $\frac{15}{64}$. (*d*) $2\frac{10}{27}$. (*e*) $7\frac{7}{18}$. (*f*) 125. (*g*) 15. (*h*) $7\frac{1}{2}$.

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54. Short Methods of Multiplying by a Mixed Number.—In all business transactions, the multiplication of a mixed number by an integer, an integer by a mixed number, or a mixed number by a mixed number, is of very frequent occurrence. Unless the numbers are quite small, which is not usually the case, it is very inconvenient to reduce the mixed numbers to improper fractions, multiply, and then reduce the product to a mixed number. A better way is to use one of the methods given below:

55. EXAMPLE.—Multiply $126\frac{7}{8}$ by 27.

SOLUTION.—

$$\begin{array}{r} 126\frac{7}{8} \\ 27 \\ \hline 3\frac{3}{8} \\ 18\frac{18}{8} \\ 882 \\ 252 \\ \hline 3425\frac{5}{8} \end{array}$$ Ans.

EXPLANATION.—First multiply the $\frac{7}{8}$ by 27; this may be conveniently done as follows: Multiply $\frac{1}{8}$ by 27; to do this all that is necessary is to divide 27 by 8 (this is evidently correct, since $\frac{1}{8} \times 27 = \frac{27}{8} = 27 \div 8$), obtaining $3\frac{3}{8}$. Now multiply the result just obtained ($3\frac{3}{8}$) by the numerator of the fraction less 1, or in this case, by $7 - 1 = 6$, getting $18\frac{18}{8}$ for the product, which write under $3\frac{3}{8}$, as shown. Then multiply 126 by 27 in the usual manner, placing the unit figure under the unit figures of the two mixed numbers, which may be

regarded as partial products. That this method is correct is readily seen. For $3\frac{3}{8}$ is $\frac{1}{8}$ of 27, and $18\frac{1}{8}^{8}$ is $\frac{6}{8}$ of 27; therefore, $3\frac{3}{8} + 18\frac{1}{8}^{8} = \frac{1}{8}$ of $27 + \frac{6}{8}$ of $27 = \frac{7}{8}$ of 27.

56. EXAMPLE.—Multiply 825 by $29\frac{3}{4}$.

SOLUTION.—*First Method.*

$$\begin{array}{r} 825 \\ 29\frac{3}{4} \\ \hline 4)2475 \\ 618\frac{3}{4} \\ 7425 \\ 1650 \\ \hline 24543\frac{3}{4} \end{array}$$ Ans.

Second Method.

$$\begin{array}{r} 825 \\ 29\frac{3}{4} \\ \hline 206\frac{1}{4} \\ 412\frac{2}{4} \\ 7425 \\ 1650 \\ \hline 24543\frac{3}{4} \end{array}$$ Ans.

EXPLANATION.—*First Method:* Since $\frac{3}{4}$ of 825 is the same as $\frac{1}{4}$ of 3 times 825, we first find 3 times 825 and take $\frac{1}{4}$ of the product. The remainder of the operation needs no explanation. The *second method* is similar to that used in the preceding example.

57. EXAMPLE.—Multiply $89\frac{2}{3}$ by $75\frac{3}{8}$.

SOLUTION.—

First Method.

$$\begin{array}{r} 89\frac{2}{3} \\ 75\frac{3}{8} \\ \hline 8)267 \\ 3)150 \\ \hline 33\frac{3}{8} \\ 50\frac{1}{4} \\ 445 \\ 623 \\ \hline 6758\frac{5}{8} \end{array}$$ Ans.

$$\frac{2}{3} \times \frac{3}{8} = \frac{1}{4}$$

Second Method.

$$\begin{array}{r} 89\frac{2}{3} \\ 75\frac{3}{8} \\ \hline 11\frac{1}{8} \\ 22\frac{2}{3} \\ 25 \\ 25 \\ 445\frac{1}{4} \\ 623 \\ \hline 6758\frac{5}{8} \end{array}$$ Ans.

EXPLANATION.—*First Method:* In this example there are four operations. (1) To multiply the fraction by the fraction. (2) To multiply the upper number by the lower fraction. (3) To multiply the lower number by the upper fraction. (4) To multiply the whole number by the whole number. We first multiply 89 by $\frac{3}{8}$, but in order to save space the division of 3 times 89 by 8 is merely indicated for the present. $\frac{2}{3}$ is multiplied by 75 in the same manner, multiplying

75 by 2 and indicating the division by 3. 267 (i. e., 3×89) is now divided by 8, obtaining $33\frac{3}{8}$, which is written as shown. 150 (i. e., 2×75) is divided by 3, obtaining 50, which is written under the $33\frac{3}{8}$. The two fractions are multiplied and the product, $\frac{1}{4}$, placed alongside of the 50. The two integers are now multiplied and all the separate products added together, as shown.

Second Method: This is a combination of the method of Art. **55,** and the second method of Art. **56,** and should be understood without further explanation. The product of the two fractions, $\frac{1}{4}$, is written alongside of the 445 and added to the other two fractions, $\frac{1}{8}$ and $\frac{2}{8}$, as shown.

58. EXAMPLE.—At $3\frac{5}{8}$ cents per pound, what will $66\frac{1}{2}$ pounds of sugar cost?

SOLUTION.—*First Method.*

$$\begin{array}{r} 66\tfrac{1}{2} \\ 3\tfrac{5}{8} \\ \hline 8)330 \\ 2)3 \\ \hline 41\tfrac{1}{4} \\ 1\tfrac{1}{2} \\ 198\tfrac{5}{16} \\ \hline 241\tfrac{1}{16} \end{array}$$

cents. Ans.

$$\frac{1}{2}\times\frac{5}{8}=\frac{5}{16}$$

Second Method.

$$\begin{array}{r} 66\tfrac{1}{2} \\ 3\tfrac{5}{8} \\ \hline 8\tfrac{1}{4} \\ 33 \\ 1\tfrac{1}{2} \\ 198\tfrac{5}{16} \\ \hline 241\tfrac{1}{16} \end{array}$$

EXPLANATION.—First multiply 66 by $\frac{5}{8}$, then 3 by $\frac{1}{2}$, then 66 by 3, and finally $\frac{1}{2}$ by $\frac{5}{8}$. The $\frac{5}{16}$ is written under and added to the other fractions.

EXAMPLES FOR PRACTICE.

59. Solve the following examples:

1. Find the cost of $89\frac{3}{8}$ yards of silk velvet at $\$4\frac{3}{4}$ per yard.
Ans. $\$425\frac{11}{12}$.

2. How much must be paid for $83\frac{1}{2}$ tons of hay at $\$16\frac{5}{8}$ a ton?
Ans. $\$1{,}388\frac{3}{16}$.

3. How far can a man ride on a bicycle in $14\frac{5}{8}$ hours at the rate of $9\frac{3}{4}$ miles per hour?
Ans. $144\frac{5}{8}$ miles.

4. Find the following products: (*a*) $287\times17\frac{7}{8}$. (*b*) $447\times16\frac{5}{8}$. (*c*) $127\frac{4}{5}\times69\frac{2}{7}$. (*d*) $86\frac{5}{12}\times784$. (*e*) $53\frac{3}{4}\times27\frac{1}{2}$.
Ans. (*a*) $510\frac{1}{8}$. (*b*) $7443\frac{1}{2}$. (*c*) $8{,}854\frac{5}{7}$. (*d*) $6{,}809\frac{13}{30}$. (*e*) $1{,}478\frac{1}{8}$.

DIVISION OF FRACTIONS.

60. In division of fractions it is not necessary to reduce the given fractions to fractions having a common denominator.

61. *Dividing the numerator or multiplying the denominator of a fraction, divides the fraction.*

EXAMPLE.—Divide $\frac{6}{8}$ by 3.

SOLUTION.—When *dividing* the *numerator*, we have

$$\tfrac{6}{8} \div 3 = \frac{6 \div 3}{8} = \tfrac{2}{8} = \tfrac{1}{4}. \quad \text{Ans.}$$

When *multiplying* the *denominator*, we have

$$\tfrac{6}{8} \div 3 = \frac{6}{8 \times 3} = \tfrac{6}{24} = \tfrac{1}{4}. \quad \text{Ans.}$$

EXAMPLE.—Divide $\frac{3}{16}$ by 2.

SOLUTION.— $\tfrac{3}{16} \div 2 = \dfrac{3}{16 \times 2} = \tfrac{3}{32}$. Ans.

EXAMPLE.—Divide $\frac{14}{32}$ by 7.

SOLUTION.— $\tfrac{14}{32} \div 7 = \dfrac{14 \div 7}{32} = \tfrac{2}{32} = \tfrac{1}{16}$. Ans.

62. To **invert** a fraction is to *turn it upside down*, that is, make the numerator and denominator change places.

Invert $\frac{3}{4}$ and it becomes $\frac{4}{3}$.

63. EXAMPLE.—Divide $\frac{9}{16}$ by $\frac{3}{16}$.

SOLUTION.—1. The fraction $\frac{3}{16}$ is contained in $\frac{9}{16}$, 3 times, for the denominators are the same, and one numerator is contained in the other 3 times. 2. If we now invert the divisor, $\frac{3}{16}$, and multiply, the solution is

$$\frac{9}{16} \times \frac{16}{3} = \frac{\overset{3}{\cancel{9}} \times \cancel{16}}{\cancel{16} \times \cancel{3}} = 3. \quad \text{Ans.}$$

This gives the same quotient as in the first case.

64. Example.—Divide $\frac{3}{8}$ by $\frac{1}{4}$.

Solution.—We cannot divide $\frac{3}{8}$ by $\frac{1}{4}$, as in the first case above, for the denominators are not the same; therefore, we must solve as in the second case.

$$\frac{3}{8} \div \frac{1}{4} = \frac{3}{8} \times \frac{4}{1} = \frac{3 \times \cancel{4}^{1}}{\underset{2}{\cancel{8}} \times 1} = \frac{3}{2} \text{ or } 1\tfrac{1}{2}. \quad \text{Ans.}$$

65. Example.—Divide 5 by $\frac{10}{16}$.

Solution.— $\frac{10}{16}$ inverted becomes $\frac{16}{10}$.

$$5 \times \frac{16}{10} = \frac{\cancel{5} \times \overset{8}{\cancel{16}}}{\underset{\cancel{2}}{\cancel{10}}} = 8. \quad \text{Ans.}$$

66. Example.—How many times is $3\frac{3}{4}$ contained in $7\frac{7}{16}$?

Solution.— $3\frac{3}{4} = \frac{15}{4}$; $7\frac{7}{16} = \frac{119}{16}$.

$\frac{15}{4}$ inverted equals $\frac{4}{15}$.

$$\frac{119}{16} \times \frac{4}{15} = \frac{119 \times \cancel{4}^{1}}{\underset{4}{\cancel{16}} \times 15} = \frac{119}{60} = 1\tfrac{59}{60}. \quad \text{Ans.}$$

67. **Rule.**—*Invert the divisor and proceed as in multiplication.*

68. We have learned that a line placed between two numbers indicates that the number above the line is to be divided by the number below it. Thus, $\frac{18}{3}$ denotes that 18 is to be divided by 3. This is also true if a fraction or a fractional expression be placed above or below a line.

$\frac{9}{\frac{3}{8}}$ means that 9 is to be divided by $\frac{3}{8}$; $\dfrac{\frac{3 \times 7}{8+4}}{16}$ means that 3×7 is to be divided by the value of $\dfrac{8+4}{16}$.

$\frac{\frac{1}{4}}{\frac{3}{8}}$ is the same as $\frac{1}{4} \div \frac{3}{8}$.

69. It will be noticed that there is a heavy line between the 9 and the $\frac{3}{8}$. This is necessary, since otherwise there would be nothing to show whether 9 is to be divided by $\frac{3}{8}$, or

$\frac{2}{3}$ is to be divided by 8. Whenever a heavy line is used, as shown here, it indicates that *all above the line* is to be divided by *all below it.*

EXAMPLES FOR PRACTICE.

70. Divide:

(*a*) 15 by $6\frac{3}{7}$.
(*b*) 30 by $\frac{6}{8}$.
(*c*) 172 by $\frac{4}{5}$.
(*d*) $\frac{14}{18}$ by $1\frac{7}{16}$.
(*e*) $\frac{103}{6}$ by $14\frac{3}{8}$.
(*f*) $\frac{142}{27}$ by $17\frac{1}{9}$.
(*g*) $\frac{14}{18}$ by $\frac{145}{72}$.
(*h*) $\frac{128}{18}$ by $72\frac{1}{3}$.

Ans. (*a*) $2\frac{1}{3}$. (*b*) 40. (*c*) 215. (*d*) $\frac{112}{207}$. (*e*) $1\frac{15}{88}$. (*f*) $\frac{71}{231}$. (*g*) $\frac{56}{145}$. (*h*) $\frac{64}{651}$.

ORDER OF SIGNS.

71. In any series of numbers connected by the signs +, −, ×, ÷, the operations indicated by the signs must be performed in order from left to right, except when a sign of multiplication or division *follows* the number on the *right* of a sign of addition or subtraction, the multiplication or division indicated by the sign should be performed before that of addition or subtraction. In all cases, the sign of multiplication takes precedence, the sign of division next, and the sign either of addition or of subtraction next.

NOTE.—Some recognized authorities claim that the multiplication and division signs have equal weight, and take precedence to the sign of either addition or subtraction, which also have equal weight.

EXAMPLE 1.—What is the value of $4 \times 24 - 8 + 17$?

SOLUTION.—Performing the operations in order from left to right, $4 \times 24 = 96$; $96 - 8 = 88$; $88 + 17 = 105$. Ans.

EXAMPLE 2.—What is the value of $1,296 \div 12 + 160 - 21 \times 3$?

SOLUTION.— $1,296 \div 12 = 108$; $108 + 160 = 268$; here we cannot subtract 21 from 268, because the sign of multiplication follows 21; hence, multiplying 21×3 we get 63, and $268 - 63 = 205$. Ans.

Had example 2 been written $1,296 \div 12 + 160 - 21 \times 3 \div 7 + 25$, it would have been necessary to divide 21×3 by 7 before subtracting from 268, and the final result would then have been: $21 \times 3 = 63$; $63 \div 7 = 9$; $268 - 9 = 259$;

$259 + 25 = 284$. Ans. In other words, it is necessary before adding or subtracting to perform first all the multiplications included between the signs of + and − or − and +, and next perform all the divisions included between these signs.

SYMBOLS OF AGGREGATION.

72. If it is desired to change the order of precedence of the signs of multiplication and division, the **symbols of aggregation** are used. They are the vinculum $\overline{\ }$, parentheses (), brackets [], and brace {}. These symbols are used to include numbers that are to be considered together. For example, $13 \times \overline{8 - 3}$, $13 \times (8 - 3)$, $13 \times [8 - 3]$, and $13 \times \{8 - 3\}$ all indicate that 13 is to be multiplied by the difference between 8 and 3. Thus,

$$13 \times \overline{8 - 3} = 65. \text{ Ans.}$$
$$13 \times (8 - 3) = 65. \text{ Ans.}$$
$$13 \times [8 - 3] = 65. \text{ Ans.}$$
$$13 \times \{8 - 3\} = 65. \text{ Ans.}$$

When the signs of aggregation are not used, we have

$$13 \times 8 - 3 = 101. \text{ Ans.}$$

In problems where one sign of aggregation occurs within another, the sign within must be removed by performing the operations indicated within the inner sign according to the order of precedence of signs. The remainder of the problem is performed in the manner described in Art. **71.**

EXAMPLE.— $2 + 6 \times [8 + 5 - 3 + 10 \times (6 - 2 \times 2)] \div 2 = ?$

SOLUTION.—The expression included in the sign of parenthesis should be first simplified by performing the multiplication $2 \times 2 = 4$, and the subtraction $6 - 4 = 2$. The whole expression now becomes $2 + 6 \times [8 + 5 - 3 + 10 \times 2] \div 2 = ?$ The remainder of the problem is performed in the manner described in Art. **71** and the expression becomes

$2 + 6 \times [8 + 5 - 3 + 20] \div 2$; or $2 + 6 \times 30 \div 2$; or $2 + 180 \div 2$; or $2 + 90 = 92$. Ans.

EXAMPLES FOR PRACTICE.

73. What is the value of:

(*a*) $7 + 14 + 27 \times 4 \div 9 + 5$?
(*b*) $18 \div 2 \times 3 + 17 - 4$?
(*c*) $(7 + 24 - 26 + 30) \div 7$?
(*d*) $8 + 4 \times [(7 - 2) \times (3 + 9)]$?
(*e*) $9 \times (3 + 2) \div [(7 + 8) \div 3]$?

Ans. (*a*) 38. (*b*) 16. (*c*) 5. (*d*) 248. (*e*) 9.

COMPLEX FRACTIONS.

74. Whenever an expression like one of the following three is obtained, it may always be simplified by transposing the denominator from *above* to *below* the line, or from *below* to *above*, as the case may be, taking care, however, to indicate that the denominator, when so transferred, is a multiplier. These expressions are called **complex fractions.**

1. $\dfrac{\frac{3}{4}}{9} = \dfrac{3}{9 \times 4} = \dfrac{3}{36} = \dfrac{1}{12}$; for, regarding the fraction above the heavy line as the numerator of a fraction whose denominator is 9, $\dfrac{\frac{3}{4} \times 4}{9 \times 4} = \dfrac{3}{9 \times 4}$, as before.

2. $\dfrac{9}{\frac{3}{4}} = \dfrac{9 \times 4}{3} = 12$. The proof is the same as in the first case.

3. $\dfrac{\frac{5}{9}}{\frac{3}{4}} = \dfrac{5 \times 4}{3 \times 9} = \dfrac{20}{27}$; for, regarding $\frac{5}{9}$ as the numerator of a fraction whose denominator is $\frac{3}{4}$, $\dfrac{\frac{5}{9} \times 9}{\frac{3}{4} \times 9} = \dfrac{5}{\frac{3 \times 9}{4}}$; and $\dfrac{\frac{5}{3 \times 9}}{4} \times 4 = \dfrac{5 \times 4}{3 \times 9} = \dfrac{20}{27}$, as above.

This principle may be used to great advantage in cases like $\dfrac{\frac{1}{4} \times 310 \times \frac{27}{12} \times 72}{40 \times 4\frac{1}{2} \times 5\frac{1}{6}}$. Reducing the mixed numbers to fractions, the expression becomes $\dfrac{\frac{1}{4} \times 310 \times \frac{27}{12} \times 72}{40 \times \frac{9}{2} \times \frac{31}{6}}$. Now,

transferring the denominators of the fractions and canceling,

$$\frac{1\times310\times27\times72\times2\times6}{40\times9\times31\times4\times12} = \frac{1\times\overset{5}{\cancel{310}}\times\overset{3}{\cancel{27}}\times\overset{3}{\cancel{72}}\times\cancel{2}\times\overset{3}{\cancel{6}}}{\underset{4}{\cancel{40}}\times\cancel{9}\times\cancel{31}\times\underset{2}{\cancel{4}}\times\cancel{12}} = \frac{27}{2} = 13\tfrac{1}{2}.$$

Greater exactness in results can usually be obtained by using this principle than can be obtained by reducing the fractions to decimals. The principle, however, should not be employed if a sign of addition or subtraction occurs either above or below the dividing line.

EXAMPLES FOR PRACTICE.

75. Simplify the following:

(*a*) $\dfrac{4\frac{2}{5}\times10\frac{2}{5}\times26\frac{1}{4}}{8\frac{2}{5}\times4\frac{7}{8}\times12\frac{3}{5}}$. (*b*) $\dfrac{10\frac{2}{3}\times12\frac{4}{7}\times15\frac{3}{4}}{4\frac{7}{12}\times\frac{24}{35}\times8\frac{2}{5}}$.

(*c*) $\dfrac{20\frac{2}{5}\times34\frac{2}{3}\times8\frac{5}{8}}{14\frac{1}{6}\times3\frac{9}{10}\times18\frac{2}{5}}$. (*d*) $\dfrac{15\frac{3}{11}\times32\frac{2}{3}\times25\frac{3}{5}}{28\frac{4}{5}\times38\frac{1}{4}\times17\frac{7}{9}}$.

Ans. (*a*) $2\frac{38}{81}$. (*b*) 80. (*c*) 6. (*d*) $\frac{5488}{8415}$.

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76. Examples similar to the following, in which the expressions above and below the heavy line involve the principles of addition, subtraction, multiplication, and division of fractions, are more complicated forms of the complex fraction. Such fractions are simplified by performing successively the operations indicated in the numerator and denominator.

EXAMPLE 1.—Simplify $\dfrac{\frac{2}{3}+\frac{3}{4}}{\frac{4}{5}-\frac{1}{3}}$.

SOLUTION.— $\dfrac{\frac{2}{3}+\frac{3}{4}}{\frac{4}{5}-\frac{1}{3}} = \dfrac{\frac{2}{3}\times\frac{4}{4}+\frac{3}{4}\times\frac{3}{3}}{\frac{4}{5}\times\frac{3}{3}-\frac{1}{3}\times\frac{5}{5}} = \dfrac{\frac{8}{12}+\frac{9}{12}}{\frac{12}{15}-\frac{5}{15}} = \dfrac{\frac{17}{12}}{\frac{7}{15}} = \dfrac{17}{12}\div\dfrac{7}{15}$

$= \dfrac{17}{\underset{4}{\cancel{12}}}\times\dfrac{\overset{5}{\cancel{15}}}{7} = \dfrac{85}{28} = 3\tfrac{1}{28}$. Ans.

EXAMPLE 2.—Simplify $\dfrac{\frac{2}{3}\div\frac{2}{5}}{\frac{3}{10}\times\frac{2}{7}}$.

SOLUTION.— $\dfrac{\frac{2}{3}\div\frac{2}{5}}{\frac{3}{10}\times\frac{2}{7}} = \dfrac{\frac{2}{3}\times\frac{5}{2}}{\frac{3}{10}\times\frac{2}{7}} = \dfrac{\frac{10}{6}}{\frac{6}{70}} = \dfrac{10}{6}\div\dfrac{6}{70} = \dfrac{\overset{5}{\cancel{10}}}{\underset{3}{\cancel{6}}}\times\dfrac{\overset{35}{\cancel{70}}}{\underset{3}{\cancel{6}}} = \dfrac{175}{9}$

$= 19\frac{4}{9}$. Ans.

EXAMPLE 3.—Simplify $\frac{\frac{2}{3}+\frac{3}{4}\times\frac{2}{5}-\frac{5}{6}\div\frac{7}{8}}{\frac{7}{8}-\frac{1}{4}\div\frac{2}{7}+\frac{2}{3}\times\frac{5}{9}}$.

SOLUTION.—First simplify, according to the order of signs, the numerator of this complex fraction, and we have: $\frac{2}{3}+\frac{3}{4}\times\frac{2}{5}-\frac{5}{6}\div\frac{7}{8}=\frac{2}{3}+\frac{3}{10}-\frac{5}{6}\div\frac{7}{8}=\frac{2}{3}+\frac{3}{10}-\frac{5}{6}\times\frac{8}{7}=\frac{2}{3}+\frac{3}{10}-\frac{20}{21}=\frac{140}{210}+\frac{63}{210}-\frac{200}{210}=\frac{3}{210}=\frac{1}{70}$. Next, simplify the denominator in the same manner, and we have: $\frac{7}{8}-\frac{1}{4}\div\frac{2}{7}+\frac{2}{3}\times\frac{5}{9}=\frac{7}{8}-\frac{1}{4}\div\frac{2}{7}+\frac{10}{27}=\frac{7}{8}-\frac{1}{4}\times\frac{7}{2}+\frac{10}{27}=\frac{7}{8}-\frac{7}{8}+\frac{10}{27}=\frac{10}{27}$. The fraction is now reduced to the form

$$\frac{\frac{1}{70}}{\frac{10}{27}}=\frac{1}{70}\div\frac{10}{27}=\frac{1}{70}\times\frac{27}{10}=\frac{27}{700}.$$ Ans.

EXAMPLES FOR PRACTICE.

77. What is the value of:

(*a*) $\frac{\frac{2}{3}+\frac{3}{8}-\frac{1}{6}}{\frac{3}{4}+\frac{5}{12}-\frac{2}{3}}$?

(*b*) $\frac{(4\frac{1}{2}-1\frac{3}{4})\div 5\frac{1}{2}}{4\frac{1}{2}-2\frac{3}{5}}$?

(*c*) $\frac{\frac{11}{12}+\frac{7}{4}-\frac{5}{6}+\frac{17}{8}}{\frac{5}{2}+\frac{2}{3}+\frac{7}{8}-\frac{11}{4}}$?

(*d*) $\frac{\frac{3}{4}-\frac{2}{3}\times\frac{1}{2}-\frac{1}{6}}{(4\frac{1}{2}-2\frac{1}{3})\div(3\frac{1}{3}-1\frac{1}{6})}$?

Ans. (*a*) $1\frac{3}{4}$. (*b*) $\frac{5}{19}$. (*c*) $3\frac{2}{31}$. (*d*) $\frac{1}{4}$.

ARITHMETIC.

(PART 3.)

DECIMALS.

NOTATION AND NUMERATION.

1. A **decimal,** or a **decimal fraction,** is a fraction whose denominator is 10, 100, 1,000, etc.

2. The denominator is always 10 or a power of 10, and is not expressed as in common fractions, by writing it under the numerator, with a line between them; as $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$. The denominator is always understood, the numerator consisting of the figures on the right of the *unit* figure of the number. In order to distinguish the unit figure, a period (.), called the **decimal point,** is placed between the unit figure and the next figure on the right. The decimal point may be regarded in two ways: first, as indicating that the number on the right is the numerator of a fraction whose denominator is 10, 100, 1,000, etc.; and, second, as a part of the Arabic system of notation, each figure on the right being 10 times as large as the next succeeding figure, and 10 times as small as the next preceding figure, serving merely to point out the unit figure.

3. The reading of a *decimal* depends upon the number of decimal places in it; i. e., upon the number of figures to the *right* of the unit figure.

The first figure to the right of the unit figure expresses *tenths.*

The second figure to the right of the unit figure expresses *hundredths.*

The third figure to the right of the unit figure expresses *thousandths.*

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The fourth figure to the right of the unit figure expresses *ten-thousandths.*

The fifth figure to the right of the unit figure expresses *hundred-thousandths.*

The sixth figure to the right of the unit figure expresses *millionths.*

Thus:

.3	=	$\frac{3}{10}$	= 3 tenths.
.03	=	$\frac{3}{100}$	= 3 hundredths.
.003	=	$\frac{3}{1000}$	= 3 thousandths.
.0003	=	$\frac{3}{10000}$	= 3 ten-thousandths.
.00003	=	$\frac{3}{100000}$	= 3 hundred-thousandths.
.000003	=	$\frac{3}{1000000}$	= 3 millionths.

The first figure to the right of the unit figure is called the *first decimal place;* the second figure, the *second decimal place*, etc. We see in the above that the number of decimal places in a decimal equals the *number of ciphers to the right of the figure 1 in the denominator of its equivalent fraction.* This fact kept in mind will be of much assistance in reading and writing decimals.

Whatever may be written to the *left* of a decimal point is a whole number. The decimal point affects only the figures to its *right.*

When a whole number and decimal are written together, the expression is a *mixed number.* Thus, 8.12 and 17.25 are mixed numbers.

The relation of decimals and whole numbers to each other is clearly shown by the following table:

hundreds of millions.	tens of millions.	millions.	hundreds of thousands.	tens of thousands.	thousands.	hundreds.	tens.	units.	decimal point.	tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.	ten-millionths.	hundred-millionths.
9	8	7	6	5	4	3	2	1	.	2	3	4	5	6	7	8	9

The figures to the left of the decimal point represent whole numbers; those to the right are decimals.

In both the decimals and whole numbers, the *units* place is made the starting point of notation and numeration. The *decimals decrease* on the scale of ten to the right, and the *whole numbers increase* on the scale of ten to the left. The first figure to the left of units is *tens*, and the first figure to the right of units is *tenths*. The second figure to the left of units is *hundreds*, and the second figure to the right is *hundredths*. The third figure to the left is *thousands*, and the third to the right is *thousandths*, and so on. The figures equally distant from units place correspond in name, but the decimals have the ending *ths*, which distinguishes them from whole numbers. The following is the numeration of the number in the above table: nine hundred eighty-seven million, six hundred fifty-four thousand, three hundred twenty-one, and twenty-three million, four hundred fifty-six thousand, seven hundred eighty-nine hundred-millionths.

The decimals increase to the left, on the scale of ten, the same as whole numbers; for, beginning at, say, 4 thousandths, in the table, the next figure to the left is hundredths, which is ten times as great, and the next tenths, or ten times the hundredths, and so on through both decimals and whole numbers.

PRINCIPLES OF DECIMALS.

4. I. *The value of a decimal is not changed by annexing or rejecting a cipher at the right.*

Thus, .5 = .50. For $.5 = \frac{5}{10} = \frac{1}{2}$, and $.50 = \frac{50}{100} = \frac{1}{2}$.

II. *A decimal is divided by 10 by inserting a cipher after the decimal point.*

Thus, $.5 = \frac{5}{10}$; $\frac{5}{10} \div 10 = \frac{5}{100} = .05$.

III. *A decimal is multiplied by 10 by rejecting a cipher from its left.*

Thus, $.05 = \frac{5}{100}$; $\frac{5}{100} \times 10 = \frac{5}{10} = .5$.

EXAMPLES FOR PRACTICE.

5. Read the following numbers:

1. .00707.
2. .100707.
3. 114.75206.
4. 40032.890001.
5. .004001008.
6. 75040.0742.
7. 234.542000.
8. 961724.009.

Write the following numbers:

9. Five hundred forty-six ten-millionths.

10. Three thousand, four, and three thousand four hundred seventeen hundred-thousandths.

11. Five hundred four, and three-tenths.

12. Nineteen thousand thirteen, and one hundred four thousand, five hundred one ten-billionths.

13. Six hundred thirty-eight million, four hundred twenty-five thousand, six hundred seventy-two, and thirty-two million, six hundred seventy-two thousand, five hundred forty-five hundred-millionths.

ADDITION OF DECIMALS.

6. To add decimals, they must be written so that the units of the same order are in the same column. That this may be, it is necessary only to see that the decimal points are in the same vertical column.

whole numbers	*decimals*	*mixed decimals*
342	.342	342.032
4234	.4234	4234.5
26	.26	26.6782
3	.03	3.06
sum 4605 Ans.	*sum* 1.0554 Ans.	*sum* 4606.2702 Ans.

7. Example.—What is the sum of 242, .36, 118.725, 1.005, 6, and 100.1?

Solution.—

242.
.36
118.725
1.005
6.
100.1
sum 468.190 Ans.

8. Rule.—*Place the numbers to be added so that the decimal points shall be directly under each other. Add as in whole numbers, and place the decimal point in the sum directly under the decimal points above.*

EXAMPLES FOR PRACTICE.

9. Find the sum of:

(*a*)	.2143, .105, 2.3042, and 1.1417.	(*a*)	3.7652.
(*b*)	783.5, 21.473, .2101, and .7816.	(*b*)	805.9647.
(*c*)	21.781, 138.72, 41.8738, .72, and 1.413.	(*c*)	204.5078.
(*d*)	.3724, 104.15, 21.417, and 100.042.	(*d*)	225.9814.
(*e*)	200.172, 14.105, 12.1465, .705, and 7.2.	(*e*)	234.3285.
(*f*)	1,427.16, .244, .32, .032, and 10.0041.	(*f*)	1,437.7601.
(*g*)	2,473.1, 41.65, .7243, 104.067, and 21.073.	(*g*)	2,640.6143.
(*h*)	4,107.2, .00375, 21.716, 410.072, and .0345.	(*h*)	4,539.02625.

Ans.

SUBTRACTION OF DECIMALS.

10. For the same reason as in addition of decimals, the numbers are placed so that the decimal points shall be in the same vertical column.

EXAMPLE.—Subtract .132 from .3063.

SOLUTION.—

minuend .3 0 6 3
subtrahend .1 3 2
difference .1 7 4 3 Ans.

11. EXAMPLE.—What is the difference between 7.895 and .725?

SOLUTION.—

minuend 7.8 9 5
subtrahend .7 2 5
difference 7.1 7 0 or 7.17 Ans.

12. EXAMPLE.—Subtract .625 from 11.

SOLUTION.—

minuend 1 1.0 0 0
subtrahend .6 2 5
difference 1 0.3 7 5 Ans.

13. Rule.—*Place the subtrahend under the minuend, so that the decimal points shall be in the same vertical column. Subtract as in whole numbers, and place the decimal point in the remainder directly under the decimal points above.*

When there are more decimal places in the subtrahend than in the minuend, place ciphers in the minuend above them, and subtract as before.

EXAMPLES FOR PRACTICE.

14. From:

(*a*)	407.385 take 235.0004.	Ans. (*a*)	172.3846.
(*b*)	22.718 take 1.7042.	(*b*)	21.0138.
(*c*)	1,368.17 take 13.6817.	(*c*)	1,354.4883.
(*d*)	70.00017 take 7.000017.	(*d*)	63.000153.
(*e*)	630.630 take .6304.	(*e*)	629.9996.
(*f*)	421.73 take 217.162.	(*f*)	204.568.
(*g*)	1.000014 take .00001.	(*g*)	1.000004.
(*h*)	783652 take .542314.	(*h*)	.241338.

MULTIPLICATION OF DECIMALS.

15. In multiplication of decimals, no attention is paid for the time being to the decimal points. Write the multiplier under the multiplicand, so that the right-hand figure of the one is under the right-hand figure of the other, and proceed *exactly as in multiplication of whole numbers.* After multiplying, *count the number of decimal places in both multiplicand and multiplier, and point off the same number in the product.*

EXAMPLE.—Multiply .825 by 13.

SOLUTION.—

multiplicand	.8 2 5
multiplier	1 3
	2 4 7 5
	8 2 5
product	1 0.7 2 5 Ans.

In this example there are 3 decimal places in the multiplicand and none in the multiplier; therefore, $3 + 0 = 3$ decimal places are pointed off in the product.

16. EXAMPLE.—What is the product of 426 and the decimal .005?

SOLUTION.—

multiplicand	4 2 6
multiplier	.0 0 5
product	2.1 3 0 or 2.13 Ans.

In this example there are 3 decimal places in the multiplier and none in the multiplicand; therefore, 3 decimal places are pointed off in the product.

17. It is not necessary to multiply by the ciphers on the *left* of a decimal; they merely *determine the number of decimal places.* Ciphers to the *right* of a decimal should be removed, as they only make more figures to deal with, and do not change the value.

18. EXAMPLE.—Multiply 1.205 by 1.15.

SOLUTION.—

multiplicand	1.205	
multiplier	1.15	
	6025	
	1205	
	1205	
product	1.38575	Ans.

In this example there are 3 decimal places in the multiplicand, and 2 in the multiplier; therefore, $3+2$, or 5 decimal places must be pointed off in the product.

19. EXAMPLE.—Multiply .232 by .001.

SOLUTION.—

multiplicand	.232	
multiplier	.001	
product	.000232	Ans.

In this example we multiply the multiplicand by the digit in the multiplier, which makes 232 in the product, but since there are 3 decimal places in the multiplier and 3 in the multiplicand, we must prefix 3 ciphers to the 232, to make $3+3$, or 6 decimal places in the product.

20. **Rule.**—*Place the multiplier under the multiplicand, disregarding the position of the decimal points. Multiply as in whole numbers, and in the product point off as many decimal places as there are decimal places in both multiplier and multiplicand, prefixing ciphers if necessary.*

EXAMPLES FOR PRACTICE.

21. Find the product of:

(*a*)	.000492 × 4.1418.	(*a*)	.0020377656.
(*b*)	4,003.2 × 1.2.	(*b*)	4,803.84.
(*c*)	78.6531 × 1.03.	(*c*)	81.012693.
(*d*)	.3685 × .042.	(*d*)	.015477.
(*e*)	178,352 × .01.	(*e*)	1,783.52.
(*f*)	.00045 × .0045.	(*f*)	.000002025.
(*g*)	.714 × .00002.	(*g*)	.00001428.
(*h*)	.00004 × .008.	(*h*)	.00000032.

Ans.

DIVISION OF DECIMALS.

22. In division of decimals we pay no attention to the decimal point until after the division is performed. Divide exactly as in whole numbers. *If the divisor contains more decimal places than the dividend, annex ciphers to the dividend until the number of decimal places in the dividend equals the number of decimal places in the divisor, before dividing. Subtract the number of decimal places in the divisor from the number of decimal places in the dividend, and point off as many decimal places in the quotient as there are units in the remainder thus found.*

23. EXAMPLE.—Divide .625 by 25.

divisor dividend quotient

SOLUTION.— 25).625(.025 Ans.

50

125

125

remainder 0

In this example there are no decimal places in the divisor, and 3 decimal places in the dividend; therefore, there are 3 minus 0, or 3 decimal places in the quotient. One cipher has to be prefixed to the 25, to make the 3 decimal places.

24. EXAMPLE.—Divide 6.035 by .05.

divisor dividend

SOLUTION.— .05)6.035

quotient 120.7 Ans.

In this example we divide by 5, as if the cipher were not before it. There is one more decimal place in the dividend

than in the divisor; therefore, one decimal place is pointed off in the quotient.

25. Example.—Divide .125 by .005.

Solution.—

divisor *dividend*
.0 0 5) .1 2 5
quotient 2 5 Ans.

In this example there are the same number of decimal places in the dividend as in the divisor; therefore, the quotient has no decimal places, and is a whole number.

26. Example.—Divide 326 by .25.

Solution.—

divisor *dividend* *quotient*
.2 5) 3 2 6.0 0 (1 3 0 4 Ans.
2 5
7 6
7 5
1 0 0
1 0 0
remainder 0

In this problem two ciphers were annexed to the dividend to make the number of decimal places equal to the number in the divisor. The quotient is a whole number.

27. Example.—Divide .0025 by 1.25.

Solution.—

1.2 5) .0 0 2 5 0 (.0 0 2 Ans.
2 5 0
remainder 0

Explanation.—In this example we are to divide .0025 by 1.25. Consider the dividend as a whole number or 25 (disregarding the two ciphers at its left, for the present); also, consider the divisor as a whole number, or 125. It is evident that the dividend 25 will not contain the divisor, 125; we must, therefore, annex one cipher to the 25, thus making the dividend 250. 125 is contained twice in 250, so we place the figure 2 in the quotient. In pointing off the decimal places in the quotient, it must be remembered that there were only four decimal places in the dividend; but one cipher was annexed, making 4 + 1, or 5, decimal places. Since there are 5 decimal places in the dividend and 2 decimal places in the

divisor, we must point off $5-2$, or 3, decimal places in the quotient. In order to point off 3 decimal places, two ciphers must be prefixed to the figure 2, thereby making .002 the quotient. It is not necessary to consider the ciphers at the left of a decimal when dividing, except when determining the position of the decimal point in the quotient.

28. Rule.—I. *Place the divisor to the left of the dividend, and proceed as in division of whole numbers; in the quotient, point off as many decimal places as the number of decimal places in the dividend exceeds those in the divisor, prefixing ciphers to the quotient, if necessary.*

II. *If in dividing one number by another there is a remainder, the remainder can be placed over the divisor, as a fractional part of the quotient, but it is generally better to annex ciphers to the remainder, and continue dividing until there are 3 or 4 decimal places in the quotient, and then if there is still a remainder, terminate the quotient by the plus sign* (+), *to show that the division can be carried farther.*

29. Example.—What is the quotient of 199 divided by 15?

Solution.—

```
                15)199(13+4/15  Ans.
                   15
                   --
                    49
                    45
                    --
remainder            4
```

```
Or,  15)199.000(13.266+  Ans.
        15
        --
         49
         45
         --
          40
          30
          --
          100
           90
           --
           100
            90
            --
remainder    10
```

$$13\tfrac{4}{15} = 13.266+$$
$$\tfrac{4}{15} = \quad .266+$$

30. It frequently happens, as in the above example, that the division will never terminate. In such cases, decide how many decimal places are desired in the quotient and then carry the work one place farther. If the last figure of the quotient thus obtained is 5 or a greater number, increase the preceding figure by 1, and write after it the minus sign (−), thus indicating that the quotient is not quite so great as indicated; if the figure thus obtained is less than 5, write the plus sign (+) after the quotient, thus indicating that the number is slightly greater than indicated. In the last example, had it been desired to obtain the answer correct to four decimal places, the work would have been carried to five places, obtaining 13.26666, and the answer would have been given as 13.2667−. This remark applies to any other calculation involving decimals, when it is desired to omit some of the figures in the decimal. Thus, if it is desired to retain three decimal places in the number .2471253, it would be expressed as .247+; if it were desired to retain five decimal places, it would be expressed as .24713−. Both the + and − signs are frequently omitted; they are seldom used in this connection outside of arithmetic, except in exact calculations, when it is desired to call particular attention to the fact that the result obtained is not *quite* exact.

EXAMPLES FOR PRACTICE.

31. Divide:

(*a*)	101.6688 by 2.36.	Ans. (*a*)	43.08.
(*b*)	187.12264 by 123.107.	(*b*)	1.52.
(*c*)	.08 by .008.	(*c*)	10.
(*d*)	.0003 by 3.75.	(*d*)	.00008.
(*e*)	.0144 by .024.	(*e*)	.6.
(*f*)	.00375 by 1.25.	(*f*)	.003.
(*g*)	.004 by 400.	(*g*)	.00001.
(*h*)	.4 by .008.	(*h*)	50.

TO REDUCE A FRACTION TO A DECIMAL.

32. EXAMPLE.— $\frac{3}{4}$ equals what decimal?

SOLUTION.— $4\,)\,\underline{3.0\,0}$ or $\frac{3}{4} = .75$. Ans.
$.7\,5$

EXAMPLE.—What decimal is equivalent to $\frac{7}{8}$?

SOLUTION.— $8\,)\,\underline{7.000}$ or $\frac{7}{8} = .875$. Ans.
$\quad\;\; .875$

33. Rule.—*Annex ciphers to the numerator and divide by the denominator. Point off as many decimal places in the quotient as there are ciphers annexed.*

EXAMPLES FOR PRACTICE.

34. Reduce the following common fractions to decimals:

(*a*)	$\frac{15}{32}$.	Ans. (*a*)	.46875.
(*b*)	$\frac{7}{8}$.	(*b*)	.875.
(*c*)	$\frac{21}{32}$.	(*c*)	.65625.
(*d*)	$\frac{51}{64}$.	(*d*)	.796875.
(*e*)	$\frac{4}{25}$.	(*e*)	.16.
(*f*)	$\frac{5}{8}$.	(*f*)	.625.
(*g*)	$\frac{10}{200}$.	(*g*)	.05.
(*h*)	$\frac{4}{1000}$.	(*h*)	.004.

35. To reduce inches to decimal parts of a foot.

EXAMPLE.—What decimal part of a foot is 9 inches?

SOLUTION.—Since there are 12 inches in one foot, 1 inch is $\frac{1}{12}$ of a foot, and 9 inches is $9 \times \frac{1}{12}$ or $\frac{9}{12}$ of a foot. This reduced to a decimal by the above rule, shows what decimal part of a foot 9 inches is.

```
12 ) 9.00 ( .75 of a foot.  Ans.
     84
     --
      60
      60
      --
```

36. Rule.—I. *To reduce inches to a decimal part of a foot, divide the number of inches by 12.*

II. *Should the resulting decimal be an unending one and it is desired to terminate the division at some point, say the fourth decimal place, carry the division one place farther, and if the fifth figure is 5 or greater, increase the fourth figure by 1. Omit the signs + and −.*

EXAMPLES FOR PRACTICE.

37. Reduce to the decimal part of a foot:

(*a*) 3 in.
(*b*) $4\frac{1}{2}$ in.
(*c*) 5 in.
(*d*) $6\frac{5}{8}$ in.
(*e*) 11 in.

Ans. (*a*) .25. (*b*) .375. (*c*) .4167. (*d*) .5521. (*e*) .9167.

TO REDUCE A DECIMAL TO A FRACTION.

38. Example.—Reduce .125 to a fraction.

Solution.— $.125 = \frac{125}{1000} = \frac{5}{40} = \frac{1}{8}$.

Example.—Reduce .875 to a fraction.

Solution.— $.875 = \frac{875}{1000} = \frac{35}{40} = \frac{7}{8}$. Ans.

39. **Rule.**—*Under the figures of the decimal, place 1 followed by as many ciphers at its right as there are decimal places in the decimal, and reduce the resulting fraction to its lowest terms.*

EXAMPLES FOR PRACTICE.

40. Reduce the following to common fractions:

(*a*) .125.
(*b*) .625.
(*c*) .3125.
(*d*) .04.
(*e*) .06.
(*f*) .75.
(*g*) .15625.
(*h*) .875.

Ans. (*a*) $\frac{1}{8}$. (*b*) $\frac{5}{8}$. (*c*) $\frac{5}{16}$. (*d*) $\frac{1}{25}$. (*e*) $\frac{3}{50}$. (*f*) $\frac{3}{4}$. (*g*) $\frac{5}{32}$. (*h*) $\frac{7}{8}$.

41. **To express a decimal approximately as a fraction having a given denominator.**

Example.—Express .5827 in 64ths.

Solution.— $.5827 \times \frac{64}{64} = \frac{37.2928}{64}$, say, $\frac{37}{64}$.

Hence, $.5827 = \frac{37}{64}$, nearly. Ans.

Example.—Express .3917 in 12ths.

Solution.— $.3917 \times \frac{12}{12} = \frac{4.7004}{12}$, say, $\frac{5}{12}$.

Hence, $.3917 = \frac{5}{12}$, nearly. Ans.

42. Rule.—*Reduce 1 to a fraction having the given denominator. Multiply the given decimal by the fraction so obtained, and the result will be the fraction required.*

EXAMPLES FOR PRACTICE.

43. Express:

(*a*) .625 in 8ths.
(*b*) .3125 in 16ths.
(*c*) .15625 in 32ds.
(*d*) .77 in 64ths.
(*e*) .81 in 48ths.
(*f*) .923 in 96ths.

Ans. (*a*) $\frac{5}{8}$. (*b*) $\frac{5}{16}$. (*c*) $\frac{5}{32}$. (*d*) $\frac{49}{64}$. (*e*) $\frac{39}{48}$. (*f*) $\frac{89}{96}$.

UNITED STATES MONEY.

44. The sign for dollars is $. It is read dollars. $25 is read 25 dollars.

Since there are 100 cents in a dollar, one cent is 1 one-hundredth of a dollar; the first two figures of a decimal part of a dollar represent *cents*. Since a mill is $\frac{1}{10}$ of a cent, or $\frac{1}{1000}$ of a dollar, the third figure represents mills. Thus, $25.16 is read twenty-five dollars and sixteen cents; $25.168 is read twenty-five dollars, sixteen cents, and eight mills.

45. To change dollars to cents, move the decimal point *two* places to the *right;* to change dollars to mills, move the decimal point *three* places to the right. Thus, to change $143.75 to cents, we have $143.75 = 14,375 cents, or 143,750 mills. The decimal point is always understood as following the unit figure, whether written or not; hence, to change $100 to cents, write it thus, $100.; to move the decimal point two places to the right, it is necessary to annex two ciphers, thus, 10000; in other words, $100 = 10,000 cents.

Moving the decimal point two places to the right is evidently the same thing as multiplying by 100, since it changes the unit figure from the first order to the third order. Thus, in 143.75, 3 is a figure of the first order; but, in 14375., 3 is a figure of the third order; and, since all the other figures have also been advanced two orders, the number has been multiplied by 100.

46. To change cents to dollars, move the decimal point *two* places to the *left;* or, to change mills to dollars, move the decimal point *three* places to the left. Thus, to change 143,750 mills to dollars, move the decimal point (understood to follow the 0) three places to the left, obtaining \$143.750 = \$143.75. Similarly, 14,375 cents = \$143.75, and 10,000 cents = \$100.00.

ALIQUOT PARTS.

47. An **aliquot part** of a number is a number that will divide it without a remainder. For example, 15 is an aliquot part of 60, because 15 is an exact divisor of 60. For the same reason, 5, 6, 10, 12, etc. are also aliquot parts of 60.

48. About the only case of any importance to the business man is that concerning the aliquot parts of 100, which are given in the following table:

$2\frac{1}{2} = \frac{1}{40}$	$12\frac{1}{2} = \frac{1}{8}$	$50 = \frac{1}{2}$
$3\frac{1}{3} = \frac{1}{30}$	$16\frac{2}{3} = \frac{1}{6}$	$62\frac{1}{2} = \frac{5}{8}$
$4 = \frac{1}{25}$	$20 = \frac{1}{5}$	$66\frac{2}{3} = \frac{2}{3}$
$5 = \frac{1}{20}$	$25 = \frac{1}{4}$	$75 = \frac{3}{4}$
$6\frac{1}{4} = \frac{1}{16}$	$33\frac{1}{3} = \frac{1}{3}$	$87\frac{1}{2} = \frac{7}{8}$
$10 = \frac{1}{10}$	$37\frac{1}{2} = \frac{3}{8}$	

The aliquot parts given in the above table should be carefully memorized; in many cases calculations may be shortened by using them. Thus, in order to multiply any number by one of the numbers in the above table:

49. **Rule.**—*Move the decimal point of the multiplicand two places to the right, and divide the multiplicand by the denominator of the fraction opposite the multiplier in the table. Then multiply the result by the numerator of the fraction.*

EXAMPLE.—Multiply 478.4 by 25.

SOLUTION.—Applying the rule, the fraction opposite 25 is $\frac{1}{4}$. Hence, $478.4 \times 25 = 47{,}840 \div 4 \times 1 = 11{,}960$. Ans.

The result may be proved to be true by actual multiplication.

50. The reasoning on which the rule is based is this: The last example required the multiplication of 478.4 by 25.

But 25 is $\frac{1}{4}$ of 100; that is, $25 = \frac{100}{4}$. Hence, in multiplying by 25, $\frac{100}{4}$ can be used as a multiplier, and the operation becomes $478.4 \times \frac{100}{4} = \frac{47840}{4} = 11{,}960$. Since, in order to multiply by 100, it is necessary only to move the decimal point two places to the right, the correctness of the rule is evident.

EXAMPLE.—Multiply (*a*) 50.64 by $16\frac{2}{3}$; (*b*) 1,894 by $37\frac{1}{2}$.

SOLUTION.—(*a*) Since $16\frac{2}{3}$ is $\frac{1}{6}$ of 100, $50.64 \times 16\frac{2}{3} = \frac{5064}{6} = 844$. Ans. (*b*) Since $37\frac{1}{2} = \frac{3}{8}$ of 100, $1{,}894 \times 37\frac{1}{2} = \frac{189400}{8} \times 3 = 23{,}675 \times 3 = 71{,}025$. Ans.

51. It matters not where the decimal point is placed in the number denoting the aliquot part, the principle can still be applied with a slight modification. Thus, suppose it is required to multiply 72 by 625. Now, $62\frac{1}{2}$, or 62.5, is $\frac{5}{8}$ of 100; that is, $62.5 = 100 \times \frac{5}{8} = \frac{500}{8}$. It is an axiom in mathematics that if equals be multiplied or divided by equals, the result will be equal; that is, if 4 = 4, and both 4's be multiplied or divided by the same number, the results will be equal. Thus, multiplying by 10, 40 = 40; dividing by 10, .4 = .4. Hence, if in the expression $62.5 = \frac{500}{8}$, both numbers are multiplied by 10, the result is $625 = \frac{5000}{8}$, or $625 = 1{,}000 \times \frac{5}{8}$. Therefore, to multiply 72 by 625, move the decimal point three places to the right, divide by 8 and multiply by 5. (It makes no difference whether we divide by the denominator or multiply by the numerator first.) Then, $72 \times 625 = \frac{72000}{8} \times 5 = 45{,}000$. Had it been required to multiply 72 by .0625, we note that $62.5 = 100 \times \frac{5}{8}$, and that moving the decimal point in 62.5 three places to the left will give .0625. Hence, moving the decimal point in the other number (the 100) three places to the left, we have $.0625 = .1 \times \frac{5}{8} = \frac{.5}{8}$; whence, $72 \times .0625 = \frac{72}{8} \times .5 = 4.5$.

To multiply any number by 5, multiply by 10 and divide by 2.

52. To divide by one of the aliquot parts of 100, simply reverse the rule. Thus:

Rule.—*Move the decimal point two places to the left, multiply by the denominator of the equivalent fraction in the table, and divide by the numerator.*

EXAMPLE 1.—Divide 1,844 by $16\frac{2}{3}$.

SOLUTION.—Since $16\frac{2}{3} = 100 \times \frac{1}{6}$, $1{,}844 \div 16\frac{2}{3} = 1{,}844 \times \frac{6}{100} = 110.64$. Ans. Or, applying the rule, $1{,}844 \div 16\frac{2}{3} = 18.44 \times 6 \div 1 = 110.64$. Ans.

EXAMPLE 2.—Divide 71,025 by 37.5.

SOLUTION.—Applying the rule, $71{,}025 \div 37.5 = 710.25 \times 8 \div 3 = 5{,}682 \div 3 = 1{,}894$. Ans.

53. To divide by a number having all the figures of the aliquot part, but with the decimal point in a different place, proceed in the same manner as in Art. **52.** Thus, to divide 45,000 by 625, we have $62.5 = 100 \times \frac{5}{8}$; hence, $625 = 1{,}000 \times \frac{5}{8} = \frac{5000}{8}$, and $45{,}000 \div 625 = 45{,}000 \div \frac{5000}{8}$

$$= \overset{9}{\cancel{45}},000 \times \frac{8}{\cancel{5},000} = 72.$$ Or, $45{,}000 \times \frac{8}{5000} = 45{,}000 \times \frac{1}{1000}$

$$\times \frac{8}{5} = \overset{9}{\cancel{45}} \times \frac{8}{\cancel{5}} = 72.$$ Ans.

54. The student will find the principle of aliquot parts extremely convenient for accurate and rapid work. Such numbers as $12\frac{1}{2}$, $16\frac{2}{3}$, 25, $33\frac{1}{3}$, $37\frac{1}{2}$, $62\frac{1}{2}$, 75, and $87\frac{1}{2}$ are of very frequent occurrence in business accounts, and the method can be readily employed in such cases. If these numbers are given as $.12\frac{1}{2}$, $.16\frac{2}{3}$, etc., as is usually the case, the example becomes even easier, as they are then equivalent to $\frac{1}{8}$, $\frac{1}{6}$, etc.

EXAMPLE.—What will be the cost of 48 yards of carpeting at $\$1.33\frac{1}{3}$ per yard?

SOLUTION.—Since $.33\frac{1}{3} = \frac{1}{3}$, $\$1.33\frac{1}{3} = 1\frac{1}{3}$, and $48 \times 1\frac{1}{3} = \$64$. Ans.

$$\begin{array}{r} 48 \\ 1\frac{1}{3} \\ \hline 16 \\ 48 \\ \hline \$64 \end{array}$$

EXAMPLES FOR PRACTICE.

55. Multiply:

(*a*)	5,427 by 25.	Ans. (*a*)	135,675.
(*b*)	6,301 by $12\frac{1}{2}$.	(*b*)	78,762.5.
(*c*)	42,078 by .0375.	(*c*)	1,577.925.
(*d*)	750 by $6.6\frac{2}{3}$.	(*d*)	5,000.
(*e*)	89.14 by 8,750.	(*e*)	779,975.

Divide:

(*f*)	542.7 by 75.	Ans. (*f*)	7.236.
(*g*)	90,309 by 125.	(*g*)	722.472.
(*h*)	3.1416 by $66\frac{2}{3}$.	(*h*)	.047124.
(*i*)	20,412 by 50.	(*i*)	408.24.
(*j*)	1,729 by 875.	(*j*)	1.976.

1. Find the cost of 12 dozen hats at $\$4.12\frac{1}{2}$ per dozen. Ans. \$49.50.

2. Find the cost of 24 boxes of note paper at $16\frac{2}{3}$ cents per box. Ans. \$4.

3. Find the cost of 75 books at 25 cents each. Ans. \$18.75.

4. Find the cost of 30.19 hundredweight of bran at $62\frac{1}{2}$ cents per hundredweight. Ans. $\$18.86\frac{7}{8}$.

5. Find the cost of 36 pairs of shoes at \$2.25 per pair. Ans. \$81.

6. Find the cost of 87 pounds of sugar at 5 cents per pound. Ans. \$4.35.

ARITHMETIC.

(PART 4.)

COMPOUND NUMBERS.

1. Numbers may be expressed according to a *uniform* or a *varying scale*. By **scale** is meant the relation of a unit of one order to the unit of the next higher or lower order. When the relation is the same for any two consecutive orders, the scale is said to be **uniform**; otherwise, it is **varying.** For example, the scale by which numbers are expressed in the Arabic notation is a uniform scale, since a unit of any order is 10 times as great as the unit of the next lower order; for 100 is 10 times 10, and 10 is 10 times 1, etc. The Arabic notation, the metric system, and United States money are the leading examples of the application of a uniform scale. All other numbers in commercial use in English-speaking countries require the use of a varying scale. Thus, to express 4 yards 2 feet 7 inches, it is necessary to write the words *yards*, *feet*, and *inches*, or their abbreviations, since 12 inches equal 1 foot, and 3 feet equal 1 yard.

2. A **simple number** is one which expresses one or more units of the same name or denomination; as, 5, 6 yards, etc.

3. A **compound number** is one which expresses units of two or more denominations of the *same kind*, the denominations increasing or decreasing according to a varying scale; as, 4 yards 2 feet 7 inches. But 4 yards and 5 ounces is not a compound number, since there is no relation between yards and ounces; that is, no number of ounces can equal a yard. Compound numbers are also frequently called **denominate numbers.**

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4. Compound numbers form a very important section of arithmetic. It is necessary for the student to obtain a clear idea of their use, and to be able to add, subtract, multiply, and divide them. The only real difficulty that arises is the memorizing of the tables. The best way to do this is to read them over very carefully several times, and then, after carefully studying the sections on Reduction, Addition, etc., work the examples (all of them), constantly referring to the tables for help. But, before leaving the subject to take up the next paper, he should thoroughly memorize *all* of the tables, so that when any one asks him how many square rods there are in an acre, or a similar question, he can answer instantly, without being obliged to stop and think.

5. A **measure** is a *standard unit* established by law or custom, by which the length, surface, capacity, and weight of things are estimated.

6. Measures are of six kinds, as follows:

(1) Extension.
(2) Weight.
(3) Capacity.
(4) Time.
(5) Angles or Arcs.
(6) Money or Value.

We shall consider them in the above order.

MEASURES OF EXTENSION.

7. **Measures of extension** are used in measuring lengths (distances), surfaces (areas), and solids (volumes), and are divided, accordingly, into *linear measure*, *square measure*, and *cubic measure*.

LINEAR MEASURE.

8. The standard to which all measures of extension are referred is the *yard*, which is the distance between two points on a brass bar kept at Washington. The yard is subdivided into feet and inches; and multiples of the yard are termed rods and miles. The relations between the different units are shown in the following table; the letters in Italics are the abbreviations of the names of the units.

LINEAR MEASURE.

TABLE I.

12	inches (*in.*)	= 1	foot	*ft.*
3	feet	= 1	yard	*yd.*
5½	yards	= 1	rod	*rd.*
320	rods	= 1	mile	*mi.*

in.	ft.	yd.	rd.	mi.
12 =	1			
36 =	3 =	1		
198 =	16½ =	5½ =	1	
63,360 =	5,280 =	1,760 =	320 =	1

9. The inch is usually divided into halves, quarters, eighths, and sixteenths; by civil engineers and scientists, into tenths, hundredths, thousandths, etc., and in other ways. In measuring cloth, ribbons, and other goods that are sold by the yard, the yard is divided into halves, quarters, eighths, and sixteenths.

A furlong is one-eighth of a mile, or 40 rods. The mile of 5,280 feet is a **statute mile,** so called to distinguish it from the geographical or sea mile, which equals 6,081 feet.

10. Another abbreviation frequently used for inches and feet is (″) and (′). Thus, instead of writing 4 feet 6 inches as 4 ft. 6 in., it may be written 4′ 6″; but when so written, it is customary to place a dash between the feet and inches; thus, 4′–6″. Still another way of writing the above is 4 ft. 6″.

SURVEYOR'S LINEAR MEASURE.

TABLE II.

7.92	inches (*in.*)	= 1	link	*li.*
25	links	= 1	rod	*rd.*
4 rods / 100 links		= 1	chain	*ch.*
80	chains	= 1	mile	*mi.*

in.	li.	ft.	rd.	ch.	mi.
7.92 =	1				
198 =	25 =	16½ =	1		
792 =	100 =	66 =	4 =	1	
63,360 =	8,000 =	5,280 =	320 =	80 =	1

11. Surveyor's linear measure is used in measuring land, roads, etc. The unit is a steel chain 66 feet long, and made of 100 links, all of equal length; therefore, the length of a link is $66 \times 12 \div 100 = 7.92$ inches. For railroad surveying and other purposes, civil engineers use a steel tape 100 feet long, the feet being divided into tenths and hundredths. In computations, the links are written as so many hundredths of a chain.

SQUARE MEASURE.

TABLE III.

144	square inches (*sq. in.*)	= 1	square foot	*sq. ft.*
9	square feet	= 1	square yard	*sq. yd.*
30¼	square yards	= 1	square rod	*sq. rd.*
160	square rods	= 1	acre	*A.*
640	acres	= 1	square mile	*sq. mi.*

sq. in.	sq. ft.	sq. yd.	sq. rd.	A.	sq. mi.
144 =	1				
1,296 =	9 =	1			
39,204 =	272¼ =	30¼ =	1		
6,272,640 =	43,560 =	4,840 =	160 =	1	
4,014,489,600 =	27,878,400 =	3,097,600 =	102,400 =	640 =	1

12. Square measure is used in estimating the *area* of surfaces. In commercial use, the square yard is the largest unit employed, the square rod, acre, and square mile being used for measuring land. The unit of square measure is a *square* whose sides are equal in length to the linear unit. The units of square measure are *derived units;* that is, they depend for their value upon the values of some other units, which, in this case, are the units of linear measure. If the large square in Fig. 1 measures 1 foot on each side, as indicated

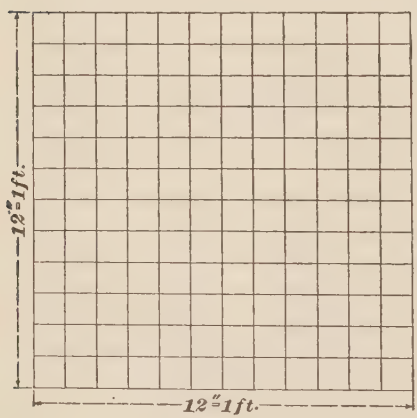

FIG. 1.

by the dimension lines, then the space which the square covers, on a flat surface, is *1 square foot*. If horizontal and vertical lines be drawn 1 inch apart, as shown, the small squares so formed will measure 1 inch on each side; and if these small squares be counted, they will be found to number 144. Hence, there are 144 square inches in 1 square foot. In the same way, it can be shown that 1 square yard contains 9 square feet. The *acre* is the only unit that forms an exception—it cannot be expressed as an exact square of any unit. A piece of land 208.71 feet square contains almost exactly an acre.

Roofers, plasterers, and carpenters frequently call 100 square feet a **square.**

SURVEYOR'S SQUARE MEASURE.

TABLE IV.

625 square links (*sq. li.*)	= 1 square rod	*sq. rd.*
16 square rods	= 1 square chain	*sq. ch.*
10 square chains	= 1 acre	*A.*
640 acres	= 1 square mile	*sq. mi.*
36 square miles (6 miles square)	= 1 township	*Tp.*

13. **Surveyor's square measure** is used only by civil engineers and surveyors. For this reason, no further remarks will be made concerning this measure.

CUBIC MEASURE.

TABLE V.

1,728 cubic inches (*cu. in.*)	= 1 cubic foot	*cu. ft.*
27 cubic feet	= 1 cubic yard	*cu. yd.*
128 cubic feet = 1 cord of wood.		

cu. in.	cu. ft.	cu. yd.
1,728 =	1	
46,656 =	27 =	1

14. **Cubic measure** is used in measuring the volumes of solids or bodies which have length, breadth, and thickness. The units of cubic measure are also derived units, since they depend upon linear measurements for their values.

The unit of cubic measure is a cube whose edges are equal in length to the corresponding linear unit. Fig. 2 represents a cube whose sides are all 3 feet long. By dividing it into equal parts, as shown, it is readily seen that 27 small cubes, measuring 1 foot on each edge, will be formed; hence, 1 cubic yard contains 27 cubic feet. In a similar manner, it can be shown that 1 cubic foot contains 1,728 cubic inches.

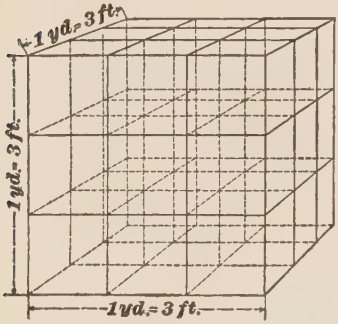

FIG. 2.

15. The **cord** is used in measuring wood. A pile of wood 8 feet long, 4 feet wide, and 4 feet high contains 1 cord, since $8 \times 4 \times 4 = 128$ cubic feet. A **cord foot** is 1 foot in length of such a pile; that is, it is 1 foot long, 4 feet wide, and 4 feet high. The cord foot contains $1 \times 4 \times 4 = 16$ cubic feet.

16. Masons use what is termed a perch. A **perch** of masonry is 1 rod ($16\frac{1}{2}$ ft.) long, $1\frac{1}{2}$ feet thick, and 1 foot high, and contains $16\frac{1}{2} \times 1\frac{1}{2} \times 1 = 24\frac{3}{4}$ cubic feet. The perch is going out of use, the cubic yard being used instead; but, when used, it is generally considered to be 25 cubic feet.

MEASURES OF WEIGHT.

AVOIRDUPOIS WEIGHT.

TABLE VI.

16 ounces (*oz.*) ..... = 1 pound ..... *lb.*
100 pounds ..... = 1 hundredweight ..... *cwt.*
20 hundredweight / 2,000 pounds ..... = 1 ton ..... *T.*

oz.	lb.	cwt.	T.
16 =	1		
1,600 =	100 =	1	
32,000 =	2,000 =	20 =	1

17. **Avoirdupois weight** is used in nearly all commercial transactions. The ton of 2,000 pounds is always understood, unless the *long* or *gross* ton is specified. Formerly, the long ton of 2,240 pounds was used exclusively, and is still used in Great Britain, by U. S. Custom Houses, in ocean freights, and by wholesale dealers in coal and iron, and iron ores. The **long ton table** is as follows:

LONG TON TABLE.

TABLE VII.

16 ounces (*oz.*) ..... = 1 pound..... *lb.*
28 pounds..... = 1 quarter..... *qr.*
4 quarters ..... = 1 hundredweight. *cwt.*
20 hundredweight / 2,240 pounds ..... = 1 ton..... *T.*

oz.	lb.	qr.	cwt.	T.
16 =	1			
448 =	28 =	1		
1,792 =	112 =	4 =	1	
35,840 =	2,240 =	80 =	20 =	1

18. Formerly, the ounce was divided into 16 parts, called drams. The dram is now seldom or never used.

TROY WEIGHT.

TABLE VIII.

24 grains (*gr.*)..... = 1 pennyweight..... *pwt.*
20 pennyweights..... = 1 ounce..... *oz.*
12 ounces ..... = 1 pound ..... *lb.*

gr.	pwt.	oz.	lb.
24 =	1		
480 =	20 =	1	
5,760 =	240 =	12 =	1

19. **Troy weight** is used by jewelers, and for the weighing of gold, silver, coins, and jewels. The Troy pound contains 5,760 grains, while the avoirdupois pound contains

7,000 grains. Hence, the Troy ounce contains $5{,}760 \div 12 = 480$ grains, and the avoirdupois ounce, $7{,}000 \div 16 = 437\frac{1}{2}$ grains. The student will find it useful to remember these facts. When it is stated that an ounce of gold is worth, say, twenty dollars, the Troy ounce, which contains 480 grains, is meant.

APOTHECARIES' WEIGHT.

TABLE IX.

20 grains (*gr.*) ..... = 1 scruple ..... *sc.* or ℈
3 scruples ..... = 1 dram ..... *dr.* or ʒ
8 drams ..... = 1 ounce ..... *oz.* or ℥
12 ounces ..... = 1 pound ..... *lb.* or ℔

gr.	℈	ʒ	℥	℔
20 =	1			
60 =	3 =	1		
480 =	24 =	8 =	1	
5,760 =	288 =	96 =	12 =	1

20. Apothecaries' weight is used by physicians and druggists in prescribing and compounding medicines not in the liquid state. When the *symbols* are used instead of the abbreviations, they are placed before the figures denoting the number of units. Thus, ʒ5 ℈2 means 5 drams 2 scruples. The pound, ounce, and grain are the same as in Troy weight.

Drugs and medicines are sold, when in large quantities, by avoirdupois weight.

MEASURES OF CAPACITY.

LIQUID MEASURE.

TABLE X.

4 gills (*gi.*) ..... = 1 pint ..... *pt.*
2 pints ..... = 1 quart ..... *qt.*
4 quarts ..... = 1 gallon ..... *gal.*
31½ gallons ..... = 1 barrel ..... *bbl.*
2 barrels, 63 gallons ..... = 1 hogshead ..... *hhd.*

gi.	pt.	qt.	gal.	bbl.	hhd.
4 =	1				
8 =	2 =	1			
32 =	8 =	4 =	1		
1,008 =	252 =	126 =	31½ =	1	
2,016 =	504 =	252 =	63 =	2 =	1

21. **Liquid measure** is used for measuring liquids. The standard gallon used in the United States is the *wine gallon*, so called to distinguish it from the *beer gallon*. The beer gallon was formerly used for measuring beer, milk, etc., but has now passed out of use. The wine gallon contains 231 cubic inches; the beer gallon contained 282 cubic inches. The gallon used in Great Britain is called the British imperial gallon; it contains 277.274 cubic inches. One wine gallon of water weighs 8.355 pounds, and 1 cubic foot contains 7.481 wine gallons. One British imperial gallon weighs 10 pounds. The student should carefully memorize these facts; he will find them very useful.

22. The barrel and hogshead are used in estimating the capacity of tanks, cisterns, reservoirs, etc. The gallon is the unit most commonly used when estimating in large quantities. The ordinary barrels and hogsheads used in commerce vary greatly in size, and their contents can be determined only by gauging or actual measurement.

APOTHECARIES' FLUID MEASURE.

TABLE XI.

60 minims, or drops, (♏)	= 1 fluidrachm.....	f℥.
8 fluidrachms	= 1 fluidounce	f℥.
16 fluidounces	= 1 pint.....	*O.*
8 pints	= 1 gallon	*Cong.*

23. **Apothecaries' fluid measure** is used in prescribing and compounding medicines. The gallon and pint are the same as the wine gallon and pint. As in apothecaries' weight, the symbols precede the numbers to which they refer. For example, Cong. 2 O. 7 f℥ 12 means 2 gallons 7 pints 12 fluidounces. *Cong.* is the abbreviation of the Latin word *congius*, gallon; *O.* is the abbreviation of the Latin word *octavius*, one-eighth.

DRY MEASURE.

TABLE XII.

2 pints (*pt.*)..... = 1 quart..... *qt.*
8 quarts..... = 1 peck..... *pk.*
4 pecks..... = 1 bushel ..... *bu.*

pt.	qt.	pk.	bu.
2 =	1		
16 =	8	= 1	
64 =	32	= 4	= 1

24. **Dry measure,** as its name implies, is used in measuring dry articles, as fruits, vegetables, grain, etc. The unit is the so called Winchester bushel, which contains 2,150.42 cubic inches, or very nearly 9.31 wine gallons. A box 14 inches long, 12.8 inches wide, and 12 inches deep (all inside measurements) contains almost exactly 1 bushel.

25. It is becoming the custom to sell by weight many articles that would ordinarily be sold by dry measure. The number of pounds of various commodities that are taken as equivalent to one bushel, varies greatly in different states; but the Boards of Trade of the principal cities of the United States use the equivalents given in the following table:

AVOIRDUPOIS POUNDS IN A BUSHEL.

TABLE XIII.

Commodities.	Lb.	Commodities.	Lb.	Commodities.	Lb.
Barley.....	48	Corn (shelled)..	56	Potatoes.....	60
Beans.....	60	Corn (in the ear)	70	Rye.....	56
Buckwheat.....	48	Malt.....	34	Timothy Seed..	45
Clover Seed.....	60	Oats.....	32	Wheat.....	60

26. The following units are also in commercial use:

1 Quintal of fish..... = 100 lb.
1 Barrel of flour..... = 196 lb.
1 Barrel of pork or beef..... = 200 lb.
1 Gallon of petroleum ..... = $6\frac{1}{2}$ lb.
1 Keg of nails..... = 100 lb.

27. **Stricken measure** means measuring the vessel even full, and striking off the surplus with a stick. Grain, seeds, berries, etc. are sold by stricken measure. The standard bushel mentioned above is a stricken bushel. The **heaped bushel** means the contents of the measuring vessel heaped up in the form of a cone. Corn in the ear, large fruits and vegetables, coal, lime, and other bulky articles, are sold by the heaped bushel. It is customary to allow 5 stricken bushels for 4 heaped ones.

In San Francisco and some other markets, produce is bought and sold by the cental, a **cental** being 100 pounds. In computing freight charges the hundredweight of 100 pounds is taken as the unit.

MEASURES OF TIME.

TABLE XIV.

60 seconds (*sec.*)	=	1 minute	*min.*
60 minutes	=	1 hour	*hr.*
24 hours	=	1 day	*da.*
7 days	=	1 week	*wk.*
4 weeks	=	1 month	*mo.*
12 months	=	1 year	*yr.*
100 years	=	1 century	*C.*

sec.	min.	hr.	da.	wk.	yr.
60 =	1				
3,600 =	60 =	1			
86,400 =	1,440 =	24 =	1		
604,800 =	10,080 =	168 =	7 =	1	
31,556,930 =	525,948 =	8,765 =	365 =	$52\frac{5}{28} = 1$	

28. The divisions of time are peculiar. The only units that may be called *natural* are the day, the lunar month, and the year, the other divisions being artificial. The time in which the earth makes one complete revolution around the sun is called a **solar year,** and it equals 365 days, 5 hours, 48 minutes, 47.5 seconds, or $365\frac{1}{4}$ days, very nearly. A **solar day** is the interval between two consecutive returns of the sun to the meridian. On account of the earth moving in an

elliptical path around the sun, the length of the solar day varies; hence, for civil purposes, the average of all the days in the year is taken as a unit. The months contain from 28 to 31 days.

In order to make the *calendar*, or *civil*, year agree as nearly as possible with the solar year, it is customary to add 1 day to the year (making 366 days) every fourth year. Years containing 366 days are called **leap years.** Every leap year is exactly divisible by 4. Thus 1896, which was a leap year, is divisible by 4 without a remainder. Any year which ends with two or more ciphers, as 1800, 1900, 2000, etc., is called a **secular year.** Since the year falls short of $365\frac{1}{4}$ days by 11 minutes and 12.5 seconds, the addition of 1 day every 4 years makes about $\frac{3}{4}$ of a day too much in a century; hence, to correct this, secular years are not leap years except when exactly divisible by 400. Thus, 1900 will not be a leap year but 2000 will. This last correction makes an error of about 1 day in 3,526 years.

29. The following is a list of months, in regular order, with the number of days which each contains:

	Days.		Days.
1. January (Jan.)	31	7. July	31
2. February (Feb.)	28	8. August (Aug.)	31
3. March (Mar.)	31	9. September (Sept.)	30
4. April (Apr.)	30	10. October (Oct.)	31
5. May	31	11. November (Nov.)	30
6. June	30	12. December (Dec.)	31

In leap years, one day is added to February, giving it 29 days. The following lines will assist the student in remembering the number of days in each month:

"Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Save February, which alone
Hath but twenty-eight, in fine,
But leap year gives it twenty-nine."

In many business transactions, the year is regarded as 360 days, or 12 months of 30 days each.

MEASURES OF ANGLES OR ARCS.

CIRCULAR MEASURE.

TABLE XV.

60 seconds (″)..... = 1 minute..... ′
60 minutes..... = 1 degree..... °
360 degrees..... = 1 circle..... ⊙

60″ = 1′
3,600″ = 60′ = 1°
1,296,000″ = 21,600′ = 360° = 1⊙

30. A **quadrant** is one-fourth of a circle, or 90°; a **sextant** is one-sixth of a circle, or 60°. A right angle (⌊) contains 90°. The unit of measurement is the degree, or $\frac{1}{360}$ of the circumference of a circle.

31. **Circular** or **angular measure** is used principally by surveyors, navigators, astronomers, and by technical men generally, for measuring angles and arcs of circles.

LONGITUDE AND TIME.

32. The earth is very nearly spherical in shape, and has a rotary movement about an imaginary line (called the axis) which is assumed to pass through the center. The two points where the axis emerges from the earth are called **poles**—one the **north pole,** the other the **south pole.** The time required for the earth to make one rotation, or complete turn, about this axis, is called one day (see Art. **28**). Midway between the poles, the earth is supposed to have a circle passing around it, called the **equator.** The equator is divided into 360 equal parts called degrees (see Table XV); each degree is divided into 60 equal parts, called minutes; and each minute is again divided into 60 equal parts, called seconds. Through each point of division a circle, called a **meridian of longitude,** is supposed to pass, the meridian also passing through the poles, as shown in Fig. 3. Other circles, called **parallels of latitude,** are supposed to be drawn parallel to the equator, and between it and the poles. By means of these imaginary circles the position of any place on the surface of the earth may be determined, this will be made plain by observing where the

parallels and meridians intersect in Fig. 3. It is evident that if we know what parallel and what meridian pass through the place, the position of the place is determined by their point of intersection. The distance between the equator and either pole is divided into 90°. The **latitude** of a place is its distance from the equator, measured in degrees,

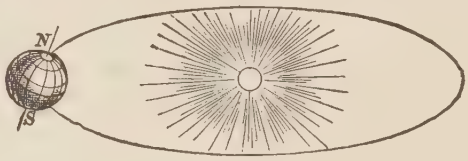

FIG. 3.

minutes, and seconds along the meridian passing through the place. The **longitude** of a place is its distance from some fixed meridian (called the **prime meridian**) in degrees, minutes, and seconds, measured along the parallel passing through the place. There are three prime meridians: that of Greenwich, that of Paris, and that of Washington, that of Greenwich being the one principally used by navigators. Longitudes reckoned east of the meridian passing through Greenwich to 180° are called **east longitudes,** and those reckoned west, **west longitudes.** Latitudes reckoned north from the equator are called **north latitudes,** and those reckoned south from the equator, **south latitudes.**

The astronomical day begins at noon, when the meridian of any place comes in direct line with the sun; but, for business and other reasons, the civil day begins twelve hours earlier, bringing what we call **noon** in the middle of the day. A little consideration of these facts will make it evident to the student that for any place situated on a meridian east of where he lives, noon will occur earlier, and for any place west of him, later, than for the place in which he is. If the longitudes of any two places are known, their difference of time, as it is called, is easily determined. For, since the earth rotates from *west* to *east* 360° in 24 hours, the sun appears to go from *east* to *west* $\frac{1}{24}$ of 360°, or 15°, in 1 hour of time. Hence, in 1 minute of time it seems to go $\frac{1}{60}$ of 15°, or 15′, and in 1 second of time, $\frac{1}{60}$ of 15′, or 15″. This is shown in the following table:

360° in longitude correspond to 24 hours of time.
15° in longitude correspond to 1 hour of time.
15′ in longitude correspond to 1 minute of time.
15″ in longitude correspond to 1 second of time.

33. **Rule.**—*To find the difference of time between two places, find the difference in longitude* in ° ′ ″, and divide by 15; the result will be their difference in time in hours, minutes, and seconds.*

34. **Rule.**—*To find the difference in longitude of two places, multiply their difference in time (solar time) expressed in hours, minutes, and seconds by 15; the result will be their difference in longitude in ° ′ ″.*

35. In the following table is given the difference in time between some important cities and Greenwich; also, the longitudes of those cities reckoned from the prime meridian passing through Greenwich:

	hr.	min.	sec.	Longitudes.
Albany	4	54	59.2	73° 44′ 48″ W.
Ann Arbor	5	34	55.1	83° 43′ 46.5″ W.
Boston	4	44	15.3	71° 3′ 49.5″ W.
Berlin	0	53	34.9	13° 23′ 43.5″ E.
Calcutta	5	55	20.7	88° 50′ 10.5″ E.
New York	4	55	53.6	Columbia College. 73° 58′ 24″ W.
New Orleans	6	0	12.9	U. S. Mint. 90° 3′ 13.9″ W.
Paris	0	9	20.9	2° 20′ 13.5″ E.
Philadelphia	5	0	38.5	75° 9′ 37.5″ W.
Rome	0	49	54.7	12° 28′ 40.5″ E.
Cincinnati	5	37	41.3	84° 25′ 19.5″ W.
Chicago	5	50	26.7	87° 36′ 40.5″ W.
Jefferson City	6	8	36	92° 9′ 0″ W.
London	0	0	22.5	5′ 37.5″ W.
City of Mexico	6	36	26.7	99° 6′ 40.5″ W.
Richmond	5	9	44	77° 26′ 0″ W.
San Francisco	8	9	38.1	122° 24′ 31.5″ W.
St. Paul	6	12	20	93° 5′ 0″ W.
St. Louis	6	0	41.1	90° 10′ 16.5″ W.
Washington	5	8	12	77° 3′ 0″ W.

Examples relating to the above will be given later. See Arts. **82–84.**

* To add, subtract, multiply, or divide compound numbers, see rules, Arts. **60, 63, 73,** and **78.**

MEASURES OF MONEY.

UNITED STATES MONEY.

TABLE XVI.

10 mills (*m.*) ..... = 1 cent ..... *ct.*
10 cents ..... = 1 dime ..... *d.*
10 dimes ..... = 1 dollar ..... *$.*
10 dollars ..... = 1 eagle ..... *E.*

m.	ct.	d.	$	E.
10 =	1			
100 =	10 =	1		
1,000 =	100 =	10 =	1	
10,000 =	1,000 =	100 =	10 =	1

36. The unit of value is the gold dollar, which weighs 25.8 grains. Since gold and silver are so soft that they would rapidly lose in weight if circulated as money in their natural state, they are alloyed with 1 part of alloy to every 9 parts of the pure metal. In other words, a gold or silver coin contains only .9 of its weight of pure gold or silver. Since 9 parts in 10 are equivalent to 900 parts in every thousand, a gold dollar is stated to contain 25.8 grains of gold 900 *fine*, the word "fine" meaning the number of parts of the pure metal in 1,000 parts by weight of the coin or alloyed metal. In business operations, the terms dollar and cent only are used, the terms eagle and dime being merely names of coins. The mill is not coined, and is rarely used except in referring to tax rates.

37. The term **legal tender** is applied to money which may be legally offered in payment of debts. All coins are legal tender up to a certain amount, depending upon the value of the coins. By legal tender is meant that if the debtor offers to pay his obligation in legal-tender money, the creditor *must* accept it. All gold coins are legal tender for their face value to any amount, provided their weight has not diminished more than $\frac{1}{200}$. Silver dollars are also legal tender to any amount; but silver coins of lower denomination than one dollar are legal tender only for sums not exceeding $10. Nickel and copper coins are legal tender for sums not exceeding 25 cents.

38. The legal coins of the United States are:

GOLD.

	Weight in grains.
1-dollar piece,	25.8
2½-dollar piece, or quarter-eagle,	64.5
3-dollar piece,	77.4
5-dollar piece, or half-eagle,	129.0
10-dollar piece, or eagle,	258.0
20-dollar piece, or double-eagle,	516.0

SILVER.

	Weight.
Standard dollar,	412½ grains.
Half-dollar, or 50-cent piece,	192.9 grains, or 12½ grams.
Quarter-dollar, or 25-cent piece,	96.45 grains, or 6¼ grams.
Dime, or 10-cent piece,	38.58 grains, or 2½ grams.

COPPER AND NICKEL.

5-cent piece,	77.16 grains, or 5 grams.
3-cent piece,	30.00 grains.
1-cent piece,	48.00 grains.

39. United States money is expressed and read decimally. Eagles and dimes are never read; only dollars and hundredths, or dollars and cents are read. Thus, 5 eagles 4 dollars 7 dimes and 3 cents would be written \$54.73 and read "fifty-four dollars and seventy-three cents." If there are more than two decimal places, the extra ones are read as decimal parts of a cent. Thus, \$54.7329 is read "fifty-four dollars and seventy-three and twenty-nine hundredths cents."

When the number of cents is less than ten, a cipher must be written in the first place to the right of the decimal point. Thus, four dollars and five cents is written \$4.05.

In checks, notes, drafts, etc., to prevent forgery and mistakes, the cents are written as hundredths of a dollar in the form of a fraction. Thus, \$13.47 would be written $\$13\frac{47}{100}$.

ENGLISH MONEY.

TABLE XVII.

4 farthings (*far.*)..... = 1 penny..... *d.*
12 pence..... = 1 shilling..... *s.*
20 shillings..... = 1 pound, or sovereign... £.

far.	d.	s.	£.
4 =	1		
48 =	12 =	1	
960 =	240 =	20 =	1

40. The unit of English money is the **pound sterling,** the value of which in United States money is $4.8665. The fineness of English silver is .925; of the gold coins, .916⅔. What is called sterling silver when applied to *solid* silver articles has the same fineness. Hence the name—sterling silver.

The other coins of Great Britain are the *florin* (= 2 shillings), the *crown* (= 5 shillings), the *half-crown* (= 2½ shillings), and the *guinea* (= 21 shillings). The largest silver coin is the crown, and the smallest, the threepence. The shilling is worth 25 cents (24.3+ cents) in United States money. The guinea is no longer coined. The abbreviation £ is written before the number, while s. and d. follow. Thus, £25 4 s. 6 d. = 25 pounds 4 shillings 6 pence.

MISCELLANEOUS TABLES.

41. The following table is used in counting certain articles:

TABLE XVIII.

12 of anything..... = 1 dozen..... *doz.*
12 dozen..... = 1 gross..... *gr.*
12 gross..... = 1 great gross... *g. gr.*
20 of anything..... = 1 score.

units	doz.	gr.	g. gr.
12 =	1		
144 =	12 =	1	
1,728 =	144 =	12 =	1

42. The following table is used in the paper trade:

TABLE XIX.

24 sheets	= 1 quire	*qr.*
20 quires	= 1 ream	*rm.*
2 reams	= 1 bundle	*bdl.*
5 bundles	= 1 bale	*B.*

sheets	qr.	rm.	bdl.	B.
24 =	1			
480 =	20 =	1		
960 =	40 =	2 =	1	
4,800 =	200 =	10 =	5 =	1

It is now becoming customary to consider 500 sheets as a ream, and to discard the higher denominations.

REDUCTION OF COMPOUND NUMBERS.

43. **Reduction** of compound numbers is the process of changing their denomination without changing their value. Reduction is divided into two cases. In one case, we reduce the number to units lower than the highest named in the number; in the other case, we reduce units of a low denomination to a higher one. The first case is called *reduction descending*, and the second, *reduction ascending*.

REDUCTION DESCENDING.

44. **Reduction descending,** or reducing units of a higher to those of a lower denomination, is performed by multiplication. Thus, to reduce 4 miles (4 mi.) to rods, yards, feet, and inches, successively, it is known that there are 320 rd. in 1 mi. (see Table I); hence, in 4 mi., there are evidently $4 \times 320 = 1{,}280$ rd. Again, since there are $5\frac{1}{2}$ yd. in 1 rd., in 1,280 rd. there are $1{,}280 \times 5\frac{1}{2} = 7{,}040$ yd. In 1 yd. there are 3 ft.; hence, in 7,040 yd., there are $7{,}040 \times 3$

= 21,120 ft. And, since in 1 ft. there are 12 in., in 21,120 ft., there are $21,120 \times 12 = 253,440$ in. All examples in reduction descending are performed in the same manner.

45. If more than one unit is given, reduce the highest unit to the next lower denomination mentioned, and then add the units of this next lower denomination to the result previously obtained. So proceed until the lowest denomination is reached. An example, which will serve as a model for the student, is here given:

EXAMPLE.—Reduce 5 mi. 47 rd. 3 yd. $1\frac{1}{2}$ ft. to feet.

SOLUTION.—

mi.	rd.	yd.	ft.
5	47	3	$1\frac{1}{2}$
320			
1600 rd.			
47			
1647 rd.			
$5\frac{1}{2}$			
8235			
$8235\frac{1}{2}$			
$9058\frac{1}{2}$ yd.			
3			
$9061\frac{1}{2}$ yd.			
3			
$1\frac{1}{2}$			
27183			
$27184\frac{1}{2}$ ft.			
$1\frac{1}{2}$			
27186 ft. Ans.			

EXPLANATION.—Since there are 320 rd. in 1 mi., multiplying 5 by 320 and adding 47 to the product will give the number of rods in 5 mi. 47 rd., or 1,647 rd. Since there are $5\frac{1}{2}$ yd. in 1 rd., multiplying 1,647 by $5\frac{1}{2}$ and adding 3 to the product will give the number of yards in 5 mi. 47 rd. 3 yd., or $9,061\frac{1}{2}$ yd. Since there are 3 ft. in 1 yd. multiplying

9,061½ by 3, and adding the 1½ to the product, will give the required result, or 27,186 ft. Ans.

46. **Rule.**—*Multiply the number of units of the highest denomination in the given compound number by the number of units of the lower denomination required to make one unit of the higher denomination, and to the product add the given number of units of the lower denomination. Proceed in this manner until the given compound number is reduced to the required denomination.*

47. In order to avoid mistakes, if any denomination be omitted between the highest and lowest denominations of the given number, represent it by a cipher.

EXAMPLE.—Reduce 40 A. 21 sq. yd. 6 sq. ft. 93 sq. in. to square inches.

SOLUTION.—

A.	sq. rd.	sq. yd.	sq. ft.	sq. in.
40	0	21	6	93
160				
6400 sq. rd.				
30¼				
1600				
192000				
193600 sq. yd.				
21				
193621 sq. yd.				
9				
1742589 sq. ft.				
6				
1742595 sq. ft.				
144				
6970380				
6970380				
1742595				
250933680 sq. in.				
93				
250933773 sq. in. Ans.				

The absence of any square rods is denoted by the cipher, as shown.

EXAMPLES FOR PRACTICE.

48. Reduce:

(*a*)	7 mi. 4 rd. 2 yd. 2 ft. to feet.	(*a*)	37,034 ft.
(*b*)	4 bu. 2 pk. 2 qt. to quarts.	(*b*)	146 qt.
(*c*)	8 lb. 4 oz. 6 pwt. 12 gr. to grains.	(*c*)	48,156 gr.
(*d*)	52 hhd. 24 gal. 1 pt. to pints.	(*d*)	26,401 pt.
(*e*)	5 ⊙ 16° 20′ 46″ to seconds.	(*e*)	6,538,846″.
(*f*)	14 bu. to quarts.	(*f*)	448 qt.
(*g*)	11 T. 9 cwt. 57 lb. to pounds.	(*g*)	22,957 lb.
(*h*)	£9 13 s. 10 d. to pence.	(*h*)	2,326 d.
(*i*)	16 cd. 112 cu. ft. to cubic feet.	(*i*)	2,160 cu. ft.
(*j*)	97 sq. rd. to square feet. Ans.	(*j*)	26,408¼ sq. ft.
(*k*)	4 yr. 8 mo. 1 wk. 3 da. to days.	(*k*)	1,578 da.
	(Count 12 mo. to the year, and 4 wk. to the month.)		
(*l*)	19 perches to cubic feet.	(*l*)	470¼ cu. ft.
(*m*)	21 cu. yd. to cubic feet.	(*m*)	567 cu. ft.
(*n*)	Cong.2 O.6 f℥10 f℥5 to fluidrachms.	(*n*)	f℥2,901.
(*o*)	13 long tons to pounds.	(*o*)	29,120 lb.
(*p*)	℔1 ℥9 ʒ6 ℈2 to grains.	(*p*)	10,480 gr.
(*q*)	Find the actual number of days in example (*k*).	(*q*)	1,711 da.
	(Count 1 leap year and 30 days to the month.)		

REDUCTION ASCENDING.

49. Reduction ascending, or reducing units of a lower to those of a higher denomination, is performed by division. Thus, to reduce 253,440 in. to feet, we must evidently divide by 12, since there are 12 in. in 1 ft. That is, 253,440 in. $= 253{,}440 \div 12 = 21{,}120$ ft. To reduce this last number to yards, we divide by 3; or, 21,120 ft. $= 21{,}120 \div 3 = 7{,}040$ yd. Continuing the process, 7,040 yd. $= 7{,}040 \div 5\frac{1}{2} = 1{,}280$ rd. $= 1{,}280 \div 320 = 4$ mi. It will be noticed that this is the reverse of the reduction of 4 mi. to inches, as in Art. **44.**

50. If, in reducing a number, as in the last article, to a higher denomination there is a remainder, reserve it and divide the quotient obtained by the division by the number of units required to make one unit of the next higher denomination. Or, carry the division farther by annexing ciphers to the remainder, thus obtaining a decimal part of the next higher unit.

EXAMPLE.—Reduce 27,186 feet to miles, rods, etc.

SOLUTION.—Instead of giving all the work of division, we shall use the short method of division to express the various steps. This will save space and make the work plainer.

$$\begin{array}{r} 3\,)\,\underline{2\,7\,1\,8\,6}\text{ ft.} \\ 5\tfrac{1}{2}\,)\,\underline{9\,0\,6\,2}\text{ yd.} \\ 3\,2\,0\,)\,\underline{1\,6\,4\,7}\text{ rd.} + 3\tfrac{1}{2}\text{ yd.} \\ 5\text{ mi.} + 47\text{ rd.} \end{array} \qquad 3\tfrac{1}{2}\text{ yd.} = 3\text{ yd. } 1\tfrac{1}{2}\text{ ft.}$$

Hence, 27,186 ft. = 5 mi. 47 rd. 3 yd. $1\frac{1}{2}$ ft. Ans. (See Art. **45.**)

EXPLANATION.—Since there are 3 ft. in 1 yd., divide 27,186 by 3 and obtain 9,062 yd. Dividing this by $5\frac{1}{2}$ to reduce yards to rods, the result is, 1,647 rd. and $3\frac{1}{2}$ yd. remaining. Write as shown. Dividing 1,647 rd. by 320, the number of rods in a mile, the result is 5 mi. and 47 rd. remaining. Hence, 27,186 ft. = 5 mi. 47 rd. $3\frac{1}{2}$ yd. But $\frac{1}{2}$ yd. = $\frac{1}{2}\times 3$ = $1\frac{1}{2}$ ft.; consequently, instead of the above, 5 mi. 47 rd. 3 yd. $1\frac{1}{2}$ ft. may be written.

51. When dividing a whole number by a mixed number, as in the above example, where 9,062 was divided by $5\frac{1}{2}$, a simple method is the following: *Multiply both dividend and divisor by the denominator of the fraction, and then divide the new dividend by the new divisor.* This will make the divisor a whole number and avoid decimals. In the above case, the denominator of the fraction was 2; $5\frac{1}{2}\times 2 = 11$, and $9{,}062\times 2 = 18{,}124$. Then, $18{,}124 \div 11 = 1{,}647 + 7$ remainder. This remainder, however, is 2 times too large; since, when the dividend was multiplied by 2, any remainder that might occur in the division was also multipled by 2. Therefore, the true remainder is $7 \div 2 = 3\frac{1}{2}$, as may be proved by dividing 9,062 by 5.5. That this method of division is correct may be easily shown. Thus, $9{,}062 \div 5\frac{1}{2} = \dfrac{9{,}062}{5\frac{1}{2}}$, which corresponds exactly to a fraction having a mixed number for a denominator. Since multiplying both numerator and denominator by the same number does not alter the *value* of the fraction, it follows that the method is correct.

52. Instead of retaining the lower units, the preceding number may be reduced to miles and decimals of a mile, as in the following example:

EXAMPLE.—Reduce 27,186 feet to miles and decimals of a mile.

SOLUTION.—The process is essentially the same as the preceding.

$$
\begin{array}{r|l}
3 & 27186 \text{ ft.} \\
5\tfrac{1}{2} & 9062 \\
11 & 18124 \\
320 & 1647.63+ \\
\end{array}
$$

5.14886+ mi. Ans.

EXPLANATION.—The work should be evident from what has preceded. Before dividing by $5\frac{1}{2}$ we multiply both dividend and divisor by 2. (See Art. **51.**) When obtaining the decimal it is not necessary to remember that the remainder 7 is $\frac{7}{2}$ yd. We simply annex a cipher to the 7 and continue the division. This is evidently correct; for $\frac{\frac{7}{2}}{5\frac{1}{2}}$ is the same as $\frac{7}{11}$. The quotient contains the figures 63 repeated indefinitely; and, when dividing by 320, we bring down first a 6 and next a 3, etc. instead of ciphers.

53. **Rule.**—*Divide the number of units of the denomination given by the number of units of that denomination that are required to make one unit of the next higher denomination. The remainder (if any) will be of the same denomination, but the quotient will be of the next higher denomination. Divide this quotient by the number of units of its denomination that are required to make one unit of the next higher denomination. Continue thus until the required denomination is reached. If it is desired to obtain a decimal fraction of the required denomination, instead of a series of remainders of lower denomination, proceed as explained in Art.* **52.**

54. The rule will be illustrated by two examples.

EXAMPLE.—Reduce 250,933,773 sq. in. to higher denominations.

SOLUTION.—

144)2509337 73 sq. in.
9)17425 95 sq. ft. + 93 sq. in.
30¼)1936 21 sq. yd. + 6 sq. ft.
121)774 484
160)64 00 sq. rd. + $\frac{84}{4}$ sq. yd. = 21 sq. yd.
40 A. 21 sq. yd. 6 sq. ft. 93 sq. in. Ans.

EXPLANATION.—Multiplying the third divisor and the third dividend by 4 to get rid of the fraction, the remainder, after division, is 84; that is, it is $\frac{84}{4}$ sq. yd. = 21 sq. yd. The remainder of the work should be clear from the preceding explanations.

EXAMPLE.—Reduce f ʒ 2,901 to gallons.

SOLUTION.—Understanding by the wording of the example that gallons and decimals of a gallon are required, the solution is as follows:

8)f ʒ 2901
16)f ʒ 362.625
8)O. 22.664 0625
Cong. 2.833 0078125, say Cong. 2.833. Ans.

EXAMPLES FOR PRACTICE.

55. Reduce all answers to the "Examples for Practice," in Art. **48**, to units of higher denomination. Also solve the following, reducing as required, and expressing the result in decimals, as in Arts. **52** and **54** (second example).

Reduce the following:

(*a*) 875 pt. to gallons.
(*b*) 10,000 pt. to bushels.
(*c*) 147,368 cu. in. to cubic yards.
(*d*) 10,000 gr. to Troy pounds.
(*e*) 8,375 d. to pounds.
(*f*) 28,140 sq. yd. to acres.
(*g*) 49,175 in. to miles.
(*h*) 380,421″ to degrees.

Ans. (*a*) 109.375 gal. (*b*) 156.25 bu. (*c*) 3.1586+ cu. yd. (*d*) 1.7361+ lb. (*e*) £34.8958+. (*f*) 5.814+ A. (*g*) 0.776+ mi. (*h*) 105.6725°.

56. In solving examples (*a*) to (*h*), Art. **55**, the table of equivalents, which accompanies every table of measures,

will be found very useful. Thus, in solving (*g*), for instance, it will be seen, by referring to Table I (lower half), that 1 mi. contains 63,360 in. Hence, (*g*) may be solved by simply dividing 49,175 by 63,360. Similarly, to solve (*f*) we find, by Table III, that 1 A. contains 4,840 sq. yd.; hence, divide 28,140 sq. yd. by 4,840. The student should practise both methods.

ADDITION OF COMPOUND NUMBERS.

57. Addition of compound numbers is similar in every respect to addition of whole numbers or of decimals, so far as the principles involved are concerned. The points of difference arise from the use of a varying scale of notation instead of a uniform scale of 10. An example will serve to show the process.

EXAMPLE.—Find the sum of 4 T. 3 cwt. 46 lb. 12 oz.; 8 cwt. 12 lb. 13 oz.; 2 T. 12 cwt. 50 lb. 13 oz.; 1 T. 27 lb. 4 oz.

SOLUTION.—

	T.	cwt.	lb.	oz.
	4	3	46	12
		8	12	13
	2	12	50	13
	1		27	4
	7	23	135	42

or 8 T. 4 cwt. 37 lb. 10 oz. Ans.

EXPLANATION.—Write the units of the same denomination in the same column, as shown, with the abbreviation of the name of the unit at the head of each column, and decreasing in order from left to right. Beginning with the right-hand column, the sum of the numbers in that column is 42, i. e., 42 oz. The sum of the numbers in the second column is 135 lb.; in the third column, 23 cwt.; in the fourth column, 7 T. Now, since 42 oz. are more than 1 lb., reduce the 42 oz. to pounds and ounces, obtaining 2 lb. 10 oz. Reserve the 10 oz., add the 2 lb. to the 135 lb. in the second column, obtaining 137 lb.; this reduced to hundredweight and pounds gives 1 cwt. 37 lb. The 1 cwt. added to the 23 cwt. in the next column gives 24 cwt., or 1 T. 4 cwt. Finally, 7 T. + 1 T. = 8 T. Hence, the sum is 8 T. 4 cwt. 37 lb. 10 oz.

58. That the process just described is the same in all respects as the addition of simple numbers will be manifest after a little consideration. Thus, suppose that we represent *thousands* by *thds.*, *hundreds* by *h.*, *tens* by *t.*, and *units* by *u.*; then the addition of 2,046, 812, 2,151, and 1,707 might be performed as follows:

	thds.	h.	t.	u.
	2		4	6
		8	1	2
	2	1	5	1
	1	7		7
	5	16	10	16

or 6 thds. 7 h. 1 t. 6 u., which equals 6,716.

It is easily seen that the same principles govern both cases.

59. Instead of writing the sum of each column separately and then reducing, as in the preceding example (Art. **57**), it is customary to add the right-hand column, and if the sum contains more units than are required to make one unit of the next higher denomination, to reduce it to the next higher denomination, placing the remainder (if any) under the right-hand column and carrying the quotient to the next column. The student will recognize this as corresponding exactly to the ordinary process of addition.

EXAMPLE.—What is the sum of 2 rd. 3 yd. 2 ft. 5 in.; 6 rd. 1 ft. 10 in.; 17 rd. 1 yd. 11 in.; 1 rd. 4 yd. 1 ft.?

SOLUTION.—

	rd.	yd.	ft.	in.
	2	3	2	5
	6		1	10
	17	1		11
	1	4	1	
	27	$4\frac{1}{2}$	0	2

or 27 rd. 4 yd. 1 ft. 8 in. Ans.

EXPLANATION.—The sum of the units in the first column is 26 in., or 2 ft. 2 in. Writing the 2 in. under the right-hand column, and carrying the 2 ft. to the next column, the result is 2 (carried) + 1 + 1 + 2 = 6 ft. = 2 yd. 0 ft. Carrying the 2 yd. to the third column, the sum is 10 yd. = 1 rd. $4\frac{1}{2}$ yd. Carrying the 1 rd. to the fourth column, the sum is 27 rd.

Now, to avoid fractions, the $\frac{1}{2}$ yd. is reduced to feet and inches, giving 1 ft. 6 in., which added to the 0 ft. and 2 in. gives for the answer 27 rd. 4 yd. 1 ft. 8 in.

60. Rule.—*Place the numbers in vertical columns so that like denominations are under each other. Begin at the right-hand column, and add. If the sum contains more units than are necessary to make one unit of the next higher denomination, reduce the sum to the next higher denomination, placing the remainder under the column added, and carrying the unit (or units) of the next higher denomination so obtained to the second column. Continue in this manner until the required denomination is reached. Should fractions of a unit be obtained, or should they occur in the numbers to be added, reduce them to units of lower denomination and add them to the sum first obtained.*

EXAMPLES FOR PRACTICE.

61. Find the sum of the following:

(*a*) 25 lb. 7 oz. 15 pwt. 23 gr.; 17 lb. 16 pwt.; 15 lb. 4 oz. 12 pwt.; 18 lb. 16 gr.; 10 lb. 2 oz. 11 pwt. 16 gr.

(*b*) 9 mi. 13 rd. 4 yd. 2 ft.; 16 rd. 5 yd. 1 ft. 5 in.; 16 mi. 2 rd. 3 in.; 14 rd. 1 yd. 9 in.

(*c*) £16 5 s. 4 d.; £12 8 s. 9 d.; £13 14 s. 8 d.; £42 7 d.; 18 s. 6 d.

(*d*) 13 cwt. 46 lb. 12 oz.; 12 cwt. $9\frac{1}{2}$ lb.; $2\frac{1}{4}$ cwt. $21\frac{5}{8}$ lb.

(*e*) 4 bu. 3 pk. 6 qt. 1 pt.; 10 bu. 2 pk. 7 qt. 1 pt.; 11 bu. 3 pk. 1 qt. 1 pt.; 9 bu. 2 pk. 5 qt. 1 pt.

(*f*) 10 yr. 8 mo. 5 wk. 3 da.; 42 yr. 6 mo. 7 da.; 7 yr. 5 mo. 18 wk. 4 da.; 17 yr. 17 da.

(*g*) 14 sq. yd. 8 sq. ft. 19 sq. in.; 105 sq. yd. 16 sq. ft. 240 sq. in.; 42 sq. yd. 28 sq. ft. 165 sq. in.

(*h*) 16 gal. 3 qt. 1 pt.; 45 gal. 2 qt.; 17 gal. 1 qt. 1 pt.; 4 gal. 3 qt.; 15 gal. 1 pt.; 24 gal. 3 qt. 1 pt.

(*i*) 16 hr. 43 min. 48 sec.; 3 hr. 12 min. 40 sec.; 1 hr. 49 min. 13 sec.; 5 hr. 19 sec.

Ans.
(*a*) 86 lb. 3 oz. 16 pwt. 7 gr.
(*b*) 25 mi. 47 rd. 1 ft. 5 in.
(*c*) £85 7 s. 10 d.
(*d*) 1 T. 8 cwt. 2 lb. 14 oz.
(*e*) 37 bu. 5 qt.
(*f*) 78 yr. 1 mo. 3 wk. 3 da.
(*g*) 5 sq. rd. 15 sq. yd. 7 sq. ft. 100 sq. in.
(*h*) 124 gal. 2 qt.
(*i*) 26 hr. 46 min.

SUBTRACTION OF COMPOUND NUMBERS.

62. The operation of subtraction is the same in principle for compound numbers as for simple numbers, the difference being due wholly to the varying scales.

EXAMPLE.—From 21 rd. 2 yd. 2 ft. $6\frac{1}{2}$ in. take 9 rd. 4 yd. $10\frac{1}{4}$ in.

SOLUTION.—

rd.	yd.	ft.	in.
21	2	2	$6\frac{1}{2}$
9	4		$10\frac{1}{4}$
11	$3\frac{1}{2}$	1	$8\frac{1}{4}$

or 11 rd. 3 yd. 2 ft. $14\frac{1}{4}$ in. = 11 rd. 4 yd. $2\frac{1}{4}$ in. Ans.

EXPLANATION.—Writing the numbers as for addition, with units of like denomination under each other, we begin with the right-hand column and subtract. Since $10\frac{1}{4}$ inches cannot be subtracted from $6\frac{1}{2}$ inches, 1 foot (= 12 inches) is borrowed from the column of feet, reduced to inches, and added to the inches in the minuend, giving $18\frac{1}{2}$ inches. Then, $18\frac{1}{2}$ in. $-10\frac{1}{4}$ in. = $8\frac{1}{4}$ in. Since 1 foot was borrowed from the 2 feet, only 1 foot remains, and, as there are no feet in the subtrahend, the 1 foot is brought down in the remainder. As 4 yards cannot be taken from 2 yards, 1 rod is borrowed from the rod column, reduced to yards, and added to the 2 yards, giving $7\frac{1}{2}$ yards; then, $7\frac{1}{2}$ yd. -4 yd. = $3\frac{1}{2}$ yd. Finally, 20 rd. -9 rd. = 11 rd., and the remainder is 11 rd. $3\frac{1}{2}$ yd. 1 ft. $8\frac{1}{4}$ in. Reducing the $\frac{1}{2}$ yd., the remainder becomes 11 rd. 3 yd. 2 ft. $14\frac{1}{4}$ in.; or, since $14\frac{1}{4}$ in. = 1 ft. $2\frac{1}{4}$ in., and 2 ft. + 1 ft. = 3 ft. = 1 yd., the remainder is 11 rd. 4 yd. $2\frac{1}{4}$ in.

63. **Rule.**—*Place the less quantity under the greater quantity, with units of like denomination under each other. Beginning at the right, subtract successively the units in the subtrahend in each denomination from those above, and place the several remainders beneath, as in simple subtraction. If the units of any denomination in the minuend are fewer than units of the same denomination in the subtrahend, borrow one unit from the next higher denomination in the minuend, reduce it to the next lower denomination, and add it to the proper number in the minuend; then subtract as before. Reduce fractional results in the remainder if there are any.*

64. Example.—From ℔4 ℈1 take ℔1 ℥5 ℈2 16 gr.

Solution.—

℔	℥	ʒ	℈	gr.	
4	0	0	1	0	
1	5	0	2	16	
2	6	7	1	4	Ans.

Explanation.—Since 16 grains cannot be taken from 0 grain, ℈1 is borrowed from the second column. ℈1 = 20 gr., and 20 gr. − 16 gr. = 4 gr. As ℈2 cannot be taken from ℈0, and there are no drams or ounces, ℔1 is borrowed from the fifth column. ℔1 = ℥11 ʒ7 ℈3; hence, subtracting ℈2 from ℈3, ʒ0 from ʒ7, ℥5 from ℥11, and ℔1 from ℔3, the remainder, or answer, is ℔2 ℥6 ʒ7 ℈1 4 gr.

65. The only case of subtraction that need cause any trouble is the finding of the difference between two dates. If great exactness is not required, put down the year, the number of the month, and the day of the month in the case of both minuend and subtrahend. Then, counting 30 days to the month, subtract as above.

Example.—How many years, months, and days between September 28, 1868, and June 15, 1891?

Solution.—

yr.	mo.	da.	
1891	6	15	
1868	9	28	
22	8	17	Ans.

Explanation.—June of the minuend is the 6th month, and September of the subtrahend is the 9th month. Writing the minuend and subtrahend as shown, we subtract as above. The result is 22 years, 8 months, 17 days. Ans.

66. Had it been required to find the exact number of days between the two given dates, the above method would not give the correct result.

67. The following method is employed by most banks and by the United States government:

From Sept. 28, 1868, to Sept. 28, 1890 = 22 yr.
From Sept. 28, 1890, to June 15, 1891, in days is
Sept. Oct. Nov. Dec. Jan. Feb. Mar. Apr. May June
2 + 31 + 30 + 31 + 31 + 28 + 31 + 30 + 31 + 15 = 260 da.

Ans.

EXPLANATION.—The exact whole number of years between the dates is first found; it equals 22 years. The remaining days are then found as shown.

68. The following table will be of great assistance in determining the actual number of days between two dates. The table gives the number of days between the same dates of any two months. Thus, to find the number of days between Mar. 12 and Sept. 12 of any year, we find opposite Mar. in the left-hand column and in the column headed Sept. the number 184, the required number of days. Had it been required to find the number of days between Mar. 12 and

TABLE XX.

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Jan.	365	31	59	90	120	151	181	212	243	273	304	334
Feb.	334	365	28	59	89	120	150	181	212	242	273	303
Mar.	306	337	365	31	61	92	122	153	184	214	245	275
Apr.	275	306	334	365	30	61	91	122	153	183	214	244
May.	245	276	304	335	365	31	61	92	123	153	184	214
June.	214	245	273	304	335	365	30	61	92	122	153	183
July.	184	215	243	274	304	335	365	31	62	92	123	153
Aug.	153	184	212	243	273	304	334	365	31	61	92	122
Sept.	122	153	181	212	242	273	303	334	365	30	61	91
Oct.	92	123	151	182	212	243	273	304	335	365	31	61
Nov.	61	92	120	151	181	212	242	273	304	334	365	30
Dec.	31	62	90	121	151	182	212	243	274	304	335	365

Sept. 25, we should have found the number of days between Mar. 12 and Sept. 12, or 184 days; then subtracting 12 from 25, the difference, 13, must be added to 184, obtaining 197 days, the number of days between Mar. 12 and Sept. 25. Had it been required to find the number of days between

Sept. 25 and Mar. 12, we should find opposite Sept., and in the column headed Mar., 181; then subtracting 12 from 25, we subtract the difference from 181, because 181 days is the number of days between Sept. 25 and Mar. 25, instead of Mar. 12, which occurs 13 days earlier. Hence, there are $181 - 13 = 168$ days between Sept. 25 and Mar. 12. Had Mar. 12 occurred in a leap year, there would have been one day more, or 169 days between the two dates, on account of Feb. 29.

The table will also be useful in those cases where a certain number of days is to be added to a given date. Thus, to find the date of 90 days after Feb. 18, we see, on referring to the table, that 89 days after Feb. 18 is May 18; hence, 90 days after Feb. 18 is May 19, or, if it is a leap year, May 18. Again, 127 days after Feb. 19 is June 26; because, referring to the table, 120 days after Feb. 19 is June 19, and $127 - 120 + 19 = 26$.

If it is desired to subtract a certain number of days from a given date, the process is simply reversed. To find, for example, the date 120 days previous to Sept. 21, we look down the column headed Sept. and find opposite May the number 123; hence, from May 21 to Sept. 21 is 123 days, and therefore from May 24 to Sept. 21 is 120 days.

EXAMPLES FOR PRACTICE.

69. Solve the following examples:

(*a*) From £10 6 s. 4 d. take £8 15 s. 3 d.

(*b*) From 125 lb. 8 oz. 14 pwt. 18 gr. take 96 lb. 9 oz. 10 pwt. 4 gr.

(*c*) From 16 yr. 8 mo. 10 da. take 12 yr. 5 mo. 8 da.

(*d*) From 126 hhd. 27 gal. take 104 hhd. 14 gal. 1 qt. 1 pt.

(*e*) Find the exact number of days between Sept. 20, 1895, and Mar. 17, 1897, inclusive.

(*f*) From 65 T. 14 cwt. 64 lb. 10 oz. take 16 T. 11 cwt. 14 oz.

(*g*) From 148 sq. yd. 16 sq. ft. 102 sq. in. take 132 sq. yd. 136 sq. in.

(*h*) Subtract 28 bu. 2 pk. 5 qt. 1 pt. from 100 bu.

(*i*) Subtract 3 mi. 27 rd. 11 yd. 4 ft. 10 in. from 14 mi. 34 rd. 16 yd. 13 ft. 11 in.

(*j*) Subtract 27° 19′ 47″ from 126° 37′ 23″.

(*k*) Referring to Art. **35**, find difference of longitude between Boston and San Francisco.

(*l*) Find difference of longitude between Albany and Calcutta.

Answers.—(*a*) £1 11s. 1d.; (*b*) 28 lb. 11 oz. 4 pwt. 14 gr.; (*c*) 4 yr. 3 mo. 2 da.; (*d*) 22 hhd. 12 gal. 2 qt. 1 pt.; (*e*) 545 da.; (*f*) 49 T. 3 cwt. 63 lb. 12 oz.; (*g*) 16 sq. yd. 15 sq. ft. 110 sq. in.; (*h*) 71 bu. 1 pk. 2 qt. 1 pt.; (*i*) 11 mi. 8 rd. 2 yd. 1 ft. 7 in.; (*j*) 99° 17′ 36″; (*k*) 51° 20′ 42″; (*l*) 162° 34′ 58.5″.

MULTIPLICATION OF COMPOUND NUMBERS.

70. The multiplication of compound numbers is similar in all respects to multiplication of simple numbers. The process will be illustrated by an example.

EXAMPLE.—A merchant divided his syrup into 12 equal parts, each containing 2 gal. 2 qt. 1½ pt.; how much did he have altogether?

SOLUTION.—He evidently had 12 times 2 gal. 2 qt. 1½ pt., or

gal.	qt.	pt.	
2	2	1½	
		12	
24	24	18	= 32 gal. 1 qt. Ans.

EXPLANATION.—Multiplying the units of each denomination by 12, and reducing the units of each denomination of the products to higher denominations, the result is 32 gal. 1 qt.

71. Such examples are usually solved as follows:

gal.	qt.	pt.	
2	2	1½	
		12	
32	1	0	Ans.

EXPLANATION.— 1½ pt. × 12 = 18 pt. = 9 qt.; reserve this, and add to the quarts product. 2 qt. × 12 + 9 qt. = 33 qt. = 8 gal. 1 qt. Write the 1 qt. and reserve the 8 gal. 2 gal. × 12 + 8 gal. = 32 gal. Hence, the answer is 32 gal. 1 qt.

72. When the multiplier contains a fraction, it is usually easier to reduce the multiplicand to the *lowest* denomination before multiplying, and then reduce the product to higher denominations. In any case, the method used in Art. **70** is to be preferred to that given in Art. **71**, when the multiplier contains a fraction. All three methods will be applied to an example to illustrate the point.

EXAMPLE.—Multiply 7 lb. 5 oz. 13 pwt. 15 gr. by $5\frac{1}{2}$.

SOLUTION.—*First Method*—

	lb.	oz.	pwt.	gr.	
	7	5	13	15	
				$5\frac{1}{2}$	
	$38\frac{1}{2}$	$27\frac{1}{2}$	$71\frac{1}{2}$	$82\frac{1}{2}$	
or	38	33	81	$94\frac{1}{2}$	
or	41 lb.	1 oz.	4 pwt.	$22\frac{1}{2}$ gr.	Ans.

Second Method—

	lb.	oz.	pwt.	gr.	
	7	5	13	15	
				$5\frac{1}{2}$	
	$40\frac{1}{2}$	$6\frac{1}{2}$	$14\frac{1}{2}$	$10\frac{1}{2}$	
or	40	12	24	$22\frac{1}{2}$	
or	41 lb.	1 oz.	4 pwt.	$22\frac{1}{2}$ gr.	Ans.

Third Method— 7 lb. 5 oz. 13 pwt. 15 gr. = 43,047 gr. (if the lowest denomination given had not been grains, we should still have reduced to grains). Hence, 43,047 gr. $\times$ 5.5 = 236,758.5 gr. = 41 lb. 1 oz. 4 pwt. $22\frac{1}{2}$ gr. Ans.

73. **Rule.**—*Multiply the units of the lowest denomination by the multiplier and reduce the product to the next higher denomination. Multiply the units of the next higher denomination by the multiplier, and add to the product the result obtained by the first operation. So continue with the remaining units. The last result, together with the various remainders, is the entire product. Should the entire product contain fractions, reduce the fractions of a unit to lower denominations, as in Art.* **72.**

Or, reduce the multiplicand to its lowest denomination; perform the multiplication, and then reduce the product to higher denominations.

EXAMPLES FOR PRACTICE.

74. Solve the following:

(*a*) Multiply £17 10 s. 8 d. by 7; by 9; by 15.

(*b*) How many cords of wood in 12 loads, each load containing 2 cd. 108 cu. ft.?

(*c*) Find the weight of 2 dozen silver spoons, each spoon weighing 1 oz. 13 pwt. What would they cost at 6 cents per pennyweight?

(*d*) If 15 men perform a certain piece of work in 3 da. 16 hr. 52 min., how long would it take one man to perform the work?

(*e*) Multiply 3 T. 15 cwt. 90 lb. by 5.

(*f*) Multiply 4 hhd. 3 gal. 1 qt. 1 pt. by 12.

(*g*) At $2.16 per gallon what would be the cost of Cong.2 O.6 f℥10 of a certain drug?

(*h*) What would be the cost of 5 bu. 3 pk. 6 qt. of potatoes at 48 cents per bushel?

(*i*) Multiply 6 A. 114 sq. rd. 19 sq. yd. 53 sq. ft. by 13.

Ans. (*a*) £122 14 s. 8 d.; £157 16 s.; £263.
(*b*) 34 cd. 16 cu. ft.
(*c*) 3 lb. 3 oz. 12 pwt.; $47.52.
(*d*) 55 da. 13 hr.
(*e*) 18 T. 19 cwt. 50 lb.
(*f*) 48 hhd. 1 bbl. 9 gal.
(*g*) $6.11.
(*h*) $2.85.
(*i*) 87 A. 52 sq. rd. 21 sq. yd. ½ sq. ft.

DIVISION OF COMPOUND NUMBERS.

75. There are two cases of division of compound numbers. In the first case, the divisor is an abstract number; in the second case, the divisor is itself a compound number. When the divisor is an abstract number, the division may be conveniently performed as in the following examples, Arts. **76–78,** inclusive.

76. Example.—Divide 48 lb. 11 oz. 6 pwt. by 8.

Solution.—

	lb.	oz.	pwt.	gr.
8)	48	11	6	0
	6 lb.	1 oz.	8 pwt.	6 gr. Ans.

Explanation.—After placing the quantities as above, proceed as follows: 8 is contained in 48 six times without a remainder. 8 is contained in 11 oz. once with 3 oz. remaining. $3 \times 20 = 60$; $60 + 6 = 66$ pwt.; 66 pwt. $\div 8 = 8$ pwt. and 2 pwt. remaining; 2×24 gr. $= 48$ gr.; 48 gr. $\div 8 = 6$ gr. Therefore, the entire quotient is 6 lb. 1 oz. 8 pwt. 6 gr. Ans.

EXAMPLE.—A silversmith melted up 2 lb. 8 oz. 10 pwt. of silver, which he made into 6 soup ladles; what was the weight of each?

SOLUTION.—

	lb.	oz.	pwt.
6)	2	8	10
		5 oz.	8 pwt. 8 gr. Ans.

EXPLANATION.—Since we cannot divide 2 lb. by 6, we reduce them to ounces. 2 lb. = 24 oz., and 24 oz. + 8 oz. = 32 oz.; 32 oz. ÷ 6 = 5 oz. and 2 oz. over. 2 oz. = 40 pwt. 40 pwt. + 10 pwt. = 50 pwt., and 50 pwt. ÷ 6 = 8 pwt. and 2 pwt. over. 2 pwt. = 48 gr., and 48 gr. ÷ 6 = 8 gr. Hence, each ladle weighs 5 oz. 8 pwt. 8 gr. Ans.

77. EXAMPLE.—Divide 820 rd. 4 yd. 2 ft. by 112.

SOLUTION.—

rd.
112) 820 (7 rd.
36 rd.
5½
198 yd.
4
112) 202 yd. (1 yd.
90 yd.
3
270 ft.
2
112) 272 ft. (2 ft.
48 ft.
12
112) 576 in. ($5\frac{16}{112} = 5\frac{1}{7}$ in.
16 in.

EXPLANATION.—We divide as in long division, using the short method. The first quotient is 7 rd. with a remainder of 36 rd., which = 198 yd. 198 yd. + 4 yd. = 202 yd.; 202 yd. ÷ 112 = 1 yd. with a remainder of 90 yd., which = 270 ft. 270 ft. + 2 ft. = 272 ft.; 272 ft. ÷ 112 = 2 ft. with a remainder of 48 ft., which = 576 in. 576 in. ÷ 112 = $5\frac{1}{7}$ in.

If desired, $\frac{1}{7}$ in. may be reduced to a decimal in the manner already explained, = .1428+ in. The common fractional form is, however, better than the decimal form, since if the quotient be multiplied by the divisor the result will then be *exactly* the same as the dividend.

78. **Rule.**—*Find how many times the divisor is contained in the first or highest denomination of the dividend. Reduce the remainder (if any) to the next lower denomination, and add to it the number in the dividend expressing that denomination. Divide this new dividend by the divisor. The quotient will be the next denomination in the quotient required. Continue in this manner until the lowest denomination is reached. The successive quotients will constitute the entire quotient.*

EXAMPLES FOR PRACTICE.

79. Divide:

(*a*) 376 mi. 276 rd. by 22; (*b*) 1,137 bu. 3 pk. 4 qt. 1 pt. by 10; (*c*) 84 cwt. 48 lb. 49 oz. by 16; (*d*) 78 sq. yd. 18 sq. ft. 41 sq. in. by 18; (*e*) 148 mi. 64 rd. 24 yd. by 12; (*f*) 100 T. 16 cwt. 18 lb. 11 oz. by 15; (*g*) 36 lb. 18 oz. 18 pwt. 14 gr. by 8; (*h*) 112 mi. 48 rd. by 100.

Ans. (*a*) 17 mi. 41 rd. 3 yd. 1 ft. 6 in.
(*b*) 113 bu. 3 pk. 1 qt. $\frac{1}{2}$ pt.
(*c*) 5 cwt. 28 lb. $3\frac{1}{16}$ oz.
(*d*) 4 sq. yd. 4 sq. ft. $2\frac{5}{18}$ sq. in.
(*e*) 12 mi. 112 rd. 2 yd.
(*f*) 6 T. 14 cwt. 41 lb. $3\frac{14}{15}$ oz.
(*g*) 4 lb. 8 oz. 7 pwt. $7\frac{3}{4}$ gr.
(*h*) 1 mi. 38 rd. 4 yd. 2 ft. 6.24 in.

80. When the divisor is a compound number, the easiest and best way to perform the division is to reduce both numbers to the lowest denomination given in either the dividend or divisor; then divide as in whole numbers.

81. EXAMPLE.—How many bottles, each holding 1 qt. $\frac{1}{2}$ pt., can be filled from a cask holding 21 gal. 3 qt.?

SOLUTION.—In 1 qt. $\frac{1}{2}$ pt. there are $2\frac{1}{2}$ pt.; in 21 gal. 3 qt. there are 174 pt. Then, 174 pt. $\div 2\frac{1}{2}$ pt. = 69 bottles and $1\frac{1}{2}$ pt. left over. Ans.

LONGITUDE AND TIME—(Continued.)

82. We are now prepared to find the difference of time, or the difference of longitude, between two places. For this purpose we use the rules given in Arts. **33** and **34.**

EXAMPLE.—What is the difference of time between New York and Chicago?

SOLUTION.—In Art. **35** the longitude of New York is stated to be 73° 58′ 24″ W., and of Chicago 87° 36′ 40½″ W. Applying the rule given in Art. **33,**

87°	36′	40½″ W.
73°	58′	24 ″ W.
13°	38′	16½″ W.

Dividing 13° 38′ 16½″ by 15 the result is

15) 13° 38′ 16.5″

54 min. 33.1 sec.

EXAMPLE.—Find the difference of time between Albany and Rome.

SOLUTION.—Applying rule, Art. **33,** difference of longitude equals

73°	44′	48″ W.
12°	28′	40.5″ E.
86°	13′	28.5″

15) 86° 13′ 28.5″

5 hr. 44 min. 53.9 sec. Ans.

We must evidently *add* to find the difference, since Rome is *east* of Greenwich, or the prime meridian, and Albany is *west*. Were both places on the *same* side, both east or both west, we should subtract.

83. The longitude of a place is determined by means of a very accurate watch, called a *chronometer*, and by observation of the sun or stars. The watch is set for Greenwich time, and an observation of the sun is taken for the place whose longitude it is desired to find. By aid of suitable instruments it can be determined when it is exactly noon at any place, and by looking at the watch, the difference of time between noon and the time indicated by the watch will be the difference of time between the place at which the observation is taken and Greenwich. If the watch appears to be *slow*, the longitude is *east;* if *fast*, it is *west*. Knowing the difference of time, the longitude is easily found by the rule given in Art. **34.**

84. EXAMPLE.—A watch set to Greenwich time appeared to be 6 hr. 10 min. 41 sec. fast at a certain place. What was the longitude of the place?

SOLUTION.—Applying rule, Art. **34,**

6 hr.	10 min.	41 sec.
		15
92°	40′	15″

Since the watch appeared fast, the longitude was 92° 40′ 15″ W.

EXAMPLES FOR PRACTICE.

85. Find the difference of time between:

(*a*) London and City of Mexico. Ans. (*a*) 6 hr. 36 min. $4\frac{1}{5}$ sec.

(*b*) Paris and Philadelphia. Ans. (*b*) 5 hr. 9 min. $59\frac{2}{5}$ sec.

(*c*) Richmond, Va., and San Francisco. Ans. (*c*) 2 hr. 59 min. $54\frac{1}{10}$ sec.

(*d*) Boston and Ann Arbor. Ans. (*d*) 50 min. $39\frac{4}{5}$ sec.

(*e*) London and Calcutta. Ans. (*e*) 5 hr. 55 min. $43\frac{1}{5}$ sec.

Find the longitude when the watch is apparently

(*f*) Fast 7 hr. 43 min. 11 sec.

(*g*) Slow 1 hr. 0 min. 49 sec.

(*h*) Slow 4 hr. 37 min. 6 sec.

(*i*) Fast 8 hr. 19 min. 24 sec.

Ans. (*f*) 115° 47′ 45″ W.; (*g*) 15° 12′ 15″ E.; (*h*) 69° 16′ 30″ E.; (*i*) 124° 51′ W.

The student should verify the table of times given in the table, Art. **35.**

ARITHMETIC.

(PART 5.)

THE METRIC SYSTEM.

1. In the metric system, a uniform scale of 10 is used throughout, as in the ordinary scale of numbers and in United States money. The name is derived from the meter (from the Greek word *metron*, a measure), the unit from which all the other units are derived. The use of the metric system was made legal in the United States in 1866, but has not yet been made compulsory; it is used by scientists throughout the world.

2. The **meter** is very nearly one ten-millionth part of the distance from the equator to the pole, and has been officially declared by the United States Government to equal 39.37 inches, which corresponds very nearly to the distance above mentioned.

3. The metric system has three principal units, the only ones we shall consider, which are: the **meter** (pronounced meeter), the unit of length; the **liter** (pronounced leeter), the unit of capacity; and the **gram**, the unit of weight. Each of the units has its multiples and subdivisions.

4. The names of the denominations higher than the leading unit are obtained by prefixing to the name of the unit the Greek names *dck'a*, *hck'to*, *kil'o*, and *myr'ia*. Thus, taking the meter as the unit, we write

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Dek'a me'ter, 10 meters, deka meaning 10;
Hek'to me'ter, 100 meters, hekto meaning 100;
Kil'o me'ter, 1,000 meters, kilo meaning 1,000;
Myr'ia me'ter, 10,000 meters, myria meaning 10,000.

5. The names of the denominations lower than the leading unit are obtained by prefixing to the name of the unit the Latin names *dec'i*, *cent'i*, and *mil'li*. Thus,

Dec'i me'ter, $\frac{1}{10}$ meter, deci meaning $\frac{1}{10}$;
Cent'i me'ter, $\frac{1}{100}$ meter, centi meaning $\frac{1}{100}$;
Mil'li me'ter, $\frac{1}{1000}$ meter, milli meaning $\frac{1}{1000}$.

The prefixes and their meanings, as given in this and the preceding article, should be carefully committed to memory.

MEASURES OF EXTENSION.

MEASURES OF LENGTH.

TABLE I.

10 millimeters (*mm.*)	= 1 centimeter....	*cm.*	= .3937 in.
10 centimeters	= 1 decimeter.....	*dm.*	= 3.937 in.
10 decimeters.....	= 1 meter.....	*m.*	= 3.28 ft.
10 meters.....	= 1 dekameter....	*Dm.*	= 32.8 ft.
10 dekameters.....	= 1 hektometer...	*Hm.*	= 328.09 ft.
10 hektometers.....	= 1 kilometer.....	*Km.*	= .62137 mi.
10 kilometers.....	= 1 myriameter...	*Mm.*	= 6.2137 mi.

It will be noticed that the abbreviations of the names of the units of the higher denominations begin with a *capital* letter, and those of the lower denominations, with a small letter. A decimeter divided into centimeters and millimeters is shown in the figure below.

6. The decimeter, dekameter, hektometer, and myriameter are rarely used. The meter is used for those measurements that would ordinarily be made in feet and yards; the centimeter and millimeter for those ordinarily made in inches and hundredths of an inch, and the kilometer for long distances, such as we ordinarily express in miles.

7. The *approximate* length of the meter is 40 inches, or, more closely, $39\frac{3}{8}$ inches (39.4 inches is a closer approximation). When great exactness is not required, 40 inches is near enough. Hence, 1 decimeter equals $40 \times \frac{1}{10} = 4$ inches; 1 centimeter = .4 inch, and 1 millimeter = .04 inch. The length of 1 kilometer = $\frac{5}{8}$ mile, approximately.

8. Since meters, centimeters, and millimeters have a decimal scale like dollars, cents, and mills, they may be read in a similar way. Thus, 41.457 meters may be read as 41 and 457-thousandths meters, or as 41 meters 45 centimeters 7 millimeters. It might also be read 41 meters 45 and 7-tenths centimeters, or 41 meters 457 millimeters.

9. To express metric numbers decimally in terms of a given unit:

EXAMPLE.—Express 4 Km. 3 Hm. 1 m., 5 cm. and 9 mm. in meters.

SOLUTION.—We write the meters in units place, at the left of the decimal point; then, writing the other units in order, supplying the missing denominations (if any) with ciphers, we have 4301.059 meters. Ans.

Had it been required to express the above in kilometers, the result would have been 4.301059 Km.

10. Rule.—*Write the number of given units; then, the numbers of the higher denominations on the left, as integers, and those of the lower denominations on the right, as decimals, supplying any missing denomination with a cipher.*

11. To reduce a metric number to higher or lower denominations, all that is necessary is to move the decimal point. Thus, 74.1026 Km. = 741.026 Hm. = 7,410.26 Dm. = 74,102.6 m. = 741,026 dm. = 7,410,260 cm. = 74,102,600 mm. Also, to reduce 97,452 cm. to kilometers, write 97,452 cm. Now, beginning with the decimal point (which, of course, follows the 2), count centi, deci, meter, deka, hekto, kilo, placing the decimal point after the last-named unit, and supplying any missing ones with ciphers. The result is 0.97452 Km.

EXAMPLES FOR PRACTICE.

12. Solve the following:

(*a*) Reduce 25.7 Km. to meters.
(*b*) Reduce 43.4 m. to millimeters.
(*c*) Reduce 4,823.6 m. to hektometers.
(*d*) Reduce 48,639 cm. to meters.
(*e*) Reduce 738.4 Dm. to centimeters.

Read the following:

(*f*) 14.5 m.
(*g*) 47.3 Dm.
(*h*) 568 Hm.
(*i*) 434.5 Km.
(*j*) 27.4 mm.
(*k*) 92.76 cm.

Answers.—(*a*) 25,700 m.; (*b*) 43,400 mm.; (*c*) 48.236 Hm.; (*d*) 486.39 m.; (*e*) 738,400 cm.; (*f*) 14 and 5-tenths meters; (*g*) 47 and 3-tenths dekameters; (*h*) 568 hektometers; (*i*) 434 and 5-tenths kilometers; (*j*) 27 and 4-tenths millimeters; (*k*) 92 and 76-hundredths centimeters.

SQUARE MEASURE.

TABLE II.

100 square millimeters (*sq. mm.* or mm^2.) =	1 square centimeter ..	*sq. cm.* or cm^2.
100 square centimeters.... =	1 square decimeter ...	*sq. dm.* or dm^2.
100 square decimeters..... =	1 square meter	*sq. m.* or m^2.
	or centare.....	*ca.*
100 square meters (*100 centares*) ..... =	1 square dekameter	*sq. Dm.* or Dm^2.
	or are.....	*A.*
100 square dekameters (*100 ares*) .. =	1 square hektometer	*sq. Hm.* or Hm^2.
	or hektare	*Ha.*
100 square hektometers (*100 hektares*) .. =	1 square kilometer....	*sq. Km.* or Km^2.

COMMON EQUIVALENTS.

1 sq. cm. = 0.1550 sq. in.
1 sq. dm. = 0.1076 sq. ft.
1 sq. m. = 1.1960 sq. yd. = 10.7637 sq. ft.
1 are = 3.954 sq. rd.
1 hektare = 2.471 A.
1 sq. Km. = 0.3861 sq. mi.

13. In square measure, the scale is 100 (10×10); hence, in reducing units of any denomination to a lower or higher denomination, the decimal point must be moved two places for each denomination. Thus, to reduce 42.09872 sq. m. to square centimeters, begin at the decimal point, point off two places to the right and say square decimeters, then two more places and say square centimeters, obtaining 420,987.2 sq. cm. Had square millimeters been desired, it would have been necessary to annex a cipher; thus, 42.09872 sq. m. = 42,098,720 sq. mm. Reduction to higher denominations is performed in the same manner, moving the decimal point to the left. Thus, 42,098,720 mm^2. = 0.4209872 ares.

14. The square meter is used in measuring floors, ceilings, and other ordinary surfaces; the are and hektare in measuring land, and the square kilometer in measuring states and territories.

EXAMPLES FOR PRACTICE.

15. Solve the following:

(*a*) Write 78.29 ares as centares; also as hektares.
(*b*) Write 9 m^2. as square decimeters; also as square centimeters.
(*c*) In 3,246 ca. how many ares?
(*d*) Express 7,041.6 sq. dm. in ares.

Answers.—(*a*) 7,829 ca. or .7829 Ha.; (*b*) 900 dm^2. or 90,000 cm^2.; (*c*) 32.46 A.; (*d*) .70416 A.

CUBIC MEASURE.

TABLE III.

1,000 cubic millimeters (*cu. mm.* or mm^3.) = 1 cubic centimeter.... *cu. cm.* or cm^3.
1,000 cubic centimeters..... = 1 cubic decimeter..... *cu. dm.* or dm^3.
1,000 cubic decimeters..... = 1 cubic meter..... *cu. m.* or m^3.

COMMON EQUIVALENTS.

1 cu. cm. = .06102 cu. in.
1 cu. dm. = 61.023 cu. in.
1 cu. m. = 61,023.4 cu. in. = 35.3145 cu. ft. = 1.308 cu. yd.

In measuring wood, the cubic meter is called a *stere*.

16. Units higher than the cubic meter are not used, except in denoting the volume of planets.

In cubic measure, the scale is 1,000 ($10 \times 10 \times 10$); hence, to reduce units from one denomination to another, apply the method given in Art. **13,** moving the decimal point three places each time, instead of two places, as in square measure.

MEASURES OF CAPACITY.

TABLE IV.

10 mil'li li'ters (*ml.*)	= 1 cen'ti li'ter	*cl.*
10 centiliters	= 1 dec'i li'ter	*dl.*
10 deciliters	= 1 li'ter	*l.*
10 liters	= 1 dek'a li'ter	*Dl.*
10 dekaliters	= 1 hek'to li'ter	*Hl.*
10 hektoliters	= 1 kil'o li'ter	*Kl.*
10 kiloliters	= 1 myr'ia li'ter	*Ml.*

COMMON EQUIVALENTS.

1 liter	= 61.023 cu. in.
1 liter	= 1.0567 liquid quarts.
1 liter	= 0.9078 dry quarts.
1 hektoliter	= 3.53144 cu. ft.
1 hektoliter	= 26.417 gallons.
1 hektoliter	= 2.8378 bushels.

17. The **liter** is equal in volume to 1 cubic decimeter, i. e., to a cube whose edges measure 1 decimeter on a side. The liter is the principal unit in measures of capacity, and is used for both *dry* and *liquid* measure; it is very nearly equal to a liquid quart.

18. One milliliter is equal in volume to 1 cubic centimeter, since 1 cu. cm. $= \frac{1}{1000}$ cu. dm. $= \frac{1}{1000}$ liter $= 1$ milliliter.

The centiliter is a little more than $\frac{1}{2}$ gill; it is used for measuring small quantities of liquids, as medicines. The liter is used for the same purposes as the quart, and the hektoliter for the same purposes as the gallon and the bushel.

The units are reduced from one denomination to another in the same way as measures of length. See Art. **11.**

EXAMPLES FOR PRACTICE.

19. Solve the following:

(*a*) Express 8.53 l. as centiliters; as deciliters.
(*b*) Express 4.64 Kl. as liters; as hektoliters.
(*c*) How many deciliters in 8 liters? In 9.35 liters?
(*d*) How many liters in 6.358 cl.?
(*e*) In 8,500 liters how many kiloliters?

Answers.—(*a*) 853 cl. or 85.3 dl.; (*b*) 4,640 l. or 46.4 Hl.; (*c*) 80 dl.; 93.5 dl.; (*d*) .06358 l.; (*e*) 8.5 Kl.

MEASURES OF WEIGHT.

TABLE V.

10 milligrams (*mg.*)..	= 1 centigram	*cg.*
10 centigrams	= 1 decigram	*dg.*
10 decigrams	= 1 gram	*g.*
10 grams	= 1 dekagram.....	*Dg.*
10 dekagrams.....	= 1 hektogram.....	*Hg.*
10 hektograms.....	= 1 kilogram, or kilo...	*Kg.* or ***K.***
10 kilograms.....	= 1 myriagram	*Mg.*
100 myriagrams.....	= 1 tonneau, or ton	*T.*

COMMON EQUIVALENTS.

1 gram	=	1 cu. cm., or 1 ml. of water.
1 kilogram...	=	1 cu. dm., or 1 liter of water.
1 metric ton.	=	1 cu. m., or 1 kiloliter of water.
1 gram	=	15.432 gr. Troy.
1 gram	=	0.03527 oz. avoirdupois.
1 kilogram ..	=	2.2046 lb. avoirdupois.
1 metric ton.	=	1.1023 tons of 2,000 lb.

20. The gram is the principal unit of weight; it is the weight of 1 cubic centimeter of pure distilled water at its temperature of maximum density, or 39.2° Fahrenheit. The gram is used in weighing gold, silver, letters (for postage), and in mixing medicines.

The kilogram is generally called the *kilo*, and is used for the same purposes as the pound avoirdupois. The tonneau

(usually called the *metric ton*) is nearly equal in weight to the long ton of 2,240 pounds.

21. Referring to Art. **38,** § 4, it will be noticed that the 5-cent piece (nickel) weighs 5 grams, that the half-dollar weighs $12\frac{1}{2}$ grams, etc.

22. The units of weight most commonly used are the milligram (in chemical analysis), the gram, the kilo, and the ton. It will be noticed that 1,000 mg. = 1 g., 1,000 g. = 1 K., and 1,000 K. = 1 ton. The milligram is, approximately, equal to $\frac{1}{65}$ of a grain; the gram, to $\frac{1}{28}$ oz. avoirdupois; the kilo, to $2\frac{1}{5}$ pounds; and the metric ton, to $1\frac{1}{10}$ (one-half of $2\frac{1}{5}$) short tons. Unless precise equivalents are desired, the values here given are accurate enough for all practical purposes. It should be firmly kept in mind that the weight of 1 liter of water is 1 kilogram, and that 1 liter of water will fill a space of 1 cubic decimeter.

OPERATIONS WITH METRIC UNITS.

23. The rules for adding, subtracting, multiplying, and dividing metric numbers are the same as for the corresponding operations on decimals. Be sure, however, that all numbers are expressed in the same units.

EXAMPLE.—What is the sum of 45.68 Dm., 63.4 Hm., and 6.845 cm. ?

SOLUTION.—Reducing all the numbers to the same unit, say meters, and adding as in decimals, the sum is 6,865.25 m.

$$\begin{array}{r} 456.8 \\ 6340. \\ \quad 68.45 \\ \hline 6865.25 \text{ m.} \end{array}$$ Ans.

EXAMPLE.—From 5.462 kilos take 7 Hg. 4 g. 9 cg.

SOLUTION.—Expressing both numbers in kilos, and subtracting, the result is 4.75791 Kg.

$$\begin{array}{r} 5.462 \\ .70409 \\ \hline 4.75791 \text{ Kg.} \end{array}$$ Ans.

EXAMPLES FOR PRACTICE.

24. Solve the following:

(*a*) What is the difference between 8.5 Kg. and 976 grams? Ans. 7.524 Kg.

(*b*) How much silk is contained in $12\frac{1}{2}$ pieces, if each piece contains 48.75 m.? Ans. 609.375 m.

(*c*) If 735 kilos of flour are equally distributed among 35 persons, how many kilos will each person receive? Ans. 21 Kg.

(*d*) In the last example, how many pounds did each person receive, counting $2\frac{1}{5}$ pounds to a kilo? Ans. 46.2 lb.

(*e*) Add 74 Hl., 147.2 l., 5,006.3 cl., and 6.5421 Kl., expressing the result in liters. Ans. 14,139.363 l.

(*f*) What is the weight in pounds of 197,862 cl. of water at 39.2° Fahrenheit? Ans. 4,353 lb., nearly.

FORMULAS.

25. A **formula** is an abridged statement of a general rule, in which symbols are used.

The symbols used are the letters of the alphabet, which represent numbers, and the signs $+$, $-$, $\times$, $\div$, $\surd$, etc., which have the same meaning as in arithmetic.

To illustrate, let the following example be taken: If a person exchanges 10 books, worth \$3 per volume, for cloth at \$2 per yard, how many yards will he obtain? A rule for solving this example, and all others like it, may be stated as follows: Multiply the number of books by the price per volume, and divide the product by the price of the cloth. The result will be the number of yards of cloth.

A more concise way of stating the rule is by using letters. Thus,

Let A = number of books;
B = price per volume;
C = price of the cloth;
D = number of yards of cloth.

Then, according to the rule,

$$\frac{\text{number of books} \times \text{price per volume}}{\text{price of cloth}} = \text{number of yards of cloth, or}$$

$$\frac{A \times B}{C} = D.$$

26. This last expression is a formula; the letters A, B, C, and D stand for the numbers given in the particular example to which it is applied; and the sign of multiplication ($\times$), and the horizontal dividing line of the fraction, which indicates division, show what operations must be performed upon the numbers to produce the answer D. In the example in question, $A = 10$, the number of books; $B = 3$, the price per volume; $C = 2$, the price of the cloth. Hence, writing for A, B, and C their values, 10, 3, and 2; D, the number of yards $= \frac{10 \times 3}{2} = 15$. Ans.

In modern technical works the rules for solving examples are commonly given by formulas, and it is important to understand how to use them. Having become accustomed to them, they will be found more convenient than rules written out in words.

27. *The multiplication sign, $\times$, is generally omitted in formulas, multiplication being indicated by simply writing the letters or expressions together.* Thus, the formula $\frac{A \times B}{C} = D$, given above, would ordinarily be written $\frac{A\,B}{C} = D$. The expression $4ab$ means the same as $4 \times a \times b$. Evidently, the sign cannot be omitted between *two figures*, as addition, instead of multiplication, would be indicated. Thus, 32 means $30 + 2$, not 3×2.

28. Formulas are usually written with the letter whose value is to be obtained standing alone at the left of the sign of equality. *To apply a formula, therefore, we have simply to substitute the given values for the letters on the right of the sign of equality and then perform the operations indicated by the signs.*

EXAMPLE.—What is the value of v, in $v = \frac{a + b\,c}{d}$ when $a = 5$, $b = 10$, $c = 4$, and $d = 20$?

SOLUTION.—Writing for a, b, c, and d their values,

$$v = \frac{5 + 10 \times 4}{20} = \frac{45}{20} = 2\tfrac{1}{4}. \quad \text{Ans.}$$

EXAMPLE.—What is the value of D, if $D = \frac{P\,c}{6\,a}$, and $P = 5$, $c = 300$, and $a = 10$?

SOLUTION.—Substituting the values of the letters,

$$D = \frac{5 \times 300}{6 \times 10} = 25. \quad \text{Ans.}$$

EXAMPLE.—The letters having the same values as before, what does x equal in the formula $x = \dfrac{c}{2\,P\,a}$?

SOLUTION.—Substituting, $x = \dfrac{300}{2 \times 5 \times 10} = \dfrac{300}{100} = 3.$ Ans.

EXAMPLE.—When $A = 10$, $B = 8$, $C = 5$, and $D = 4$, what is the value of E in the following:

$$(a)\ E = \frac{B\,C\,D}{A\left(2 + \dfrac{D}{C}\right)}? \qquad (b)\ E = \frac{A - \frac{3}{4}\,D + \dfrac{4\,B}{A + C}}{A - \dfrac{2\,B}{A + 22}}?$$

SOLUTION.—(*a*) Substituting,

$$E = \frac{8 \times 5 \times 4}{10\left(2 + \dfrac{4}{5}\right)}.$$

To simplify the denominator, notice that $2 + \dfrac{4}{5}$ is equivalent to the mixed number $2\dfrac{4}{5}$; hence, $10\left(2 + \dfrac{4}{5}\right) = 10 \times 2\dfrac{4}{5} = 10 \times \dfrac{14}{5} = \dfrac{140}{5}$.

Therefore, $$E = \frac{8 \times 5 \times 4}{\dfrac{140}{5}} = \frac{160 \times 5}{140} = 5\frac{5}{7}.$$ Ans.

(*b*) Substituting the values of the letters,

$$E = \frac{10 - \frac{3}{4} \times 4 + \dfrac{4 \times 8}{10 + 5}}{10 - \dfrac{2 \times 8}{10 + 22}} = \frac{10 - 3 + \dfrac{32}{15}}{10 - \dfrac{16}{32}} = \frac{9\dfrac{2}{15}}{9\dfrac{1}{2}} = \frac{\dfrac{137}{15}}{\dfrac{19}{2}} = \frac{274}{285}.$$ Ans.

EXAMPLES FOR PRACTICE.

29. Find the numerical values of x in the following formulas, when $a = 9$, $b = 8$, $c = 2$, $d = 10$, and $e = 3$:

(*a*) $x = \dfrac{d + c\,e}{b\,d - 40}$. Ans. $x = \frac{2}{5}$.

(*b*) $x = \dfrac{\frac{2}{3}(a + e)}{c\,e}$. Ans. $x = 1\frac{1}{3}$.

(*c*) $x = \dfrac{a\,d}{2\,c} + a\,b\,c$. Ans. $x = 166.5$.

(*d*) $x = \dfrac{a\,e\,d}{b\,c} + 4\frac{1}{8}$. Ans. $x = 21$.

INVOLUTION.

30. If a product consists of equal factors, it is called a **power** of one of those equal factors, and one of the equal factors is called a **root** of the product. The power and the root are named according to the number of equal factors in the product. Thus, 3×3, or 9, is the *second power*, or **square**, of 3; $3 \times 3 \times 3$, or 27, is the *third power*, or **cube**, of 3; $3 \times 3 \times 3 \times 3$, or 81, is the **fourth power** of 3. Also, 3 is the **second root**, or **square root**, of 9; 3 is the **third root**, or **cube root**, of 27; 3 is the **fourth root** of 81.

31. For the sake of brevity,

3×3 is written 3^2, and read **three square**,
or *three exponent two;*
$3 \times 3 \times 3$ is written 3^3, and read **three cube**,
or *three exponent three;*
$3 \times 3 \times 3 \times 3$ is written 3^4, and read **three fourth**,
or *three exponent four;*

and so on.

A number written above and to the right of another number, to show how often the latter number is used as a factor, is called an **exponent.** Thus, in 3^{12}, the number ^{12} is the exponent, and shows that 3 is to be used as a factor twelve times; so that 3^{12} is a contraction for

$$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3.$$

In an expression like 3^5, the exponent ^{5} shows how often 3 is used as a factor. Hence, if the exponent of a number is unity, the number is used once as a factor; thus, $3^1 = 3$, $4^1 = 4$, $5^1 = 5$.

32. If the side of a square contains 5 inches, the area of the square contains 5×5, or 5^2, square inches. If the edge of a cube contains 5 inches, the volume of the cube contains

$5 \times 5 \times 5$, or 5^3, cubic inches. It is for this reason that 5^3 and 5^2 are called the square and cube of 5, respectively.

33. To find any power of a number:

EXAMPLE 1.—What is the third power, or cube, of 35?

SOLUTION.— $35 \times 35 \times 35$

or

```
  35
  35
 175
105
1225
  35
6125
3675
```

cube = 42875 Ans.

EXAMPLE 2.—What is the fourth power of 15?

SOLUTION.— $15 \times 15 \times 15 \times 15$

or

```
   15
   15
   75
  15
  225
   15
 1125
 225
 3375
   15
16875
33750
```

fourth power = 50625 Ans.

EXAMPLE 3.— 1.2^3 = what?

SOLUTION.— $1.2 \times 1.2 \times 1.2$

or

```
  1.2
  1.2
 1.44
  1.2
 288
144
```

cube = 1.728 Ans.

EXAMPLE 4.—What is the third power, or cube, of $\frac{3}{8}$?

SOLUTION.— $\left(\frac{3}{8}\right)^3 = \frac{3}{8} \times \frac{3}{8} \times \frac{3}{8} = \frac{3^3}{8^3} = \frac{3 \times 3 \times 3}{8 \times 8 \times 8} = \frac{27}{512}$. Ans.

34. Rule.—I. *To raise a whole number or a decimal to any power, use it as a factor as many times as there are units in the exponent.*

II. *To raise a fraction to any power, raise both the numerator and denominator to the power indicated by the exponent.*

EXAMPLES FOR PRACTICE.

Raise the following to the powers indicated:

(*a*) 85^2.
(*b*) $(\frac{12}{13})^2$.
(*c*) 6.5^2.
(*d*) 14^4.
(*e*) $(\frac{3}{4})^3$.
(*f*) $(\frac{5}{6})^3$.
(*g*) $(\frac{7}{2})^3$.
(*h*) 1.4^5.

Ans. (*a*) 7,225.
(*b*) $\frac{144}{169}$.
(*c*) 42.25.
(*d*) 38,416.
(*e*) $\frac{27}{64}$.
(*f*) $\frac{125}{216}$.
(*g*) $\frac{343}{8}$.
(*h*) 5.37824.

ARITHMETIC.

(PART 6.)

MENSURATION.

1. **Mensuration** treats of the measurement of lines, angles, surfaces, and solids.

LINES AND ANGLES.

2. A **straight line** is one that does not change its direction throughout its whole length—it is the shortest distance between two points. To distinguish one straight line from another, two of its points are designated by letters. The line shown in Fig. 1 would be called the line *A B*.

FIG. 1.

3. A **curved line** changes its direction at every point. (Fig. 2.)

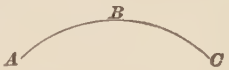

FIG. 2.

4. **Parallel lines** are equally distant from each other at all points. (Fig. 3.)

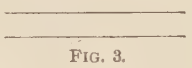
FIG. 3.

5. A line is **perpendicular** to another when it meets that line so as not to incline towards it on either side. (Fig. 4.)

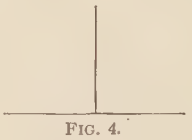
FIG. 4.

6. A **horizontal line** is a line parallel to the horizon or water level. (Fig. 5.)

7. A **vertical line** is a line perpendicular to a horizontal line; consequently, it has the direction of a plumb-line. (Fig. 5.)

FIG. 5.

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8. An **angle** is the amount of divergence between two lines that intersect, or meet; the point of meeting is called the **vertex** of the angle. Thus, in Fig. 6, the two lines form an angle whose vertex is at *B*. Angles are distinguished by naming the vertex and a point on each line. Thus, in Fig. 6, the angle formed by the lines *A B* and *C B* is called the angle *A B C*, or the angle *C B A*; the letter at the vertex is always placed at the middle. When an angle stands alone so that it cannot be mistaken for any other angle, only the vertex letter need be used. Thus, the angle referred to might be designated simply as the angle *B*.

FIG. 6.

9. A **right angle** is one of the angles formed by the intersection of two lines which are perpendicular to each other. In Fig. 7, the line *A B* is perpendicular to the line *C D*; therefore, the angles *A B C* and *A B D* are right angles.

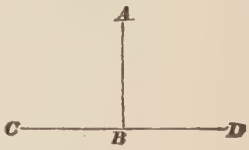

FIG. 7.

10. An **acute angle** is less than a right angle. The angle *A B C*, Fig. 8, is an acute angle.

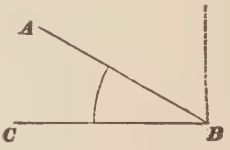

FIG. 8.

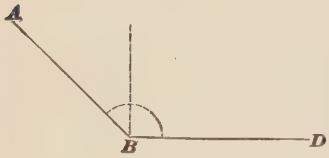

FIG. 9.

11. An **obtuse angle** is greater than a right angle. The angle *A B D*, Fig. 9, is an obtuse angle.

QUADRILATERALS.

12. A **plane figure** is any part of a plane, or flat, surface, bounded by straight or curved lines.

13. A **quadrilateral** is a plane figure bounded by four straight lines.

14. A **parallelogram** is a quadrilateral whose opposite sides are parallel.

There are four kinds of parallelograms: the **rectangle**, the **square**, the **rhomboid**, and the **rhombus**.

FIG. 10.

15. A **rectangle** is a parallelogram whose angles are all right angles. (Fig. 10.)

FIG. 11.

16. A **square** is a rectangle whose sides are all of the same length. (Fig. 11.)

FIG. 12.

17. A **rhomboid** is a parallelogram whose opposite sides are equal, and whose angles are not right angles. (Fig. 12.)

FIG. 13.

18. A **rhombus** is a rhomboid having equal sides. (Fig. 13.)

FIG. 14.

19. A **trapezoid** is a quadrilateral having only two of its sides parallel. (Fig. 14.)

20. The **altitude** of a parallelogram or trapezoid is the perpendicular distance between the parallel sides. The length of the dotted lines in Figs. 12, 13, and 14 is the altitude.

21. The **base** of a quadrilateral is the side on which it is supposed to stand. Any side may be taken as the base.

22. The **area** of a plane figure is the number of square units contained in its surface. The square unit may be a square inch, square foot, square yard, square meter, etc., as is most convenient.

23. The area of a parallelogram is equal to the product of the base and the altitude. This can be shown readily in

the case of the rectangle. Suppose, for example, the leaf of a book is 6 inches wide and 9 inches long (Fig. 15). It is a rectangle with a base of 9 inches and an altitude of 6 inches. Suppose the base to be divided into 9 equal parts, each 1 inch in length, and assume lines to be drawn through each point of division, parallel to the short sides of the rectangle. In a similar manner, suppose the altitude, or short side, to be divided into 6 equal parts, each 1 inch long, and through these points of division let lines be drawn parallel to the base. The rectangle is divided by these two sets of lines into little squares, as shown in Fig. 15. The area of one of the small squares is 1 square inch, since each of its sides is 1 inch in length. There are 9 of the squares in each horizontal row, and there are 6 rows. Hence, the total number of the little squares is $6 \times 9 = 54$, and the area of the surface is 54 square inches.

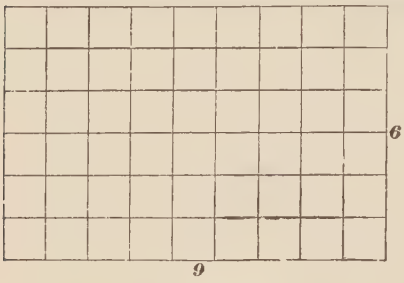

FIG. 15.

24. Rule.—*To find the area of a rectangle, multiply the base by the altitude.*

25. In ordinary language, the base and altitude of a rectangular surface are spoken of as length and breadth; the area of the surface is obtained by multiplying together the length and breadth. In applying the above rule, care must be taken that the base and altitude, or length and breadth, are reduced to the same kind of units. For example, if the base is given in feet and the altitude in inches, they cannot be multiplied together unless both are feet or both inches. This principle is of great importance, and holds good throughout the subject of Mensuration.

It must not be understood from the foregoing that *feet can be multiplied by feet* or *inches by inches*. In multiplication the multiplier is *always abstract*. In Fig. 15 there are 9 square inches in 1 row, and 6 times as many in 6 rows.

The operation in reality is 9 sq. in. $\times 6 = 54$ sq. in., or 6 sq. in. $\times 9 = 54$ sq. in.

26. EXAMPLE.—What is the area of a floor 16 feet long and $13\frac{1}{2}$ feet wide?

SOLUTION.—The base is 16 feet and the altitude is $13\frac{1}{2}$ feet.

Area = base $\times$ altitude = $16 \times 13\frac{1}{2} = 216$ sq. ft. Ans.

27. The area of any parallelogram is equivalent to the area of a rectangle of the same base and altitude. In Fig. 16, the plane figure $A\,B\,D\,C$ is a rhomboid. Suppose the corner $A\,C\,E$ is cut off, as shown, and placed at the other end in the position $B\,D\,F$. If the cutting line $A\,E$ is perpendicular to the base $C\,D$, the new figure $A\,B\,F\,E$ is a rectangle. It is plain that the base and altitude of the rectangle are the same as the base and altitude of the rhomboid, and that the areas of the two figures are the same.

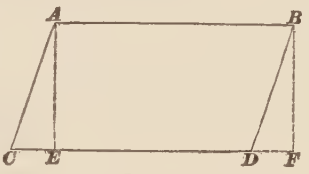

FIG. 16.

28. *To find the area of any parallelogram, multiply the base by the altitude.*

EXAMPLE.—A piece of cloth 1 yard wide is "cut on the bias," that is, it has the shape shown in Fig. 16. If the length of the strip is 8 feet, what is its area?

SOLUTION.—The altitude is 1 yd. = 3 ft., and the base is 8 ft. Hence,

Area = base $\times$ altitude = $8 \times 3 = 24$ sq. ft. Ans.

29. **Rule.**—*To find the area of a trapezoid, multiply one-half the sum of the parallel sides by the altitude.*

30. The reason for this rule will appear from an examination of Fig. 17. If E and F be the middle points of the sides that are not parallel, and if $A\,E\,3$ and $B\,F\,2$ be cut off below by *4–3* and *1–2*, perpendicular to $A\,B$, and placed above, as shown, we have a rectangle whose area is equal to $E\,F \times H\,I$. But $E\,F$ is as much less than $A\,B$ as it is greater than $D\,C$. In other words,

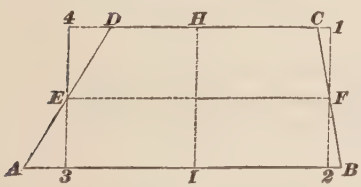

FIG. 17.

EF = half the sum of the parallel sides of the trapezoid.

Hence, area of trapezoid $= \frac{DC+AB}{2} \times HI.$

EXAMPLE.—A piece of land has the form of a trapezoid. The parallel sides are, respectively, 40 rd. and 56 rd. long, and the perpendicular distance between them is 35 rods. How many acres are contained in the piece?

SOLUTION.—One-half the sum of the parallel sides $= \frac{40+56}{2} = 48$ rd.

Area $= 48 \times 35 = 1{,}680$ sq. rd. $= \frac{1{,}680}{160} = 10.5$ acres. Ans.

31. The **perimeter** of a quadrilateral is the sum of the lengths of its four sides.

EXAMPLE.—A room is 23 ft. long and 18 ft. wide. What is its perimeter?

SOLUTION.—Perimeter $= 23+23+18+18 = 23\times2+18\times2 = 82$ ft. Ans.

PLASTERING, PAINTING, AND KALSOMINING.

32. Plastering, painting, and kalsomining are usually estimated by the square yard. Allowances for doors, windows, etc. are not regulated by any established usage. Sometimes no deduction is made for them, sometimes one-half their extent is deducted; but this is a matter usually specified in the contract.

33. EXAMPLE.—At 22 cents per square yard, what will it cost to plaster a room 65 ft. long, 22 ft. wide, and 15 ft. high; deducting in full for 8 doors 4 ft. 6 in. wide and 11 ft. 6 in. high; 10 windows 3 ft. 6 in. wide and 8 ft. high; and a baseboard $6\frac{1}{2}$ in. high extending around the room?

SOLUTION.—Perimeter of the room $= 65\times2+22\times2 = 174$ ft.

Area of walls	$= 174\times15 =$	2,610 sq. ft.
Area of ceiling	$= 65\times22 =$	1,430 sq. ft.
Total	$=$	4,040 sq. ft.
Area of doors	$= 4\frac{1}{2}\times11\frac{1}{2}\times8 =$	414 sq. ft.
Area of windows	$= 3\frac{1}{2}\times8\times10 =$	280 sq. ft.
Area of baseboard = (perimeter less the width of 8 doors) $\times \frac{6\frac{1}{2}}{12} = (174-4\frac{1}{2}\times8)\times\frac{6\frac{1}{2}}{12}$	$=$	$74\frac{3}{4}$ sq. ft.
Total, after reduction	$=$	$3{,}271\frac{1}{4}$ sq. ft.
Area in square yards	$= 3{,}271\frac{1}{4}\div9 =$	$363\frac{7}{36}$ sq. yd.
Cost	$= \$.22\times363\frac{7}{36} =$	\$79.96. Ans.

34. **Rule.**—*Multiply the perimeter of the room by the height of the ceiling for the area of the walls. To this add the area of the ceiling, and from the sum make such deductions as are specified. Reduce the results to square yards, and multiply the price per square yard by the number denoting the area in square yards.*

EXAMPLES FOR PRACTICE.

35. Solve the following examples:

1. What will it cost to plaster a room 24 ft. by 30 ft., the ceiling being 9 ft. 6 in. high, at 25 cents a square yard, if no deductions are made for openings? Ans. $48.50.

2. At 12 cents per square yard, what will it cost to paint the walls and ceiling of a hall 60 ft. long, 45 ft. wide, and 15 ft. high, deducting one-half for 4 doors, 11 ft. high and 8 ft. wide, and 8 windows, 9 ft. high and 4 ft. wide? Ans. $73.73.

3. What must be paid, at 5 cents per square yard, for kalsomining 3 rooms, each having a ceiling 8 ft. 9 in. high, and the following dimensions, respectively: 18 ft. by 20 ft., 21 ft. by 27 ft., and 24 ft. by 30 ft., there being no deductions? Ans. $22.76.

PAPERING.

36. Wall paper as made in the United States is 18 inches ($\frac{1}{2}$ yard) wide, and is sold in *single rolls* and *double rolls;* a single roll is 8 yards long, and a double roll is 16 yards long. When cutting the paper, paper hangers divide the rolls into strips of sufficient length to reach from the baseboard to a short distance (say 6 inches) above the lower edge of the border. There is always considerable waste in cutting, owing to the matching of the figures forming the design, and the fact that there is a part of a strip left over after cutting up the roll. The parts of strips thus left over are used for the surface above doors, and above and below windows, and other irregular places. Although double rolls are usually counted as two single rolls, there is a choice between them in certain cases. Thus, suppose the strips were required to be 9 feet (3 yards) long; only 2 strips could be cut from a single roll, or 4 strips from 2 single rolls, while 5 strips could be cut from a double roll. The length of a roll of border is the same as the length of a roll of paper.

37. On account of the waste in cutting, the varying sizes and shapes of rooms, the number of windows, doors, etc., it is difficult to estimate exactly the number of rolls required. We give herewith two rules, both of which are used in practice:

Rule I.—*From the perimeter of the room subtract the widths of openings (windows and doors), and reduce the result to half-yards; the number of half-yards so obtained will be the total number of strips required. Find the number of strips that can be cut from a roll and divide the first result by the second; the quotient will be the number of rolls required.*

Rule II.—*Divide the number of half-yards in the perimeter of the room by the number of strips that can be cut from a roll; the quotient will be the number of rolls required.*

38. If computed by the first rule, the number of rolls obtained may be too small, and if computed by the second rule, too large. But, since paper dealers will usually take back all rolls that are intact, the second rule will generally give the best results, as it will prevent the loss of time required to send to the dealer for extra rolls, in case they are required.

EXAMPLE.—Find how much paper will be needed to cover the walls and ceiling of a room 15 ft. by 20 ft., the border for both walls and ceiling to be 18 inches wide. The baseboard is 8 inches high, and the height of walls from floor to ceiling is 9 feet.

SOLUTION.—Since the widths of the openings are not specified, it will be necessary to use rule II.

Perimeter of room $= 2\times 15+2\times 20 = 70$ ft. $= 23\frac{1}{3}$ yd. $= 46\frac{2}{3}$ half-yards, or 47 strips. Assuming that the strips extend the height of the baseboard above the bottom edge of the border, the length of a strip is (since 18 in. $= 1\frac{1}{2}$ ft.) $9-1\frac{1}{2} = 7\frac{1}{2}$ ft. $= 2\frac{1}{2}$ yd. Hence, the number of strips in a single roll is $8\div 2\frac{1}{2} = 3$ strips, and the number of rolls required is $47\div 3 = 15\frac{2}{3}$, or 16 rolls.

In papering the ceiling, the direction in which the strips are to run must be considered. If the strips run lengthwise of the room, the distance between the edges of the border is $20-2\times 1\frac{1}{2} = 17$ ft., and the length of the strips must be at least 18 ft., or 6 yd. long; hence, but one strip can be cut from a single roll, and but two from a double roll. The width of the room in half-yards is $(15\div 3)\times 2 = 10$; hence, allowing for the border, 9 strips, or 9 single rolls will be required.

If the strips run crosswise of the room, the length of a strip between the edges of the border will be $15-2\times 1\frac{1}{2} = 12$ ft., and the length of

a strip must be at least 13 ft., or $4\frac{1}{3}$ yd.; hence, 1 strip may be obtained from a single roll, or $16 \div 4\frac{1}{3} = 3$ strips from a double roll. The length of the room in half-yards is $(20 \div 3) \times 2 = 13\frac{1}{3}$; hence, allowing the paper to extend 6 in. beyond the inner edge of the border at both ends of the room, 12 strips will be required. The number of double rolls required will be $12 \div 3 = 4$ double rolls. Consequently, there is less waste, in this case, when the paper runs crosswise than when it runs lengthwise.

Since the perimeter of the room is 70 ft., or $23\frac{1}{3}$ yd., $23\frac{1}{3} \div 8 = 3$ single rolls of border for the walls, and the same amount for the ceiling will be required. Therefore, 16 single rolls of paper are required for the walls, 4 double rolls for the ceiling, 3 single rolls of border for the walls, and 3 single rolls for the ceiling. Ans.

CARPETING.

39. Carpet is made of various widths. Ingrain carpet is usually 36 inches, or 1 yard wide; Brussels carpet is 27 inches, or $\frac{3}{4}$ yard wide. Carpet borders are $22\frac{1}{2}$ inches, or $\frac{5}{8}$ yard wide. A linear yard of ingrain carpet contains a square yard, and a linear yard of Brussels carpet contains $\frac{3}{4}$ of a square yard. If no allowance is made for cutting and matching the strips of carpet, the number of linear yards of carpet required for a room is found by dividing the area of the room in square yards by the area of a linear yard of the carpet.

EXAMPLE.—How many yards of Brussels carpet are required to cover a floor 36 ft. long and 21 ft. wide, making no allowance for cutting and matching?

SOLUTION.—Area of floor $= 36 \times 21 = 756$ sq. ft. $= \frac{756}{9} = 84$ sq. yd

A linear yard of Brussels carpet has an area of $\frac{3}{4}$ sq. yd. Hence, the number of linear yards required is $84 \div \frac{3}{4} = 112$ yd. Ans.

40. In practice, there is usually considerable loss due to cutting and matching. To find the number of yards required for a room, when allowance is made for loss, the width of the room is divided by the width of a single strip. The quotient is the number of strips required, supposing them to run lengthwise of the room. The number of strips multiplied by the length in yards of a single strip, making allowance for the loss required for matching, is the number of linear yards required.

EXAMPLE.—How many yards of Brussels carpet are required to cover a room 23 ft. long and 15 ft. wide, making an allowance of 1 ft. on each strip for matching? The carpet is supposed to run lengthwise.

SOLUTION.—Width of room = 15 ft. = 180 in. Width of carpet = 27 in. Number of strips = $180 \div 27 = 6\frac{2}{3}$. Hence, 7 strips must be used, the excess, 9 in., being cut off or turned under. Allowing 1 foot for matching, length of strip = $23 + 1 = 24$ ft. = 8 yd. Number of linear yards required = $7 \times 8 = 56$ yd. Ans.

41. The number of linear yards of carpet border required for a room is equal to the perimeter of the room in yards.

EXAMPLE.—How many yards of border are required in carpeting a room 42 ft. long and $26\frac{1}{2}$ ft. wide?

SOLUTION.—Perimeter of room = $42 \times 2 + 26\frac{1}{2} \times 2 = 137$ ft. = $\frac{137}{3}$ = $45\frac{2}{3}$ yd. Ans.

BOARD MEASURE.

42. In measuring lumber, the unit is the **board foot,** which is a board 1 foot long, 1 foot wide, and 1 inch (or less) thick. One board foot is equal to $\frac{1}{12}$ of a cubic foot. Hence, to find the number of board feet in any piece of lumber:

Rule.—*Multiply the length in feet by the breadth in feet, and this product by the thickness in inches, if it be more than one inch; or, otherwise, multiply the length in feet by the breadth in inches, and this product by the thickness in inches, and then divide by 12.*

EXAMPLE.—How many board feet are contained in a joist 18 feet long, 14 inches wide, and 12 inches thick?

SOLUTION.— $\frac{18 \times 14 \times 12}{12} = 252$ board feet. Ans.

43. Lumber is sold by the thousand (M) feet, the term foot being always used instead of the longer term, board foot. Hence, to find the cost, divide the number of feet by 1,000, and multiply by the cost per M.

EXAMPLE.—What will be the cost of 19 boards, 14 feet long, 15 inches wide, and $1\frac{1}{2}$ inches thick, at $23.50 per M?

SOLUTION.—Number of thousand feet = $\frac{19 \times 14 \times 15 \times 1\frac{1}{2}}{12 \times 1{,}000} = .498\frac{3}{4}$. Hence, $.498\frac{3}{4} \times 23.50 = \$11.72$. Ans.

44. When expressing the size of anything that is rectangular, it is customary to write the dimensions and connect them by the sign of multiplication. Thus, to express the size of a room that is 12 feet long and 10 feet wide, it would be written $12' \times 10'$, and read *12 feet by 10 feet.* In such cases the abbreviations (′) and (″) are generally used instead of feet and inches. If three dimensions are to be expressed, all three are connected by the cross (read *by*), the length being written first, then the breadth, and, lastly, the thickness or height. Thus, a room 18 feet long, 14 feet wide, and 10 feet high would be expressed as a room $18' \times 14' \times 10'$. Hence, the joist in the example, Art. **42,** would be expressed as $18' \times 14'' \times 12''$.

45. Shingles are sold in bundles of 250 ($\frac{1}{4}$ M). The lengths of all shingles in bundle are the same (usually 12″, 14″, or 16″), but the width varies. The *average* width, however, is generally 4″, the width of all bundles being alike. When laying shingles, 4″ are usually exposed to the weather, the remaining portions being concealed by the other shingles. Hence, to find the number of shingles required to cover a roof:

46. Rule.—*Compute the total area of the roof in square inches, and divide this area by the product of the average width of the shingle and the length that is exposed to the weather.*

Example.—What would it cost to shingle a roof, each side measuring $40' \times 16'$, if the shingles cost \$4.50 per M?

Solution.—Since the size of the exposed portion is not stated, it will be assumed as $4'' \times 4''$. Then, for one side, $\frac{40 \times 16 \times 144}{4 \times 4} = 5{,}760$ shingles will be required, and for both sides, $5{,}760 \times 2 = 11{,}520$ shingles. Therefore, the cost will be $11.52 \times 4.50 = \$51.84$. Ans.

We multiply by 144 in order to reduce the square feet (40×16) to square inches. Allowance should also be made for waste.

47. If the exposed portion is $4'' \times 4''$, it will take 9 shingles for each square foot; hence, in such cases it is only necessary to find the total area in square feet and multiply by

9 to find the number of shingles. Thus, in the last example, the total area in square feet is $40 \times 16 \times 2 = 1{,}280$ sq. ft., and $1{,}280 \times 9 = 11{,}520$ shingles, the same result as before.

EXAMPLES FOR PRACTICE.

48. Solve the following:

1. How many shingles are required for a roof which measures $45' \times 17'$ on one side and $45' \times 24'$ on the other side, the exposed portion of the shingles being $4'' \times 5''$? Ans. 13,284 shingles.

2. (*a*) How many thousand feet of lumber are contained in a pile having 42 layers of boards 16 feet long, the width of the layers being 11 feet, and the thickness of the boards, 1 inch? (*b*) What would be its cost at $18.75 per M. Ans. (*a*) 7.392 M. (*b*) $138.60.

3. What is the area in square feet of a parallelogram whose base is $58\frac{1}{4}''$ and altitude is $23\frac{5}{8}''$? Ans. 9.5566+ sq. ft.

4. How many square yards of oilcloth will cover a floor $15' \times 13\frac{1}{2}'$? Ans. $22\frac{1}{2}$ sq. yd.

5. If Brussels carpet costs 95 cents per yard, what will be the cost of carpeting a room $13\frac{1}{4}' \times 18'$, allowing 1 ft. on each strip for waste in matching? Ans. $36.10.

6. How many sheets of tin $20'' \times 14''$ are required to cover a roof $56' \times 30'$? Ans. 864 sheets.

7. At 18 cents per square yard, what will be the cost of plastering the ceiling and walls of a room 23 ft. long, 16 ft. wide, and 12 ft. high, making allowance for 3 doors, 3 ft. 6 in. wide by 7 ft. 6 in. high, 5 windows, 3 ft. 6 in. wide by 5 ft. 4 in. high, and a baseboard 8 in. high? Ans. $21.74.

8. At $2.50 per square yard, what is the cost of paving a street $\frac{1}{2}$ mile long and 60 feet wide? Ans. $44,000.

9. How many double rolls of paper and border are required to cover the walls of the room of example 7, assuming that the border, which is 18 in. wide, extends the height of the baseboard over the paper? Use rule I, Art. **37.** Ans. 9 rolls for walls. 2 rolls for border.

10. How many board feet in a stick of timber $27' \times 9'' \times 8''$? Ans. 162 ft.

11. How many single rolls of paper would be required to paper the ceiling of the room of example 7, assuming that there is no border, and that the paper overlaps on the walls at least 2 in.? Ans. 11 rolls.

THE TRIANGLE.

49. A **triangle** is a plane figure having three sides.

FIG. 18.

50. An **isosceles** triangle is one having two of its sides equal, as in Fig. 18.

FIG. 19.

51. An **equilateral** triangle is one having all of its sides equal. (Fig. 19.)

FIG. 20.

52. A **scalene** triangle is one having no two of its sides equal. (Fig. 20.)

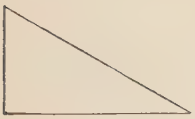

FIG. 21.

53. A **right-angled** triangle is any triangle having one right angle. The side opposite the right angle is called the **hypotenuse.** (Fig. 21.) A right-angled triangle may be isosceles or scalene.

54. The **altitude** of any triangle is a line drawn from the vertex of the angle opposite the base perpendicular to the base, or to the base extended. In Figs. 22 and 23 the vertical dotted line $A\ B$ is the altitude of the triangle. The **perimeter** of a triangle is the sum of the lengths of the three sides.

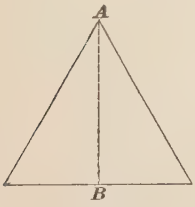

FIG. 22.

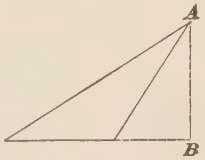

FIG. 23.

55. If in any parallelogram a straight line, called the **diagonal,** is drawn, connecting two opposite corners, the parallelogram is divided into two equal triangles, as DAB and DCB, Fig. 24. The area of each triangle, therefore, is equal to one-half the area of the parallelogram, or to one-half the product of the base and the altitude. Any side of a triangle may be taken as the base.

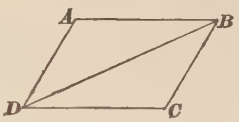

FIG. 24.

56. Rule.—*To find the area of a triangle, multiply the base by the altitude and divide the product by 2.*

EXAMPLE.—The base of a triangle is 36 inches long and its altitude is $20\frac{1}{2}$ inches. What is the area of the triangle?

SOLUTION.—Area $= \frac{\text{base} \times \text{altitude}}{2} = \frac{36 \times 20\frac{1}{2}}{2} = 369$ sq. in. **Ans.**

THE CIRCLE.

57. A **circle** is a figure bounded by a curved line, called the **circumference,** every point of which is equally distant from a point within, called the **center.** (Fig. 25.) The circumference of a circle is also called its **periphery.**

FIG. 25.

NOTE.—When a surface is bounded by straight lines, the length of the bounding line is called the *perimeter;* when the bounding line is a curve, the length of the curve is called the *periphery.* Thus, we speak of the perimeter of a polygon, and the periphery of a circle.

58. The **diameter** of a circle is a straight line passing through the center and terminated at both ends by the circumference. (See AB, Fig. 26.)

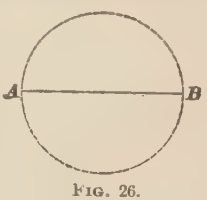

FIG. 26.

59. The **radius** of a circle is a straight line drawn from the center to the circumference. It is equal in length to one-half the diameter. The plural of radius is **radii,** and we say that all radii of a circle are equal. (OA, Fig. 27, is a radius.)

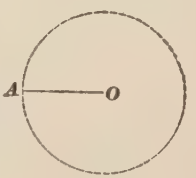

FIG. 27.

60. If a circle is divided by a diameter, each half is called a **semicircle**, and each half-circumference is called a **semi-circumference.**

61. It has been found that the length of the circumference of any circle divided by the length of the diameter gives a constant number. This number is very nearly 3.1416; it is generally denoted by the Greek letter π (pronounced *pi*).

62. Rule.—*To find the circumference of a circle, multiply the diameter by 3.1416.*

Let C = circumference of circle;
D = diameter of circle;
R = radius of circle;
π = 3.1416.

The above rule may be expressed by the formula,

$$C = \pi D = 3.1416\ D.$$

EXAMPLE.—If a car wheel is 36 inches in diameter, what is its circumference?

SOLUTION.— $C = 3.1416\ D = 3.1416 \times 36 = 113.0976$ in. Ans.

63. Rule.—*To find the diameter of a circle, divide the circumference by 3.1416.*

Formula: $$D = \frac{C}{\pi} = \frac{C}{3.1416}.$$

EXAMPLE.—The circumference of a tree is 10 feet 4 inches; what is the diameter?

SOLUTION.— 10 ft. 4 in = 124 in. Using the formula,

$$D = \frac{124}{3.1416} = 39.47 \text{ in.} \quad \text{Ans.}$$

64. Rule.—*To find the area of a circle, multiply the square of the radius by 3.1416, or multiply the square of the diameter by .7854.*

Formulas: $$A = \pi R^2 = 3.1416\ R^2,$$
$$A = \tfrac{1}{4}\pi D^2 = .7854\ D^2,$$

in which A denotes the area of the circle.

EXAMPLE.—If the diameter of a circular piston is 14 inches, what is its area?

SOLUTION.—The radius is one-half the diameter (Art. 59), or 7 in.
Hence, $A = 3.1416 \times 7^2 = 3.1416 \times 49 = 153.9384$ sq. in.
or, $A = .7854 \times 14^2 = .7854 \times 196 = 153.9384$ sq. in. Ans.

EXAMPLES FOR PRACTICE.

65. Solve the following examples:

1. Find (*a*) the circumference and (*b*) the area of a circle 34 feet in diameter.

Ans. { (*a*) 106.814 ft. (*b*) 907.92 sq. ft.

2. What is the area of a circle 4 feet $6\frac{1}{2}$ inches in diameter?

Ans. 2,332.834 sq. in.

3. (*a*) What must be the diameter, in rods, of a circular race-track 1 mile in length? (*b*) What is the area of the field enclosed?

Ans. { (*a*) 101.859 rd. (*b*) 50.93 A.

4. Find (*a*) the circumference and (*b*) the area of a locomotive driving wheel, the diameter of which is 5 feet $6\frac{1}{2}$ inches.

Ans. { (*a*) 208.916 in. (*b*) 3,473.235 sq. in.

THE PRISM AND CYLINDER.

66. A **solid**, or body, has three dimensions: length, breadth, and thickness. The sides that enclose it are called its **faces**, and the intersections of the sides are called the **edges**.

67. A **prism** is a solid whose ends are equal and parallel plane figures, and whose sides are parallelograms. Prisms take their names from the form of their bases. Thus, a triangular prism is one having a triangle for its base.

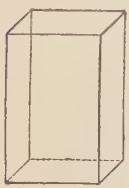

FIG. 28.

68. A **parallelopipedon** is a prism whose bases (ends) are parallelograms. (Fig. 28.)

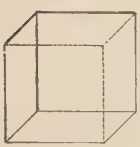

FIG. 29.

69. A **cube** is a prism whose faces are equal squares. (Fig. 29.) All the faces of a cube are equal. A cube is also a parallelopipedon.

70. A **cylinder** is a body of uniform diameter throughout its entire length, whose ends are equal parallel circles. (Fig. 30.)

FIG. 30.

71. The **altitude** of a prism, or of a cylinder, is the perpendicular distance between its bases.

72. A **right prism** is one whose sides are perpendicular to the bases.

73. A **right cylinder** is one in which the line joining the centers of the two circular bases is perpendicular to those bases.

74. In the case of plane figures, we have had to do with perimeters and areas. In the case of solids, we have to do with the areas of their outside surfaces, and with their contents or volumes.

75. The **entire surface** of any solid is the area of the whole outside of the solid.

76. The **convex surface** of a solid is the same as the entire surface, except that in the case of prisms and cylinders the areas of the ends are not included.

77. **Rule.**—*To find the convex surface of a prism or cylinder, multiply the perimeter of the base by the altitude.*

EXAMPLE.—A block of marble is 24 inches long and its ends are 9 inches square; what is the area of its convex surface?

SOLUTION.— $9 \times 4 = 36$ in. = the perimeter of the base; $36 \times 24 = 864$ sq. in., the convex area. Ans.

78. To find the entire area of the outside surface, add the areas of the two ends to the convex area. Thus, in the last example, the area of the two ends $= 9 \times 9 \times 2 = 162$ square inches; $864 + 162 = 1{,}026$ square inches.

EXAMPLE.—(*a*) What is the convex surface of a cylindrical tank with flat ends 23 feet long and 4 feet 6 inches in diameter? (*b*) What is the entire surface?

SOLUTION.—Perimeter of end $= 4\frac{1}{2} \times 3.1416 = 14.137$ ft.

(*a*) Convex surface $= 14.137 \times 23 = 325.151$ sq. ft. Ans.

Area of one end $= .7854 \times (4\frac{1}{2})^2 = 15.904$ sq. ft.

(*b*) Entire surface $= 325.151 + 2 \times 15.904 = 356.959$ sq. ft. Ans.

79. The **volume** of a solid is the quantity of space it occupies. As shown in *Arithmetic*, Part 4, the measuring unit is a cube whose edges are equal in length to a linear unit; it may be a cubic inch, cubic foot, cubic yard, or cubic meter. Fig. 31 represents a rectangular prism 4 feet long, 3 feet wide, and 2 feet thick. Dividing the prism by lines, as shown, it is seen that there are four equal slices, each of which is made up of $2 \times 3 = 6$ cubes. In all, there are $4 \times 2 \times 3 = 24$ cubes, each containing 1 cubic foot; that is, the volume of the prism is 24 cubic feet. It is seen that the number of cubes in each horizontal layer is just equal to the number of square feet in the base; and the number of layers is equal to the number of feet in the altitude. The same reasoning holds true for a prism with triangular base, or for a cylinder.

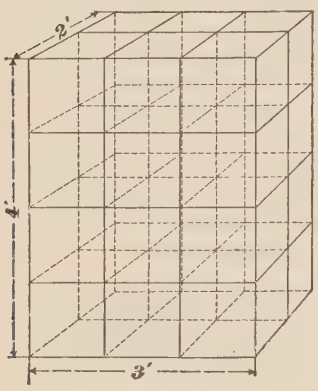

FIG. 31

80. **Rule.**—*To find the volume of a prism or cylinder, multiply the area of the base by the altitude.*

In applying this rule, all dimensions must have the same unit.

EXAMPLE 1.—A packing box is $4\frac{1}{2}$ feet long, 4 feet wide, and $3\frac{1}{4}$ feet deep; what is its volume?

SOLUTION—Area of base $= 4\frac{1}{2} \times 4 = 18$ sq. ft. Altitude $= 3\frac{1}{4}$ ft. Volume, or cubical contents $= 18 \times 3\frac{1}{4} = 58\frac{1}{2}$ cu. ft. Ans.

EXAMPLE 2.—(*a*) How many cubic feet of water will a circular cistern contain that is 8 feet in diameter and 10 feet deep? (*b*) How many gallons will the cistern hold?

SOLUTION.—(*a*) The problem is to find the volume of a cylinder whose altitude is 10 ft., and whose bases are 8 ft. in diameter.

Area of base $= .7854 \times 8^2 = 50.265$ sq. ft.

Volume $= 50.265 \times 10 = 502.65$ cu. ft. Ans.

(*b*) According to *Arithmetic*, Part 4, 1 gal. contains 231 cu. in. Hence, the cistern can hold $\frac{502.65 \times 1{,}728}{231} = 3{,}760$ gal., very nearly. Ans.

81. The dimensions of a rectangular solid are spoken of as length, breadth, and thickness. According to Art. **79**, the volume of the solid is the product of these three dimensions.

EXAMPLE.—A brick is 8 inches long, 4 inches wide, and 2 inches thick; what is its volume?

SOLUTION.—Volume = length × breadth × thickness $= 8 \times 4 \times 2 = 64$ cu. in. Ans.

MASONRY.

82. In estimating the cubical contents of stone walls, the perch of $24\frac{3}{4}$ cubic feet is used. As stated in *Arithmetic*, Part 4, the perch is often assumed to be 25 cubic feet.

83. **Rule.**—*To find the number of perches of masonry in a wall, divide the volume of the wall in cubic feet by* $24\frac{3}{4}$.

EXAMPLE.—How many perches in a wall 8 rods long, $4\frac{1}{2}$ feet high, and 2 feet thick?

SOLUTION.—Length of wall $= 8 \times 16\frac{1}{2} = 132$ ft. Cubical contents of wall $= 132 \times 4\frac{1}{2} \times 2 = 1{,}188$ cu. ft. Number of perches $= 1{,}188 \div 24\frac{3}{4} = 48$. Ans.

84. In estimating the contents of stone foundations for buildings, the length of the wall is measured on the outside, thus counting each corner twice. This is illustrated in Fig. 32. If a wall 2 feet thick measures 12 feet by 20 feet on the outside, and we assume that the corners are parts of the longer sides, we have two walls each 20 feet long, and two walls each 8 feet long. The actual length is, therefore, $2 \times 20 + 2 \times 8 = 56$ feet.

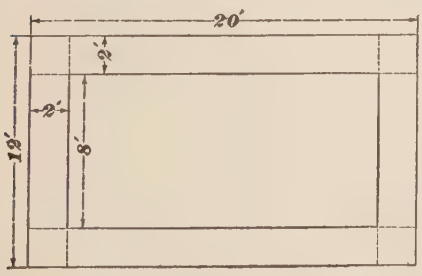

FIG. 32.

The length estimated on the outside is $2 \times 20 + 2 \times 12 = 64$ feet. To find the actual length of such a wall, subtract four times the thickness of the wall from the length measured on the outside. Thus, in the above case, actual length $= 64 - 4 \times 2 = 56$ feet.

Usually, masons make no allowance for windows or doors in estimating their work. In estimating the quantity of stone required for the wall such allowance should be made.

EXAMPLE.—(*a*) How many perches of stone are required to build the walls of a church 60 feet long by 32 feet wide, the walls being 24 feet high and $2\frac{1}{4}$ feet thick? There are 8 windows, each 5 feet wide and 11 feet high, and 2 doors, each 6 feet wide and 9 feet high. (*b*) What is the cost of laying the walls at $3.50 per perch?

SOLUTION.—

Length of wall (outside) $= 2 \times 60 + 2 \times 32 = 184$.
Actual length $= 184 - 4 \times 2\frac{1}{4} = 175$ ft.
Actual cubical contents $= 175 \times 24 \times 2\frac{1}{4} = 9{,}450$ cu. ft.
Allowance for windows $= 5 \times 11 \times 2\frac{1}{4} \times 8 = 990$ cu. ft.
Allowance for doors $= 6 \times 9 \times 2\frac{1}{4} \times 2 = 243$ cu. ft.
Net contents $= 9{,}450 - (990 + 243) = 8{,}217$ cu. ft.
(*a*) Perches required for wall $= 8{,}217 \div 24\frac{3}{4} = 332$. Ans.

(*b*) Since in estimating the cost of the work, no allowance is made for corners, doors, and windows.

Cubical contents $= 184 \times 24 \times 2\frac{1}{4} = 9{,}936$ cu. ft.
Perches of stonework $= 9{,}936 \div 24\frac{3}{4} = 401\frac{5}{11}$.
Cost of laying walls $= 401\frac{5}{11} \times \$3.50 = \$1{,}405.09$. Ans.

85. In **brickwork,** the unit of measurement is one thousand (M) bricks. The dimensions of an ordinary brick are 8 in. $\times$ 4 in. $\times$ 2 in. In some localities, they are made smaller; in others, larger. To allow for mortar, $\frac{1}{4}$ inch is added to the length and to the thickness in making calculations. On this assumption, the ordinary brick with its mortar has a volume of $8\frac{1}{4} \times 4 \times 2\frac{1}{4} = 74\frac{1}{4}$ cubic inches. Since a cubic foot contains 1,728 cubic inches, it takes $1{,}728 \div 74\frac{1}{4} = 23\frac{3}{11}$ bricks to make a cubic foot of wall.

86. **Rule.**—*To find the number of ordinary bricks in a wall, multiply its volume in cubic feet by* $23\frac{3}{11}$.

EXAMPLE.—How many bricks are required in a wall 80 feet long, $16\frac{1}{2}$ feet high, and 4 feet thick?

SOLUTION.—

Volume of wall $= 80 \times 16\frac{1}{2} \times 4 = 5{,}280$ cu. ft.

Number of bricks $= 5{,}280 \times 23\frac{3}{11} = 122{,}880$ or 122.88 M. Ans.

87. In estimating the cost of brickwork, it is customary in most localities to use the outside, or gross, length of the wall, and to allow for doors and windows. The practice, however, is not uniform, and in some cases no allowance is made for corners or openings.

EXAMPLE.—What will be the cost of erecting the walls of a building 64 feet long and 40 feet wide, the wall being 36 feet high and 3 bricks (= 1 ft.) thick? Allowance is to be made for 40 windows, each 6 ft. × 2 ft. 9 in., and 8 doors, each 8 ft. × 3 ft. 6 in. The bricks cost $5.75 per M, and the laying costs $1.40 per M, based on outside length of walls.

SOLUTION.—Outside length of wall $= 64 \times 2 + 40 \times 2 = 208$ ft.

Net length of wall $= 208 - 4 \times 1 = 204$ ft.

Contents of wall $= 204 \times 36 \times 1 = 7{,}344$ cu. ft.

Deduction for windows $= 6 \times 2\frac{3}{4} \times 1 \times 40 = 660$ cu. ft.

Deduction for doors $= 8 \times 3\frac{1}{2} \times 1 \times 8 = 224$ cu. ft.

Net contents of wall $= 7{,}344 - (660 + 224) = 6{,}460$ cu. ft.

Number of bricks $= 6{,}460 \times 23\frac{3}{11} = 150{,}342 = 150.342$ M.

Cost of bricks $= 150.342 \times \$5.75 = \$864.47$.

Cost of laying $= \{[208 \times 36 \times 1 - (660 + 224)] \times 23\frac{3}{11} \div 1{,}000\} \times 1.40 = \$215.17$.

Total cost of erecting walls $= \$864.47 + \$215.17 = \$1{,}079.64$. Ans.

EXAMPLES FOR PRACTICE.

88. Solve the following examples:

1. Find the cost of building a stone wall around a rectangular yard 160 feet long and 108 feet wide. The wall is 9 feet high and 2 feet 6 inches thick, and the price of laying is $2.25 per perch.

Ans. $1,096.36.

2. How many thousand bricks are required for a house 18 feet wide, 38 feet long, and 32 feet high, walls 3 bricks thick, making allowance for 3 doors, each 3 ft. 4 in. × 7 ft. 6 in., and 16 windows, each 3 ft. by 6 ft.?

Ans. 71.983 M.

3. Philadelphia bricks are $8\frac{1}{4}$ in. × $4\frac{1}{8}$ in. × $2\frac{3}{8}$ in. Allowing $\frac{1}{4}$ inch on length and thickness for mortar, how many of these bricks are required to make a cubic foot?

Ans. $18\frac{3}{4}$, nearly

BINS, CISTERNS, ETC.

89. It is frequently necessary to estimate the capacity of a bin, box, or vessel, in bushels, barrels, or gallons. The volume of the bin or vessel in cubic feet or cubic inches is divided by the number of cubic feet or cubic inches in a bushel, barrel, or gallon, as the case may be.

EXAMPLE.—How many bushels of wheat can be put into a bin 35 feet long, 6 feet wide, and 8 feet high?

SOLUTION.—Cubical contents of the bin $= 35 \times 6 \times 8 = 1{,}680$ cu. ft. $= 1{,}680 \times 1{,}728 = 2{,}903{,}040$ cu. in. One bushel contains 2,150.42 cu. in. (See *Arithmetic*, Part 4.) Number of bushels $= 2{,}903{,}040 \div 2{,}150.42 = 1{,}350$ bu., nearly. Ans.

90. For convenience of reference the following table of capacities is given:

TABLE I.—DRY MEASURE.

1 heaped bushel	= 2,747.71	cu. in. = 1.59 cu. ft., nearly.
1 stricken bushel	= 2,150.42	cu. in. = 1.25 cu. ft., nearly.
1 peck	= 537.6	cu. in.
1 quart	= 67.2	cu. in.
1 pint	= 33.6	cu. in.

LIQUID MEASURE.

1 hogshead	= 8.422 cu. ft.
1 barrel . .	= 4.211 cu. ft.
1 gallon . .	= 231 cu. in.
1 quart . .	= 57.75 cu. in.
1 pint . .	= 28.875 cu. in.

91. **Rule.**—*To find the capacity of a bin or other vessel in dry measure or in liquid measure, divide the volume of the bin or vessel in cubic inches by the number of cubic inches in the unit of measure.*

EXAMPLE 1.—How many liquid quarts are contained in a rectangular pail 8 in. $\times$ 5 in. $\times$ 4 in.?

SOLUTION.—Volume of pail $= 8 \times 5 \times 4 = 160$ cu. in. In 1 liq. qt. there are 57.75 cu. in. Hence, the capacity of the pail is $160 \div 57.75 = 2.77$ qt. Ans.

EXAMPLE 2.—How many gallons in a milk can 16 inches in diameter and 30 inches high?

SOLUTION.—Volume of can = area of base × altitude = $.7854 \times 16^2 \times 30 = 6{,}031.8$ cu. in. A gallon contains 231 cu. in. Number of gallons = $6{,}031.8 \div 231 = 26.11$ gal. Ans.

92. The following table of *approximate* capacities is very convenient in rough calculations:

TABLE II.

1 cu. ft. = .63 of a heaped bushel.
1 cu. ft. = .8 of a stricken bushel.
1 cu. ft. = 7.5 liquid gallons.
1 cu. ft. = $\frac{19}{80}$ of a barrel.

The following short rules are approximate, but the results are sufficiently accurate for all practical purposes.

93. Rule.—*To find the capacity of a bin in heaped bushels, multiply the volume in cubic feet by .63.*

94. Rule.—*To find the capacity of a bin in stricken bushels, multiply the volume in cubic feet by .8.*

EXAMPLE.—(*a*) How many stricken bushels in a bin 18 ft. × 13 ft. × 7 ft.? (*b*) How many heaped bushels in the same bin?

SOLUTION.—Volume = $18 \times 13 \times 7 = 1{,}638$ cu. ft.

(*a*) Stricken bushels = $1{,}638 \times .8 = 1{,}310.4$ bu. Ans.

(*b*) Heaped bushels = $1{,}638 \times .63 = 1{,}031.94$ bu. Ans.

95. Rule.—*To find the number of gallons in a cistern or other vessel, multiply the volume in cubic feet by 7.5.*

96. Rule.—*To find the number of barrels in a cistern, multiply the volume in cubic feet by* $\frac{19}{80}$.

EXAMPLE.—A rectangular cistern 9 feet 6 inches long, 6 feet wide, and 4 feet deep contains (*a*) how many gallons? (*b*) how many barrels?

SOLUTION.—Volume of cistern = $9\frac{1}{2} \times 6 \times 4 = 228$ cu. ft.

(*a*) $228 \times 7\frac{1}{2} = 1{,}710$ gal. Ans.

(*b*) $228 \times \frac{19}{80} = 54.15$ bbl. Ans.

97. Rule.—*To find the number of gallons in a cylindrical vessel, multiply the square of the diameter in inches by the height in inches, and that product by .0034.*

EXAMPLE.—An oil tank 7 feet 6 inches long and 24 inches in diameter contains how many gallons?

SOLUTION.— 7 ft. 6 in. = 90 in. Capacity = $24^2 \times 90 \times .0034 = 176\frac{1}{4}$ gal. Ans.

EXAMPLES FOR PRACTICE.

98. Solve the following examples by the exact methods:

1. A wagon body is 14 feet long, 4 feet wide, and 24 inches deep; how many bushels of shelled corn will it hold? Ans. 90 bu.

2. A rectangular can is 30 in. × 16 in. × 11½ in.; how many more liquid quarts than dry quarts will it hold? Ans. 13.44 liq. qt.

3. How many barrels are contained in a cylindrical cistern 9 feet deep and 8 feet 6 inches in diameter? Ans. 121.28 bbl.

4. A tin cup is 4 inches in diameter and 5½ inches deep; (*a*) how many liquid pints will it hold? (*b*) how many dry pints?

Ans. (*a*) 2.394 pt.
(*b*) 2.057 pt.

5. How many dry pecks can be put into a hogshead?
Ans. 27.07 pk.

Solve the following examples by the approximate rules:

6. A box that holds exactly 14 stricken bushels will hold how many liquid gallons? Ans. 131.25 gal.

7. How many bushels of wheat in a bin 21 ft. × 6½ ft. × 4½ ft.?
Ans. 491.4 bu.

8. How many barrels are contained in a cistern 11 feet 4 inches deep and 7 feet 6 inches in diameter? Ans. 118.91 bbl.

9. How many heaped bushels of potatoes are contained in a bin 30 ft. × 18 ft. × 7½ ft.? Ans. 2,551.5 bu.

10. How many gallons of water can be pumped into a cylindrical stand pipe 12 feet in diameter and 80 feet high? Ans. 67,859 gal.

COAL AND HAY

99. A *ton* (2,000 lb.) of Lehigh coal, egg size, measures 34½ cubic feet in the bin.

A ton of Schuylkill coal, egg size, measures 35 cubic feet.

A ton of pink gray and red ash coal, egg size, measures 36 cubic feet.

A ton of Wyoming coal, egg size, measures 31 cubic feet.

The bulk of a ton of hay is dependent upon the pressure to which it is subjected. Roughly speaking, a ton of hay lying

unpressed measures 500 cubic feet; when in a small stack, 400 cubic feet; and in mows compressed with grain, or in well settled stacks, 300 cubic feet.

EXAMPLES FOR PRACTICE.

100. 1. How many tons of hay are contained in a well compressed mow 30 ft. × 18 ft. × 15 ft.? Ans. 27 T.

2. How many tons of Lehigh coal will fill a bin 17 feet long, 13 feet wide, and 8 feet high? Ans. 51.2 T.

3. How many tons of Wyoming coal will fill a car 32 feet long, $6\frac{1}{2}$ feet wide, and 4 feet deep? Ans. 26.84 T.

ARITHMETIC.

(PART 7.)

PERCENTAGE.

DEFINITIONS AND PRINCIPLES.

1. In certain operations pertaining to business, it is very convenient to regard the quantity on which we are to operate as being divided into 100 equal parts; thus, instead of using the ordinary fractions $\frac{1}{4}$, $\frac{3}{5}$, $\frac{2}{7}$, we use the equivalent fractions $\frac{25}{100}$, $\frac{60}{100}$, $\frac{28\frac{4}{7}}{100}$, or their equivalent decimals, .25, .60, .28$\frac{4}{7}$. This practice is a very convenient one in all computations involving United States money, because, since $1 equals 100 cents, it is easier to comprehend what part of the whole $\frac{35}{100}$ is than some other equivalent fraction, as $\frac{49}{140}$; it is also much easier to compute with fractions whose denominators are 100 than it is to compute with fractions whose denominators are composed of other figures.

2. **Percentage** is a term applied to those arithmetical operations in which the number or quantity to be operated upon is supposed to be divided into 100 equal parts.

3. The term **per cent.** means *by the hundred.* Thus, 8 per cent. of a number means 8 hundredths, i. e., $\frac{8}{100}$, or .08, of that number; 8 per cent. of 250 is $250 \times \frac{8}{100}$, or $250 \times .08 = 20$; 47 per cent. of 75 bushels is $75 \times \frac{47}{100} = 75 \times .47 = 35.25$ bushels. The statement that the population of a city has increased 22 per cent. in a given time, say from 1880 to 1890, is equivalent to saying that the increase is 22 in every hundred, that is, for every 100 in 1880, there are 22 more, or 122, in 1890.

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4. The **sign** of per cent. is %, and is read *per cent.* Thus, 6% is read six per cent.; $12\frac{1}{2}\%$ is read twelve and one-half per cent., etc.

5. When expressing the per cent. of a number to use in calculations, it is customary to express it decimally instead of fractionally. Thus, instead of expressing 6%, 25%, and 43% as $\frac{6}{100}$, $\frac{25}{100}$, and $\frac{43}{100}$, it is usual to express them as .06, .25, and .43.

6. The following table will show how many per cent. can be expressed either as a decimal or as a fraction:

Per Cent.	Decimal.	Fraction.	Per Cent.	Decimal.	Fraction.
1%	.01	$\frac{1}{100}$	$\frac{1}{4}\%$ . . .	.0025	$\frac{1}{100}$ or $\frac{1}{400}$
2%	.02	$\frac{2}{100}$ or $\frac{1}{50}$	$\frac{1}{2}\%$ . . .	.005	$\frac{1}{100}$ or $\frac{1}{200}$
5%	.05	$\frac{5}{100}$ or $\frac{1}{20}$	$1\frac{1}{2}\%$ . . .	.015	$\frac{1\frac{1}{2}}{100}$ or $\frac{3}{200}$
10%	.10	$\frac{10}{100}$ or $\frac{1}{10}$	$6\frac{1}{4}\%$ . . .	.06$\frac{1}{4}$	$\frac{6\frac{1}{4}}{100}$ or $\frac{1}{16}$
25%	.25	$\frac{25}{100}$ or $\frac{1}{4}$	$8\frac{1}{3}\%$ . . .	.08$\frac{1}{3}$	$\frac{8\frac{1}{3}}{100}$ or $\frac{1}{12}$
50%	.50	$\frac{50}{100}$ or $\frac{1}{2}$	$12\frac{1}{2}\%$ . . .	.125	$\frac{12\frac{1}{2}}{100}$ or $\frac{1}{8}$
75%	.75	$\frac{75}{100}$ or $\frac{3}{4}$	$16\frac{2}{3}\%$ . . .	.16$\frac{2}{3}$	$\frac{16\frac{2}{3}}{100}$ or $\frac{1}{6}$
100%	1.00	$\frac{100}{100}$ or 1	$33\frac{1}{3}\%$ . . .	.33$\frac{1}{3}$	$\frac{33\frac{1}{3}}{100}$ or $\frac{1}{3}$
125%	1.25	$\frac{125}{100}$ or $1\frac{1}{4}$	$37\frac{1}{2}\%$ . . .	.37$\frac{1}{2}$	$\frac{37\frac{1}{2}}{100}$ or $\frac{3}{8}$
150%	1.50	$\frac{150}{100}$ or $1\frac{1}{2}$	$62\frac{1}{2}\%$ . . .	.625	$\frac{62\frac{1}{2}}{100}$ or $\frac{5}{8}$
500%	5.00	$\frac{500}{100}$ or 5	$87\frac{1}{2}\%$ . . .	.875	$\frac{87\frac{1}{2}}{100}$ or $\frac{7}{8}$

7. The names of the different terms used in percentage are: the *base*, the *rate* or *rate per cent.*, the *percentage*, the *amount*, and the *difference.*

8. The **base** is the number or quantity which is supposed to be divided into 100 equal parts.

9. The **rate per cent.** is that number of the 100 equal parts into which the base is supposed to be divided, that is taken or considered. The **rate** is the number of hundredths of the base, that is taken or considered. The distinction between the rate per cent. and the rate is this: the *rate per cent.* is always 100 times the *rate.* Thus, 7% of 125 and .07

of 125 amount in the end to the same thing; the former, 7, is the *rate per cent.*—the *number* of hundredths of 125 intended; the latter, .07, is the *rate*, the *part* of 125 that is to be found; 7% is used in *speech*, .07 is the form used in *computation*. So, also, $12\frac{1}{2}\% = .125$, $\frac{1}{2}\% = .005$, $1\frac{3}{4}\% = .0175$.

10. The **percentage** is the result obtained by multiplying the base by the rate. Thus, 7% of $125 = 125 \times .07 = 8.75$, the percentage.

11. The **amount** is the sum of the base and the percentage.

12. The **difference** is the remainder obtained when the percentage is subtracted from the base.

13. The terms amount and difference are ordinarily used when there is an increase or a decrease in the base. For example, suppose the population of a village is 1,500 and it increases 25 per cent. This means that for every 100 of the original 1,500 there is an increase of 25, or a total increase of $15 \times 25 = 375$. This increase added to the original population gives the *amount*, or the population after the increase. If the population had decreased 375, the final population would have been $1,500 - 375 = 1,125$, and this would be the *difference*. The original population, 1,500, is the base on which the percentage is computed; the 25 is the rate per cent., and the increase or decrease, 375, is the percentage. If the base increases, the final value is the amount, and if it decreases, its final value is the difference.

BASE, RATE, AND PERCENTAGE.

14. **Rule.**—*To find the percentage, multiply the base by the rate.*

EXAMPLE.—A farmer raised 650 bushels of wheat and sold 64% of it. How many bushels did he sell?

SOLUTION.—The base is 650 bu. Out of every 100 bu. raised 64 were sold; that is, the number of bushels sold was $\frac{64}{100}$ or .64 of the number raised.

$650 \times .64 = 416$ bu., the percentage. Ans.

15. **Rule.**—*To find the rate, divide the percentage by the base.*

EXAMPLE 1.—Bought 300 bushels of apples and sold 228 bushels; what per cent. of the number of bushels bought was sold?

SOLUTION.—Here 300 is the base and 228 is the percentage; hence, applying rule,

$$\text{rate} = 228 \div 300 = .76 = 76\%. \quad \text{Ans.}$$

EXAMPLE 2.—What per cent. of 875 is 25?

SOLUTION.—Here 875 is the base, and 25 is the percentage; hence, applying rule,

$$25 \div 875 = .02\tfrac{6}{7} = 2\tfrac{6}{7}\%. \quad \text{Ans.}$$

PROOF.— $875 \times .02\tfrac{6}{7} = 25$.

16. **Rule.**—*To find the base when the percentage and rate are given, divide the percentage by the rate.*

EXAMPLE.—Bought a certain number of bushels of apples and sold 76% of them; if I sold 228 bushels, how many bushels did I buy?

SOLUTION.—Here 228 is the percentage, and .76 is the rate; hence, applying the rule,

$$228 \div .76 = 300 \text{ bu.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE.

17. What is

(*a*)	36% of 1,762?	Ans. (*a*)	634.32.
(*b*)	19% of $89?	(*b*)	$16.91.
(*c*)	47% of 2,400 bushels?	(*c*)	1,128 bu.
(*d*)	113% of $1,640?	(*d*)	$1,853.20.

What per cent. of

(*e*)	360 is 90?	Ans. (*e*)	25%.
(*f*)	$900 is $360?	(*f*)	40%.
(*g*)	125 is 25?	(*g*)	20%.
(*h*)	150 is 750?	(*h*)	500%.
(*i*)	280 horses is 112 horses?	(*i*)	40%.
(*j*)	400 is 200?	(*j*)	50%.
(*k*)	47 is 94?	(*k*)	200%.
(*l*)	500 days is 250 days?	(*l*)	50%.
(*m*)	42 is 6% of what number?	Ans. (*m*)	700.
(*n*)	126 is 31½% of what number?	(*n*)	400.
(*o*)	198 is 36% of what number?	(*o*)	550.

AMOUNT AND DIFFERENCE.

18. To find the relations existing between the amount or difference, and the base and rate, let us consider an example.

EXAMPLE.—In a factory where 2,100 men are employed, the force is increased 8%; how many new men are employed, and how many men are at work after the increase?

SOLUTION.— 8% of 2,100 = 2,100 × .08 = 168 men = number of new men employed. Ans. The total number of men after the force is increased is 2,100 + 168 = 2,268 men. Ans.

19. **Rule.**—*To find the amount, the base and rate being given, multiply the base by 1 plus the rate.*

EXAMPLE.—The population of a city in 1880 was 43,000, and it increased 65% in the next ten years; what was its population in 1890?

SOLUTION.—In this case, 43,000 is the base, .65 is the rate, and the required population in 1890 is the amount. Applying the rule,

amount = 43,000 × (1 + .65) = 43,000 × 1.65 = 70,950. Ans.

20. **Rule.**—*To find the difference, the base and rate being given, multiply the base by 1 minus the rate.*

EXAMPLE.—A speculator invested $26,500 in a business enterprise, and lost 16% of his investment; how much had he left?

SOLUTION.—The original capital, $26,500, is the base, and .16 is the rate. Since the capital is diminished, the portion of it remaining is the difference. Applying the rule,

difference = $26,500 × (1 − .16) = $26,500 × .84 = $22,260. Ans.

21. **Rule.**—*To find the base, the amount and rate being given, divide the amount by 1 plus the rate.*

EXAMPLE.—In 1895, the population of a village was 4,130, which was 18% more than the population in 1890; what was the population in 1890?

SOLUTION.—The unknown population, in 1890, is the base, upon which the 18% is computed. The final population in 1895 is the amount, since it is an increase. Applying the rule,

base = 4,130 ÷ (1 + .18) = 4,130 ÷ 1.18 = 3,500. Ans.

22. **Rule.**—*To find the base, the difference and rate being given, divide the difference by 1 minus the rate.*

EXAMPLE.—A speculator lost 34% of an investment in stocks and had $10,560 remaining; what was the original investment?

SOLUTION.—The original investment, on which the 34% is computed, is the base. Since there has been a loss, or decrease, the remaining $10,560 is the difference. Applying the rule,

base = $10,560 ÷ (1 − .34) = $10,560 ÷ .66 = $16,000. Ans.

23. The difficulty that the student is most likely to experience in percentage is the identification of the terms or elements. When an example is given, he must first determine from it which is the base, the percentage, the amount, etc. The rate is always recognized by the words *per cent.* or by the sign %. The base is the most important element, and it can be identified by referring to the rate. The student asks himself: "Of what number do I wish to find the percentage?" The answer to this question is the base. The percentage is always a number of the same kind as the base. If, from the statement of the problem, it is seen that the base increases or decreases, the percentage is the increase or decrease, and the final value obtained, when the base has been increased or decreased by the percentage, is the amount or the difference. To be able to recognize the elements immediately requires much practice. As an exercise, several examples are given.

24. EXAMPLE 1.—Fifteen tons of iron are obtained from 282 tons of ore; what per cent. of the ore is iron?

SOLUTION.—Here the statement of the example shows that the rate is what is required. From the phrase, "what per cent. of the ore," we see that the ore is the thing of which a per cent. is taken. Therefore, the 282 T. must be the base. 15 T. is a number of the same kind as the base, and is the part of the base corresponding to the rate. It is, therefore, the percentage.

Rate = percentage ÷ base = 15 ÷ 282 = .05319 = 5.319%. Ans.

EXAMPLE 2.—Out of a cargo of oranges, 8% spoiled and 4,600 boxes remained. How many boxes were in the cargo? How many boxes spoiled?

SOLUTION.—The rate is .08; .08 of what? The number of boxes in the cargo. Therefore, the base is the original number of boxes and the less number of boxes must be the difference. The number of boxes spoiled is the percentage.

Base = difference ÷ (1 − rate)
= 4,600 ÷ (1 − .08) = 4,600 ÷ .92 = 5,000 boxes. Ans.

Percentage or number of boxes spoiled = 5,000 × .08 = 400 boxes. Ans.

EXAMPLE 3.—A farmer lost 63 sheep, which was 18% of his flock; how many had he left?

SOLUTION.—The rate is .18. The number of sheep in the flock must be the base, since it is the number on which the 18% is computed. The decrease, or number of sheep lost, is the percentage, and the number remaining is the difference.

Base = percentage ÷ rate
= 63 ÷ .18 = 350 sheep = number originally in the flock.

Difference = base − percentage = 350 − 63 = 287 sheep left. Ans.

EXAMPLES FOR PRACTICE.

25. Solve the following:

(*a*) What is $12\frac{1}{2}\%$ of \$900?
(*b*) What is $\frac{4}{5}\%$ of 627?
(*c*) What is $33\frac{1}{3}\%$ of 54?
(*d*) 101 is $68\frac{3}{4}\%$ of what number?
(*e*) 784 is $83\frac{1}{3}\%$ of what number?
(*f*) What per cent. of 960 is 160?
(*g*) What per cent. of \$3,606 is $\$450\frac{3}{4}$?
(*h*) What per cent. of 280 is 112?

Ans. (*a*) \$112.50. (*b*) 5.016. (*c*) 18. (*d*) $146\frac{10}{11}$. (*e*) 940.8. (*f*) $16\frac{2}{3}\%$. (*g*) $12\frac{1}{2}\%$. (*h*) 40%.

1. A man's salary is \$1,800 per year and he saves \$225: (*a*) What per cent. of his salary does he save? (*b*) What per cent. of it does he spend?

Ans. (*a*) $12\frac{1}{2}\%$. (*b*) $87\frac{1}{2}\%$.

2. A man has 32% of his money invested in stocks, 18% in grain, and the remainder, which is \$7,620, in real estate; what is the total value of his property? Ans. \$15,240.

3. If wool loses 32% of its weight in washing, how many pounds of unwashed wool are required to produce 35,360 pounds of washed wool? Ans. 52,000 lb.

4. In 1890, the population of a city was 85,000, which was 36% more than the population in 1880; what was the population in 1880? Ans. 62,500

5. If gunpowder contains 75% of saltpeter, 10% of sulphur, 15% of charcoal, how much of each is there in a ton of powder?

Ans. Saltpeter, 1,500 lb. Sulphur, 200 lb. Charcoal, 300 lb.

6. A man bequeathed to a charity 32% of his estate; to another charity he gave $23,100, which was 23% less than the amount given to the first charity: (*a*) What was the value of the estate? (*b*) What per cent. of the estate was given to the second charity?

Ans. { (*a*) $93,750. (*b*) 24.64%.

7. A man owning a ship worth $225,000, sells one-fourth of it to A, 20% of the remainder to B, and 35% of what then remains, to C; how much each do A, B, and C pay for their shares?

Ans. { A, $56,250. B, $33,750. C, $47,250.

PROFIT AND LOSS.

26. **Profit and loss** treats of the gains or losses arising in business transactions.

If the price for which merchandise is sold is greater than the cost of the merchandise, the difference is *profit* or *gain*. If the selling price is less than the cost, the difference is *loss*.

27. The **gross cost** of merchandise is its first cost plus the expenses of purchase, transportation, and storage. Such expenses are commission, freight, insurance, drayage, etc.

28. The **net selling price** is the gross selling price, less all discounts and expenses of sale.

29. Computations in profit and loss are made according to the rules of percentage. The gross cost of the merchandise is the *base*, upon which the *rate* of profit or loss is computed. The profit or loss is the *percentage*. If the merchandise is sold at a profit, the net selling price is the *amount;* if at a loss, the net selling price is the *difference*.

30. **Rule.**—*To find the profit or loss, multiply the gross cost by the rate of gain or loss.* (Art. **14.**)

Formula.— *Profit or loss = cost × rate.*

EXAMPLE.—A house costing $3,000 is sold for 22% above cost; what is the profit?

SOLUTION.—Profit = cost × rate = $3,000 × .22 = $660. Ans.

31. Rule.—*To find the rate of profit or loss, divide the difference between the selling price and gross cost by the gross cost; or divide the profit or loss by the gross cost.* (Art. **15.**)

Formula.— *Rate = profit or loss ÷ gross cost.*

EXAMPLE.—A merchant sold for $768 a lot of dry goods for which he paid $900; what was the per cent. loss?

SOLUTION.— Loss = $900 − $768 = $132.
Rate of loss = loss ÷ cost = $132 ÷ $900 = .14⅔ or 14⅔%. Ans.

32. Rule.—*To find the selling price, the cost and rate of gain or loss being given, multiply the cost by 1 plus the rate of gain, or by 1 minus the rate of loss.*

Formulas.—

$$\textit{Selling price} = \begin{cases} \textit{Cost} \times (1 + \textit{rate of gain}). & \text{(Art. 19.)} \\ \textit{Cost} \times (1 - \textit{rate of loss}). & \text{(Art. 20.)} \end{cases}$$

EXAMPLE.—If hay is bought for $8 per ton, and if baling and shipping costs $5.50 per ton additional, at what price must it be sold to yield a profit of 16%?

SOLUTION.— Gross cost = $8 + $5.50 = $13.50.
Selling price = cost × (1 + rate) = $13.50 × 1.16 = $15.66. Ans.

33. Rule.—*To find the cost, the selling price and rate of gain or loss being given, divide the selling price by 1 plus the rate of gain, or by 1 minus the rate of loss.*

Formulas.—

$$\textit{Cost} = \begin{cases} \textit{Selling price} \div (1 + \textit{rate of gain}). & \text{(Art. 21.)} \\ \textit{Selling price} \div (1 - \textit{rate of loss}). & \text{(Art. 22.)} \end{cases}$$

EXAMPLE.—Sold drugs for $112 and gained 75%; what was the cost of the drugs, and what was the profit?

SOLUTION.—
Cost = Selling price ÷ (1 + rate) = $112 ÷ 1.75 = $64. Ans.
Profit = $112 − $64 = $48. Ans.

EXAMPLES FOR PRACTICE.

34. What is the profit or loss
(*a*) If the gross cost is $85 and the rate of gain is 32%?
(*b*) If the gross cost is $837.50 and the rate of loss is 12%?

(*c*) If the gross cost is \$240 and the rate of gain is $16\frac{2}{3}\%$?

Ans. { (*a*) \$27.20. (*b*) \$100.50. (*c*) \$40.00.

What is the rate of gain or loss

(*d*) If the gross cost is \$6.50 and selling price is \$9.10?

(*e*) If the gross cost is \$14.00 and selling price is \$12.50?

(*f*) If the gross cost is \$3,500 and profit is \$500?

Ans. { (*d*) 40%. (*e*) $10\frac{5}{7}\%$. (*f*) $14\frac{2}{7}\%$.

What is the selling price

(*g*) If the cost is \$945 and the rate of gain is $33\frac{1}{3}\%$?

(*h*) If the cost is \$3.50 and the rate of gain is $12\frac{1}{2}\%$?

(*i*) If the cost is \$125 and the rate of loss is 18%?

Ans. { (*g*) \$1,260. (*h*) \$3.94. (*i*) \$102.50.

What is the cost

(*j*) If the selling price is \$575 and the rate of gain is 15%?

(*k*) If the selling price is \$28 and the rate of loss is $12\frac{1}{2}\%$?

(*l*) If the selling price is \$3.50 and the rate of gain is 26%?

Ans. { (*j*) \$500. (*k*) \$32. (*l*) $\$2.77\frac{7}{9}$.

1. A house and lot that cost \$3,250, is sold at a profit of 12%; what is: (*a*) the profit? (*b*) the selling price? Ans. { (*a*) \$390. (*b*) \$3,640.

2. What must be the selling price of a suit of clothes that costs \$18 in order that the profit may be $33\frac{1}{3}\%$? Ans. \$24.

3. A harvesting machine costs the hardware merchant \$90 net, and \$6 for freight and cartage; if sold for \$108, what is the gain per cent.? Ans. $12\frac{1}{2}\%$.

4. A carload of cattle is sold for \$875, which is at a loss of 16%; what was the cost of the cattle? Ans. \$1,041.67.

5. A sells a steam tug to B, gaining 14%, and B sells it to C for \$4,104 and gains 20%; how much did the tug cost A? Ans. \$3,000.

6. How much must hay sell for per ton, to gain 25%, if when sold for \$8.40 per ton, there is a gain of $16\frac{2}{3}\%$? Ans. \$9.

7. Six horses were sold at \$125 each; three of them at a profit of 25% and the others at a loss of 25%. What was the net gain or loss? Ans. \$50 loss.

TRADE DISCOUNTS.

35. **Trade discounts** are reductions made by manufacturers, jobbers, or merchants from their list, or catalog, prices.

In many branches of business, manufacturers and dealers list their goods at a fixed price for each article, and allow a rate of discount on orders of a certain amount, a second discount on orders of larger amount, and perhaps a third discount on still larger orders. If it becomes necessary to raise or lower the price of the goods, the rate of discount is decreased or increased, the list price remaining the same. The system of discounts thus saves the expense of publishing a new price list every time prices change.

36. Merchandise is frequently sold at *time prices;* that is, payment is to be made in 30, 60, or 90 days after date of sale, and a certain rate of discount is allowed if payment is made at an earlier date. Business houses usually make announcements such as the following upon their bill heads: "Terms: 4 mo., or 5% 60 days;" "Terms: 60 days net; 30 days, 3% off; 10 days, 5% off." Even when no discount is stated in the terms, sellers will usually deduct the legal interest for the time remaining, if the payment is made before it becomes due. Thus, if a payment due in 3 months is made 1 month after the sale, the seller should deduct the interest for the remaining 2 months.

37. Trade discounts are computed by the rules of percentage, the list price of the goods being the base. When several discounts are allowed, the first discount is computed on the list price, the second is computed on the remainder after deducting the first discount, and so on, each remainder being regarded as a base for the computation of the next discount. The several discounts, if there are more than one, form a **discount series.**

38. **Rule.**—*To find the selling price, multiply the list price by the rate, and subtract the discount thus obtained from the list price. If there is a discount series, compute the second discount,*

using the first remainder as a base, and subtract the discount from the remainder. Repeat the process, using each successive remainder as a base for computing the next discount. The last remainder is the selling price.

EXAMPLE 1.—The list price of an article is \$62.50 and a discount of 40% is allowed; what is the selling price?

SOLUTION.—*First Method.*—

Discount = \$62.50 × .40 = \$25.00.

Selling price = \$62.50 − \$25.00 = \$37.50. Ans.

Second Method.—Since the list price is the base, and the selling price is the base less the percentage, i. e., the difference, the selling price may be found by applying the rule of Art. **20.** Thus,

Selling price = \$62.50 × (1 − .40) = \$37.50. Ans.

EXAMPLE 2.—On a bill of goods amounting to \$720, discounts of 30%, 10%, and 5% are allowed; what is the selling price?

SOLUTION.—*First Method.*—

First discount = \$720 × .30 = \$216.
Remainder = \$720 − \$216 = \$504.
Second discount = \$504 × .10 = \$50.40.
Remainder = \$504 − \$50.40 = \$453.60.
Third discount = \$453.60 × .05 = \$22.68.
Selling price = \$453.60 − \$22.68 = \$430.92. Ans.

Second Method.—Regarding \$720 as divided into 100 parts, the first discount of 30% leaves 100 − 30 = 70 parts. The second discount of 10% is computed on the remainder after the first discount has been deducted; that is, the second discount is 10% of 70 parts. Hence, the remainder after the second discount has been deducted is 70 × (1−.10) = 63 parts. Similarly, the remainder after the third discount has been deducted is 63 × (1 − .05) = 59.85 parts, or 59.85% of \$720. Therefore,

Selling price = \$720 × .5985 = \$430.92. Ans.

Ordinarily, when applying this method, the work would be as follows:

\$720 × (1 − .30) × (1 − .10) × (1 − .05)
= \$720 × .70 × .90 × .95 = \$430.92. Ans.

39. The discounts usually allowed are aliquot parts of 100%, and the labor of computation may be shortened by using the fractions corresponding to the rates of discount.

EXAMPLE.—The gross amount of a bill of hardware is \$640, and discounts of 25%, 10%, and 5% are allowed; what is the net amount of the bill?

SOLUTION.—

$25\% = \frac{1}{4}$, $10\% = \frac{1}{10}$, $5\% = \frac{1}{20}$.

The solution is arranged as shown. To multiply \$640 by 25% or $\frac{1}{4}$, we divide by 4; then the discount \$160 is subtracted and the remainder, \$480, is divided by 10. The second discount, \$48, is subtracted, and the remainder is divided by 20. The final remainder, \$410.40, is the net amount or selling price.

4)	\$640	
	\$160	1st discount.
10)	\$480	1st remainder.
	\$ 48	2d discount.
20)	\$432	2d remainder.
	\$ 21.60	3d discount.
	\$410.40	net amount. Ans.

40. When a discount series is allowed, business men usually reduce the series to an equivalent single discount; if there is a large number of sales, much labor of computation is saved by using the equivalent discount rather than the series.

41. RULE.—*To reduce a discount series to an equivalent single discount, subtract each rate of discount from 1, and multiply the remainders together. Subtract the product from 1, and the remainder will be the single discount.* (See example 2, second method, Art. **38.**)

EXAMPLE 1.—What single discount on the gross price is equivalent to a discount series of 25%, 20%, and 10%?

SOLUTION.— $1 - .25 = .75$; $1 - .20 = .80$; $1 - .10 = .90$.

$$.75 \times .80 \times .90 = .54.$$

$1 - .54 = .46$, or 46%. Ans.

EXAMPLE 2.—The cost of a line of goods is \$350; what must they be marked to give a profit of 20% and allow a discount of 30% on the marked price?

SOLUTION.—*First Method.*—The profit is $\$350 \times .20 = \$70$; therefore, the actual selling price is $\$350 + \$70 = \$420$. This is what remains after deducting 30% from the marked price. Since the 30% discount is computed on the marked price, that price must be the base, and the less price, \$420, obtained by subtracting the discount, is the difference. According to the rule, Art. **22,** base = difference $\div$ (1 − rate); hence,

marked price = $\$420 \div (1 - .30) = \$420 \div .70 = \$600$. Ans.

Second Method.—Regarding the \$350 as divided into 100 parts, 20 parts must be added to this in order to gain 20%; that is, the selling price must be 120 parts, or 120% of \$350. Now, if a discount of 30% is

to be allowed from the list price and leave a remainder of 120 parts, it is evident that 120 parts is the difference, and the list price is the base. According to rule, Art. **22**,

list price (base) = selling price (difference) ÷ (1 − rate), or
list price = 120 ÷ (1 − .30) = $171\frac{3}{7}$ parts, or $171\frac{3}{7}\%$.
Hence, list price = \$350 × $1.71\frac{3}{7}$ = \$600. Ans.

42. Rule.—*To find the price at which goods must be marked to insure a given profit after allowing a discount, or a discount series, add to the cost the profit required, and divide the sum by 1 minus the discount or equivalent single discount.*

EXAMPLE.—The cost of manufacturing hats is \$36 per dozen; at what price per dozen must they be marked that the manufacturer may realize $16\frac{2}{3}\%$ profit after allowing the trade discounts of 20% and $12\frac{1}{2}\%$?

SOLUTION.—*First Method.*— $16\frac{2}{3}\% = \frac{1}{6}$. Profit = \$36 × $\frac{1}{6}$ = \$6; selling price = \$36 + \$6 = \$42. 1 − .20 = .80; 1 − $.12\frac{1}{2}$ = $.87\frac{1}{2}$; .80 × $.87\frac{1}{2}$ = .70. The equivalent single discount is 1 − .70 = .30. Marked price = \$42 ÷ (1 − .30) = \$60 per doz. Ans.

Second Method.—Selling price expressed as per cent. = $100 + 16\frac{2}{3}$ = $116\frac{2}{3}$. Marked price expressed as per cent. = $116\frac{2}{3}$ ÷ (1 − .20) × (1 − $.12\frac{1}{2}$) = $116\frac{2}{3}$ ÷ .70 = $166\frac{2}{3}\%$. Hence, marked price = \$36 × $1.66\frac{2}{3}$ = \$60 per doz. Ans.

EXAMPLES FOR PRACTICE.

43. Reduce the following discount series to equivalent single discounts:

(*a*) 25% and 16%.
(*b*) 30%, 20%, and 5%.
(*c*) 60%, 10%, and 5%.
(*d*) 40%, 20%, $12\frac{1}{2}\%$, and 4%.

Ans. (*a*) 37%. (*b*) 46.8%. (*c*) 65.8%. (*d*) 59.68%.

1. A musical instrument is listed at \$122 and discounts of 60% and 5% are allowed; what is the selling price? Ans. \$46.36.

2. A bill of hardware is sold at the following discounts: \$452.60 at 30% and 10%; \$216 at $33\frac{1}{3}\%$ and 5%; \$137.50 at 20%; and \$83.75 net. What is the total net amount of the bill? Ans. \$615.69.

3. A bill of goods amounting to \$836.72 was bought May 5, 1890, on the following terms: 4 months, or 5% off 30 days; how much would pay the bill June 4, 1890? Ans. \$794.88.

4. A wholesale dealer sells books at discounts of 20% and 5%; what must he mark a set of books that cost him \$24 in order to make $26\frac{2}{3}\%$ profit, after allowing discounts? Ans. \$40.

5. Plows are bought at a discount of 40% from the list price; what per cent. is gained by selling them at the list price? Ans. $66\frac{2}{3}\%$.

6. A wholesale dealer offers silks at $3.50 per yard, subject to a discount of 20%, $12\frac{1}{2}\%$, and 10%; how many yards can be bought for $352.80? Ans. 160.

7. A suit of clothing is marked 50% off; by selling at this price the clothier loses $12\frac{1}{2}\%$ of the cost of the suit, which was $12. What was the marked price of the suit? Ans. $21.

COMMISSION AND BROKERAGE.

44. **Commission,** or **brokerage,** is the sum paid an agent for transacting business for another person; as, for buying or selling merchandise or property, for collecting or investing money, etc.

45. The agent or party who transacts the business is called a **commission merchant,** or **broker;** the party for whom the business is transacted is called the **principal.** The term broker is applied to one who sells and buys stocks, bonds, bills of exchange, and money securities.

46. A **consignment** is a shipment of goods from one party to another; the party that ships the goods is called the **consignor** or **shipper,** and the party to whom they are shipped is called the **consignee.**

47. When goods are sold on credit, the agent charges an additional amount for guaranteeing the payment of the sale. This extra charge is termed **guaranty.**

48. The **gross proceeds** of a sale or collection is the total amount realized by the agent before deducting his commission and other expenses connected with the transaction. The **net proceeds** is the amount due the principal after the commission and all other charges have been deducted.

49. An **account sales** is a detailed statement made by the agent to his principal, showing the goods sold and the prices obtained, giving a list of the charges and expenses, and the net proceeds due the principal. The charges include freight, cartage, storage, insurance, inspection, advertising, commission, and guaranty.

50. The **prime cost** of a purchase is the sum paid by the agent for the goods or property. The **gross cost** is the prime cost plus the commission and expenses incident to the purchase.

51. An **account purchase** is a detailed statement made by the agent to his principal, showing the cost of goods or property bought, the expenses attending the purchase, and the gross cost.

52. The commission or brokerage is usually computed at a certain per cent. of the gross proceeds of a sale or the prime cost of a purchase. In some cases, however, it is computed at a certain price per unit of weight or measure; as, so much per ton, per bushel, or per barrel. Examples in commission are solved by the rules of percentage. Either the gross proceeds or prime cost is the *base;* the net proceeds is the *difference;* the gross cost is the *amount;* the commission is the *percentage;* and the rate of commission is the *rate per cent.* The remittance from the principal to the purchasing agent, including both the investment and the commission, is an *amount.* The following rules are derived directly from the principles of percentage:

53. **Rule.**—*To find the commission, multiply the prime cost or gross selling price by the rate of commission.*

Formula.—

Commission = cost or selling price × rate of commission.

EXAMPLE.—A real estate agent sells a house and lot for $4,375 and receives 2% commission; what is the commission and what are the net proceeds?

SOLUTION.—Commission = selling price × rate = $4,375 × .02 = $87.50. Ans.

Net proceeds = selling price − commission = $4,375 − $87.50 = $4,287.50. Ans.

54. **Rule.**—*To find the prime cost or gross proceeds, the commission being given, divide the commission by the rate of commission.*

Formula.—

Prime cost or gross proceeds = commission ÷ rate of commission.

EXAMPLE.—An agent received $319.50 commission for selling apples; if the rate of commission charged was $1\frac{1}{2}\%$, what was the selling price of the apples?

SOLUTION.—Selling price or gross proceeds = $319.50 ÷ .015 = $21,300. Ans.

55. Rule.—*To find the prime cost and commission, the remittance from the principal being given, subtract from the remittance the expenses of the purchase, if any, and divide the remainder by 1 plus the rate of commission. The quotient is the prime cost. Subtract the prime cost from the remainder and the difference is the commission.*

EXAMPLE.—A principal sends his agent $21,611 with orders to buy cotton after deducting his commission and other charges; the agent paid $124.30 for freight, $51.70 for cartage, $15 for insurance, and deducted his commission of 2%. (*a*) How much remained to invest in cotton? (*b*) What was his commission?

SOLUTION.—The expenses are first deducted. $21,611 − ($124.30 + $51.70 + $15) = $21,420, which is the sum of the prime cost and commission. According to the rule,

(*a*) Prime cost = $21,420 ÷ (1 + .02) = $21,420 ÷ 1.02 = $21,000. Ans. (*b*) Commission = $21,420 − $21,000 = $420. Ans.

NOTE.—When a charge is made for guaranty, add the per cent. of guaranty to 1 plus the rate of commission, and proceed as above.

EXAMPLES FOR PRACTICE.

56. What is the commission

(*a*) If the gross proceeds are $300 and the rate of commission is $3\frac{1}{2}\%$?

(*b*) If the gross proceeds are $9,375 and the rate of commission is 2%?

(*c*) If the prime cost is $831.75 and the rate of commission is $1\frac{1}{4}\%$?

(*d*) If the prime cost is $960 and the rate of commission is $\frac{5}{8}\%$?

Ans. (*a*) $10.50. (*b*) $187.50. (*c*) $10.40. (*d*) $6.00.

What are the net proceeds

(*e*) If the gross proceeds are \$340, rate of commission 3%, and other expenses \$4.30?

(*f*) If the gross proceeds are \$6,375, rate of commission 4%, and other expenses \$32.50?

(*g*) If the gross proceeds are \$195.40, rate of commission $1\frac{1}{2}\%$, and other expenses \$7.45?

Ans. { (*e*) \$325.50. (*f*) \$6,087.50. (*g*) \$185.02.

What is the prime cost if

(*h*) The gross cost is \$520 and the rate of commission is 4%?

(*i*) The gross cost is \$1,606 and the rate of commission is $\frac{3}{8}\%$?

(*j*) The gross cost is \$843, the rate of commission is 2%, and the expenses of buying are \$27?

Ans. { (*h*) \$500. (*i*) \$1,600. (*j*) \$800.

1. An agent receives \$13 commission for selling \$650 worth of goods; what rate of commission does he charge? Ans. 2%.

2. A commission merchant sold a quantity of wool for \$4,650; he charged $2\frac{1}{2}\%$ commission, 2% guaranty, and the transportation, storage, and other expenses amounted to \$184. How much should he send his principal? Ans. \$4,256.75.

3. An agent received \$550.50 to buy potatoes after deducting all expenses; he paid \$26.50 for drayage, \$32 for barrels, and charged $2\frac{1}{2}\%$ commission for buying. How many bushels did he buy at 60 cents per bushel? Ans. 800 bu.

4. A commission merchant has consigned to him 400 barrels of flour which he sells at \$4.75 per barrel, and charges $2\frac{1}{2}\%$ commission; with the net proceeds he buys sugar at $6\frac{1}{4}$ cents a pound and charges $2\frac{1}{2}\%$ commission for buying. (*a*) How many pounds of sugar does he buy? (*b*) What is the amount of his commissions?

Ans. { (*a*) $28{,}917\frac{1}{8}$ lb. (*b*) \$92.68.

5. A New York agent received \$1,134 with which to purchase hats. If he charges 3% commission and 2% additional for guaranty of quality, how many dozen hats can he buy at \$13.50 per dozen?

Ans. 80 doz.

6. A commission merchant sold 2,500 bushels of wheat at 64 cents a bushel and a quantity of corn at 23 cents per bushel; the rate of commission on each sale was $1\frac{3}{4}\%$, and the total commission was \$35,084. How many bushels of corn were sold? Ans. 1,760 bu.

INSURANCE.

57. **Insurance** is a contract by which one party, the **underwriter,** or **insurer,** agrees, for a consideration, to make good a loss sustained by another party.

58. Insurance is of two kinds, **property insurance** and **personal insurance.** Property insurance includes *fire insurance* (indemnity for loss or damage by fire); *marine insurance* (indemnity for losses at sea); *transit insurance* (indemnity for loss of, or damage to, merchandise during transportation); *stock insurance* (indemnity for loss of live stock); and *accident insurance* (indemnity for breakage of fragile materials, as plate glass, etc.).

Personal insurance includes *life insurance*, which secures the payment of a certain amount to a specified person at the death of the party insured, or after the lapse of a specified time; *accident insurance*, which secures the payment of a certain sum in case of accident to the insured; *health insurance*, which secures the payment of a weekly sum during sickness; and insurance against the dishonesty of employes.

59. The **policy** is the written contract between the insurance company and the party insured; it contains a description of the property insured, the conditions upon which the insurance is taken, and the amount to be paid in case of loss.

60. The **premium** is the amount paid to the insurer for assuming the risk of loss or damage. The premium is a certain per cent. of the amount of insurance, as $\frac{5}{8}\%$, $\frac{3}{4}\%$. The rate of premium depends upon the nature of the risk and upon the length of time the insurance has to run. It is customary to speak of the rate of premium as the cost per $100 of insurance; as 60 cents per $100, $1.20 per $100, etc.

61. In property insurance, all computations are based on the rules of percentage. The amount of insurance is the *base*, the premium is the *percentage*, and the rate of premium is the *rate*.

62. Rule.—*To find the premium, multiply the amount of insurance by the rate of premium.* (Art. **14.**)

Formula.—

Premium = amount of insurance × rate.

EXAMPLE.—A house and furniture are insured against fire for $3,250, the rate of premium being $\frac{3}{4}\%$, or 75 cents per $100 per year; what is the yearly premium?

SOLUTION.— $\frac{3}{4}\% = .0075$. Using the rule,

Premium = $3,250 × .0075 = 24.37\frac{1}{2}$. Ans.

63. Rule.—*To find the amount of insurance, divide the premium by the rate of premium. To find the rate of premium, divide the premium by the amount of insurance.*

Formulas.—

Amount of insurance = premium ÷ rate of premium.

Rate of premium = premium ÷ amount of insurance.

EXAMPLE 1.—What amount of insurance can be obtained for $137.50, the rate of premium being 55 cents per $100?

SOLUTION.—Rate = .55 ÷ 100 = .0055. Amount of insurance = premium ÷ rate = $137.50 ÷ .0055 = $25,000. Ans.

EXAMPLE 2.—A warehouse and contents together valued at $13,500 is insured for two-thirds of its value; if the premium is $78.75 what is the rate of premium?

SOLUTION.—Amount of insurance = $13,500 × $\frac{2}{3}$ = $9,000. Rate of premium = $78.75 ÷ $9,000 = .00875 = 87$\frac{1}{2}$ cents per $100, or $\frac{7}{8}\%$. Ans.

EXAMPLES FOR PRACTICE.

64. 1. A store and contents valued at $16,400 is insured for three-fifths of its value at $1\frac{1}{8}\%$ premium; what is the cost per year of insurance? Ans. $110.70.

2. A premium of $162.50 is paid for the insurance of a steamer worth $32,500, which is insured for four-fifths of its value; what is the premium, in cents, per $100? Ans. 62$\frac{1}{2}$ cents per $100.

3. A stock of goods is insured for two-thirds of its value; the premium paid is $43.75 and is at a rate of 75 cents per $100. What is the value of the stock of goods? Ans. $8,750.

4. A boat load of 8,600 bushels of corn, worth 32 cents per bushel, is insured for three-fourths of its value at $1\frac{5}{8}\%$ premium; if the corn is totally destroyed, what will be the owner's loss? Ans. $721.54.

5. A building worth $9,600 is insured for three-fourths of its value in three companies. The first company takes one-third of the risk at $\frac{1}{2}\%$ premium; the second two-fifths of it at $\frac{3}{4}\%$; and the third the remainder at $\frac{7}{8}\%$; what is the total premium? Ans. $50.40.

6. If it cost $324 to insure a property for $21,600, what will it cost at the same rate to insure a similar property for $35,000, if $1.50 is charged for the policy in the latter case? Ans. $526.50.

7. A fire insurance company took a risk of $42,000 at $\frac{3}{4}\%$ premium, and reinsured one-third of it in another company at $\frac{1}{2}\%$, and one-half of it in a third company at $\frac{5}{8}\%$; what did the company gain by reinsuring? Ans. $61.25.

TAXES.

65. **Taxes** are sums of money levied on persons, properties, or incomes, for public purposes. Thus, taxes are levied to support the state, county, and city governments; to support schools and charities; and to make improvements, such as paved streets and sewers.

66. A **capitation,** or **poll** tax, is a tax levied on persons; a **property** tax is a tax levied on real estate or personal property; an **income** tax is levied on incomes or salaries. The poll tax is usually a fixed amount for each citizen over 21 years of age. The property tax is estimated at a certain per cent. of the assessed valuation of the property subject to taxation.

67. **Rule.**—*To find a property tax, multiply the assessed value of the property by the rate of taxation.*

EXAMPLE.—What property tax must a person pay who owns real estate assessed at $34,000 and personal property assessed at $12,500, the rate of taxation being 8 mills per $1, or rate = .008?

SOLUTION.—Total assessed value = $34,000 + $12,500 = $46,500.

Tax = $46,500 × .008 = $372. Ans.

68. **Rule.**—*To find the rate of taxation, subtract from the total tax to be raised the poll tax, if any; divide the remainder by the total assessed valuation of the property to be taxed.*

This rule is derived directly from the rule, Art. **15**, since the tax is the *percentage*, and the assessed valuation is the *base*.

EXAMPLE.—The assessed valuation is $1,375,000 on real estate and $575,000 on personal property. The town votes to raise $12,000 for schools, $2,100 for streets and highways, $3,000 for salaries, $1,500 for support of the poor, and $650 for contingent expenses. There is a poll tax of $1 each on 3,650 persons. What is the rate of taxation and how much must A pay who has property assessed at $26,400 and pays for three polls?

SOLUTION.—Total tax = $12,000 + $2,100 + $3,000 + $1,500 + $650 = $19,250. The poll tax is $3,650; therefore, the property tax is $19,250 − $3,650 = $15,600.

Total assessed value of property = $1,375,000 + $575,000 = $1,950,000. Rate of taxation = $15,600 ÷ $1,950,000 = .008, or 8 mills per $1. Ans.

A s property tax is $26,400 × .008 = $211.20; his poll tax is $3.

$211.20 + $3 = $214.20, A's total tax. Ans.

EXAMPLES FOR PRACTICE.

69. What is the tax if the assessed valuation of the property is

(*a*) $6,300 and the rate of taxation is $1\frac{1}{8}\%$?

(*b*) $34,300 and the rate of taxation is $6\frac{1}{2}$ mills per $1?

(*c*) $9,430 and the rate of taxation is 85 cents per $100?

Ans. { (*a*) $70.87\frac{1}{2}$. (*b*) $222.95. (*c*) $80.15\frac{1}{2}$. }

What is the rate of taxation, mills per $1, if the assessed valuation is

(*d*) $625,000 and the gross tax is $5,230, including 460 polls at $.50 each?

(*e*) $1,364,000 and the gross tax is $18,130, including 1,440 polls at $.75 each?

(*f*) $34,000 and the gross tax is $469.20?

Ans. { (*d*) 8 mills. (*e*) $12\frac{1}{2}$ mills. (*f*) 13.8 mills. }

1. I own real estate worth $8,500 and personal property worth $3,750; both are assessed at three-fifths of their value. The rate of taxation is 1.2%, but I receive a discount of $2\frac{1}{2}\%$ of my taxes for prompt payment. How much do I pay the tax receiver? Ans. $86.

2. A has five lots worth $1,300 each; B has four lots worth $1,000 each; C has two lots worth $1,500 each; D has seven lots worth $800

each, and E has one lot worth $1,150. A tax of $1,417.50 for a street improvement is to be divided among them. What should each pay?

Ans. { A, $455. B, $280. C, $210. D, $392. E, $80.50.

3. The rate of taxation for a certain state is $3\frac{1}{2}$ mills per $1; how much state tax must be raised by a county whose valuation is fixed by the State Board of Equalization at $13,876,394? Ans. $48,567.38.

ARITHMETIC.

(PART 8.)

INTEREST.

SIMPLE INTEREST.

1. **Interest** is money paid for the use of money belonging to another.

2. The **principal** is the sum for which interest is paid.

3. The **rate per cent.** is the per cent. of the principal that is paid for its use for a given time, usually a year.

4. The **amount** is the sum of the principal and interest.

5. The **legal rate** is the rate established by law.

6. **Usury** is a rate that exceeds the legal rate. The penalty for usury is, in some States, the forfeiture of all interest, in others the forfeiture of both principal and interest. In a number of States, no legal notice is taken of usury.

7. In computing interest, a year is usually regarded as consisting of 12 months of 30 days each. Interest so computed is greater than it should be, unless the time is an exact number of years.

8. The elements in interest correspond with those of ordinary percentage as follows:

The *principal* is the *base.*

The *interest* is the *percentage.*

The *product of the rate per year by the time in years* is the *rate.* Thus, if the rate per cent. per year is 4% and the time is 5 years, the rate per cent. is 20%. That is, 20% of the principal

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equals the interest. It is, of course, understood that fractions of a year are included in the expression "time in years."

9. From the correspondence between percentage and interest, it is obvious that all methods of computing interest depend upon the principle expressed in the following

Formula: *Interest = principal × rate for 1 year × time in years.*

10. As in percentage, the rate is used in computation instead of rate per cent. When no other rate per cent. is specified, 6% is to be understood.

FIRST METHOD.

EXAMPLE 1.—Find the interest of $400 for 2 years 6 months at 5%.

SOLUTION.— 2 years 6 months = $2\frac{1}{2}$ years; $\$400 \times .05 \times 2\frac{1}{2} = \$50$. Ans.

EXAMPLE 2.—What is the interest of $878.60 for 3 yr. 5 mo. 15 da. at 6%?

SOLUTION.— $\$878.60 \times .06 \times \frac{83}{24} = \$182.31$. Ans.

EXPLANATION.—The time must first be changed to years. To do this, we begin with the days. 15 da. $= \frac{1}{2}$ mo.; $5\frac{1}{2}$ mo. $= \frac{11}{2}$ mo., or $\frac{11}{2 \times 12}$ yr. $= \frac{11}{24}$ yr.; $3\frac{11}{24}$ yr. $= \frac{83}{24}$ yr.

EXAMPLE 3.—Find the interest of $697.23 for 2 yr. 11 mo. 24 da. at $3\frac{1}{2}$%.

SOLUTION.— $\$697.23 \times .035 \times \frac{179}{60} = \$72.80$. Ans.

EXPLANATION.— 24 da. $= \frac{4}{5}$ mo.; $11\frac{4}{5}$ mo. $= \frac{59}{5}$ mo. $= \frac{59}{5 \times 12}$ yr. $= \frac{59}{60}$ yr.; $2\frac{59}{60}$ yr. $= \frac{179}{60}$ yr. The rate per cent. being $3\frac{1}{2}$%, the rate is $.03\frac{1}{2}$, or .035. Performing the multiplication as indicated, and pointing off the result, gives $72.80.

11. **Rule.**—*Multiply the principal by the rate, and that product by the time in years. The result will be the interest required.*

SECOND METHOD.

12. A very neat method of computing interest is the following:

EXAMPLE.—What is the interest at 5% of $248.80 for 5 yr. 9 mo. 29 da.?

SOLUTION.—

$248.80	
.05	
$12.4400	= Interest for 1 yr.
62.2000	= Interest for 5 yr. = 1 yr. × 5.
6.2200	= Interest for 6 mo. = 1 yr. ÷ 2.
3.1100	= Interest for 3 mo. = 6 mo. ÷ 2.
.5200	= Interest for 15 da. = 3 mo. ÷ 6.
.4150	= Interest for 12 da. = 6 mo. ÷ 15.
.0690	= Interest for 2 da. = 12 da. ÷ 6.
$72.5340	= Interest for 5 yr. 9 mo. 29 da. Ans.

EXPLANATION.—We first find the interest for 1 year, and then for 5 years. Having the interest for 1 year, we take half of that, which is the interest for 6 of the 9 months. Half of the interest for 6 months is the interest for 3 months. The interest for 29 days is found by taking $\frac{1}{6}$ of 3 months' interest, which gives the interest for 15 ($\frac{1}{2}$ mo.) of the 29 days. Since 12 days $= \frac{1}{15}$ of 6 months, we divide the interest for 6 months by 15* and get 12 days' interest. Only 2 of the 29 days remain; but 2 days $= \frac{1}{6}$ of 12 days. Hence, dividing 12 days' interest by 6, we have the interest for 2 days. 5 years + 6 months + 3 months + 15 days + 12 days + 2 days = 5 yr. 9 mo. 29 da., so that the sum of these several interests = the interest required.

It will be noticed that in the foregoing, 30 days are considered equal to 1 month, or 360 days to a year.

EXAMPLE.—What is the interest of $6,400 for 3 yr. 11 mo. 26 da., at $4\frac{1}{2}\%$?

SOLUTION.—

$6400	
.045	
$288.00	= Interest for 1 yr.
864.00	= Interest for 3 yr. = 1 yr. × 3.
144.00	= Interest for 6 mo. = 1 yr. ÷ 2.
72.00	= Interest for 3 mo. = 6 mo. ÷ 2.
48.00	= Interest for 2 mo. = 6 mo. ÷ 3.
16.00	= Interest for 20 da. = 2 mo. ÷ 3.
4.00	= Interest for 5 da. = 20 da. ÷ 4.
.80	= Interest for 1 da. = 5 da. ÷ 5.
$1148.80	= Interest for 3 yr. 11 mo. 26 da. Ans.

*To divide by 15, divide by 5 and then by 3.

13. Rule.—*Find the interest at the given rate for one year, and multiply this by the number of years. Find the interest for the given months by taking suitable parts of one year's interest, and for the days, suitable parts of the interest for one or more months. The sum of these partial results will be the total interest for the given time.*

EXAMPLES FOR PRACTICE.

14. Find the interest by the first method, and prove the correctness of your work by the second method:

1. Of \$600 for 1 yr. 4 mo. 15 da. at 6%.
2. Of \$2,400 for 2 yr. 2 mo. 18 da. at 5%.
3. Of \$1,800 for 2 yr. 7 mo. 20 da. at 4½%.
4. Of \$1,250 for 9 mo. 25 da. at 3½%.
5. Of \$87.50 for 5 yr. 10 mo. 27 da. at 8%.
6. Of \$675.60 for 2 yr. 8 mo. 12 da. at 4¼%.
7. Of \$1,388.84 for 3 yr. 7 mo. 17 da. at 4%.
8. Of \$725.50 for 7 yr. 5 mo. 23 da. at 7½%.
9. Of \$6,496.40 for 5 yr. 11 mo. 21 da. at 3%.
10. Of \$847.23 for 9 yr. 6 mo. 14 da. at 3¾%.

Answers.—(1) \$49.50; (2) \$266; (3) \$213.75; (4) \$35.85+; (5) \$41.36—; (6) \$77.53—; (7) \$201.69; (8) \$407.04—; (9) \$1,164.48—; (10) \$303.06.

SIX-PER-CENT. METHOD.

15. Assuming that a year is composed of 12 months, each consisting of 30 days, it is clear that at 6% the interest of \$1 for 1 year is 6 cents, or \$.06; that for 1 month it is $\frac{1}{12}$ of \$.06, which is equal to 5 mills, or \$.005; and that for $\frac{1}{5}$ of 30 days, or 6 days, it is 1 mill, or \$.001. Or, in tabular form,

Interest of \$1 for 1 year = *\$.06.*
Interest of \$1 for 1 month = *\$.005.*
Interest of \$1 for 6 days = *\$.001.*

Hence, the interest of \$1 for any other time will be \$.06 for each year, \$.005 for each month, and \$.000$\frac{1}{6}$ for each day. The sum of these three results will be the interest of \$1 for the given time.

EXAMPLE.—What is the interest of $1 for 5 yr. 7 mo. 21 da. at 6%?

SOLUTION.—

Interest of $1 for 5 yr. = $.06 × 5 = $.30
Interest of $1 for 7 mo. = .005 × 7 = .035
Interest of $1 for 21 da. = .000$\frac{1}{6}$ × 21 = .0035
Interest of $1 for 5 yr. 7 mo. 21 da. at 6% = $.3385. Ans.

Now, if we know the interest of $1, it is simply a matter of multiplication to find the interest of any other number of dollars, or of dollars and cents.

Again, if we know the interest at 6% we may obtain the interest at 1% by dividing the interest at 6% by 6; and having the interest at 1%, we may find it at 4% by multiplying the interest at 1% by 4; similarly, for any other per cent.

EXAMPLE.—What is the interest of $654 for 3 yr. 9 mo. 28 da. at 5%?

SOLUTION.—

Interest of $1 at 6% for 3 yr. = .06 × 3 = $.18
Interest of $1 at 6% for 9 mo. = .005 × 9 = .045
Interest of $1 at 6% for 28 da. = .000$\frac{1}{6}$ × 28 = .004$\frac{2}{3}$
Interest of $1 at 6% for 3 yr. 9 mo. 28 da. = $.229$\frac{2}{3}$
$.229$\frac{2}{3}$ × 654 = $150.202 = interest at 6%.
$150.202 ÷ 6 = 25.033\frac{2}{3}$ = interest at 1%.
25.033\frac{2}{3}$ × 5 = 125.168\frac{1}{3}$ = interest at 5%. Ans.

16. Rule.—*Find the interest of $1 at 6% for the given time. To do this, multiply $.06 by the number of years, $.005 by the number of months, and $.000$\frac{1}{6}$ by the number of days. Multiply the sum of these results by the number of dollars in the principal, and the result will be the interest at 6%. For any other rate, divide the interest at 6% by 6, and multiply the quotient by the given rate.*

17. In practice, it is better to find the interest at other rates than 6% by adding to, or subtracting from, the result for 6% suitable parts of itself.

The following partial table will illustrate:

Interest at 6% + { $\frac{1}{6}$ of itself = Interest at 7%.
$\frac{1}{8}$ of itself = Interest at 8%.
$\frac{1}{2}$ of itself = Interest at 9%.
$\frac{1}{4}$ of itself = Interest at 7$\frac{1}{2}$%.
$\frac{1}{12}$ of itself = Interest at 6$\frac{1}{2}$%.

$$\text{Interest at } 6\% - \begin{cases} \frac{1}{6} \text{ of itself} = \text{Interest at } 5\%. \\ \frac{1}{4} \text{ of itself} = \text{Interest at } 4\frac{1}{2}\%. \\ \frac{1}{8} \text{ of itself} = \text{Interest at } 4\%. \\ \frac{1}{9} \text{ of itself} = \text{Interest at } 5\frac{1}{8}\%. \\ \frac{1}{12} \text{ of itself} = \text{Interest at } 5\frac{1}{2}\%. \end{cases}$$

In the case of unusual rates per cent., it may be necessary to add or subtract two or more quotients.

EXAMPLE.—Suppose we have found the interest of a certain principal at 6% to be $237.68. How shall we find the interest of the same principal at $4\frac{3}{4}\%$?

SOLUTION.—

$237.68	=	Interest at 6%.
59.42	=	Interest at $1\frac{1}{2}\%$ = 6 % ÷ 4.
$178.26	=	Interest at $4\frac{1}{2}\%$ = 6 % − $1\frac{1}{2}\%$.
9.90+	=	Interest at $\frac{1}{4}\%$ = $1\frac{1}{2}\%$ ÷ 6.
$188.16	=	Interest at $4\frac{3}{4}\%$ = $4\frac{1}{2}\%$ + $\frac{1}{4}\%$. Ans.

18. When the time for which the interest is to be computed is less than a year, it is customary to use the following method for finding the interest at 6%:

Since the interest for 1 day at 6% is $\$.000\frac{1}{6}$ (see Art. **15**), the interest for any number of days may be found by multiplying the principal by the number of days, moving the decimal point *three* places to the left, and dividing the result by 6. It is usually easier, however, to divide the number of days by 6, multiply the quotient by the principal, and move the decimal point three places to the left.

EXAMPLE.—Find the interest of $1,215 for 86 days at 6%.

SOLUTION.— $86 \div 6 = 14\frac{1}{3}$; $1.215 = the principal with the decimal point moved three places to the left; $\$1.215 \times 14\frac{1}{3} = \$17.415$, or $17.42—. Ans.

When using this method, retain any fraction that may arise from dividing by 6. Thus, for 83 days, multiply by $13\frac{5}{6}$, rather than reduce the fraction to a decimal.

EXAMPLES FOR PRACTICE.

19. By the foregoing method, find the interest

1. Of $484 for 2 yr. 5 mo. 15 da. at 3%.
2. Of $768 for 1 yr. 9 mo. 20 da. at 4%.
3. Of $3,825 for 3 yr. 6 mo. 24 da. at 5%.

4. Of \$9,600 for 4 yr. 7 mo. 27 da. at $3\frac{1}{2}\%$.
5. Of \$168.75 for 2 yr. 11 mo. 23 da. at $4\frac{1}{2}\%$.
6. Of \$437.50 for 5 yr. 8 mo. 18 da. at $5\frac{1}{2}\%$.
7. Of \$627.40 for 4 yr. 10 mo. 14 da. at $7\frac{1}{2}\%$.
8. Of \$969.96 for 3 yr. 9 mo. 22 da. at 7%.
9. Of \$1,237.50 for 7 yr. 2 mo. 26 da. at $2\frac{3}{4}\%$.
10. Of \$1,875.60 for 12 yr. 3 mo. 10 da. at $3\frac{2}{3}\%$.
11. Of \$784.15 for 57 da.
12. Of \$4,225 for 126 da.

Answers.—(1) \$35.695; (2) \$55.47—; (3) \$682.13—; (4) \$1,565.20; (5) \$22.63+; (6) \$137.56—; (7) \$229.26+; (8) \$258.76+; (9) \$246.35—; (10) \$844.37—; (11) \$7.45—; (12) \$88.73—.

SIXTY-DAY METHOD.

20. For 1 year, at 6%, the interest of any principal is .06 of the principal itself, and for 2 months, or 60 days, the interest is .01 of the principal. Hence,

If the decimal point of any sum be moved two places to the left, it will give the interest of that sum for 60 days at 6%.

Thus, the interest of \$3,472.75 for 60 days at 6% is \$34.73—, and of \$692 it is \$6.92.

Having the interest for 60 days, it is easy, by operations that will suggest themselves, to find the interest for any other number of days.

EXAMPLE 1.—Find the interest of \$8,368 for 99 days at 6%.

SOLUTION.—

\$ 8 3.6 8	=	Interest for 60 days.
4 1.8 4	=	Interest for 30 days = $\frac{1}{2}$ of 60 days.
8.3 6 8	=	Interest for 6 days = $\frac{1}{10}$ of 60 days.
4.1 8 4	=	Interest for 3 days = $\frac{1}{2}$ of 6 days.
\$ 1 3 8.0 7 2	=	Interest for 99 days. Ans.

EXAMPLE 2.—What is the interest at 9% of \$1,264.76 for 49 days?

SOLUTION.—

\$ 1 2.6 4 7 6	=	Interest for 60 days at 6%.
6.3 2 3 8	=	Interest for 30 days = $\frac{1}{2}$ of 60 days.
3.1 6 1 9	=	Interest for 15 days = $\frac{1}{2}$ of 30 days.
.6 3 2 4	=	Interest for 3 days = $\frac{1}{10}$ of 30 days.
.2 1 0 8	=	Interest for 1 day = $\frac{1}{3}$ of 3 days.
\$ 1 0.3 2 8 9	=	Interest for 49 days, at 6%.
5.1 6 4 4	=	Interest for 49 days, at 3%.
\$ 1 5.4 9 3 3	=	Interest for 49 days, at 9%. Ans.

21. **Rule.**—*Take .01 of the principal for the interest at 6% for 60 days, and then, by the method of aliquot parts, find the interest for the given time at the rate specified.*

EXAMPLES FOR PRACTICE.

22. By the sixty-day method, find the interest

1. Of \$8,000 for 87 days at 6%.
2. Of \$6,050 for 96 days at 3%.
3. Of \$875.28 for 77 days at $3\frac{1}{2}$%.
4. Of \$1,468.80 for 123 days at 4%.
5. Of \$23,750 for 108 days at $4\frac{1}{2}$%.
6. Of \$42,690 for 176 days at $3\frac{7}{8}$%.
7. Of \$7,200 for 225 days at 5%.
8. Of \$468.24 for 101 days at $5\frac{1}{2}$%.
9. Of \$6,880 for 186 days at 7%.
10. Of \$7,600 for 143 days at $7\frac{1}{2}$%.

Answers.—(1) \$116; (2) \$48.40; (3) \$6.55+; (4) \$20.07+; (5) \$320.625; (6) \$765.26—; (7) \$225; (8) \$7.23—; (9) \$248.83—; (10) \$226.42—.

EXACT INTEREST.

23. When interest is to be computed for one or more entire years at a specified rate per year, the fact that 12 months of 30 days each are usually regarded as a year does not affect the result—it is only when months and days, or days alone, become an element of the given time, that the interest is greater than it should be. The average length of a month in an ordinary year is $30\frac{5}{12}$ days, and in a leap year it is $30\frac{1}{2}$ days. A day is not $\frac{1}{360}$ of a year, but $\frac{1}{365}$ of a common year, and $\frac{1}{366}$ of a leap year. Hence, 360 days $= \frac{360}{365}$, or $\frac{72}{73}$, of a common year, and $\frac{360}{366}$, or $\frac{60}{61}$, of a leap year. By the ordinary method of finding interest, the result is either $\frac{1}{73}$ or $\frac{1}{61}$ greater than it should be.

Thus, the interest of \$7,300 for 60 days at 6%, as found by the usual method, is \$73. In equity it is $\$7{,}300 \times .06 \times \frac{60}{365}$ = \$72. That is, each \$73 interest should be \$72.

24. The method practised by the government and by most banks is to compute the interest for the number of

entire years in a period, and then treat the remaining days as so many 365ths of a year.

EXAMPLE.—Find the exact interest of $8,000 from Jan. 5, 1873, to July 23, 1880, at 6%.

SOLUTION.—

From Jan. 5, 1873, to Jan. 5, 1880 = 7 yr.
From Jan. 5, 1880, to July 23, 1880 =
Jan. Feb. Mar. Apr. May June July
$26 + 29 + 31 + 30 + 31 + 30 + 23 = 200$ days $= \frac{200}{365} = \frac{100}{183}$ yr.
$\$8{,}000 \times .06 \times 7\frac{100}{183} = \$3{,}622.30$—. Ans.

25. The same result is obtained by adding to the interest for 7 years the interest for 200 days less $\frac{1}{61}$ of itself, found by the usual method. Thus,

$\$8{,}000 \times .06 \times 7 =$		$\$3{,}360.00$
$\$8{,}000 \times .06 \times \frac{200}{360} =$	$\$266.67$	
Deducting $\frac{1}{61}$ of $\$266.67$	$4.37 =$	262.30
Exact interest for 7 yr. 200 da. $=$		$\$3{,}622.30$ Ans.

EXAMPLE.—What is the exact interest of $4,800 for 198 days of an ordinary year, at 6%?

SOLUTION.—

$\$48.00$	= Interest for 60 days.
96.00	= Interest for 120 days.
12.00	= Interest for 15 days.
2.40	= Interest for 3 days.
$73)\$158.40$	= Interest for 198 days, counting 360
2.17	days of the year.
$\$156.23$	= Exact interest for 198 days. Ans.

EXPLANATION.—The interest is first found by the usual method, and $\frac{1}{73}$ of the result deducted.

26. **Rule.**—*Find the interest by the ordinary method for the whole number of years included in the period. Count the number of days that remain, and by the same method find the interest for the days. If the days are part of an ordinary year, diminish the interest for the days by $\frac{1}{73}$ of itself; if they are part of a leap year, diminish the interest by $\frac{1}{61}$ of itself. Add the interest for the years to that for the days, and the result will be the exact interest.*

EXAMPLES FOR PRACTICE.

27. Find the exact interest

1. Of \$10,000 for 123 days at 6%.
2. Of \$12,800 for 168 days at 6%.
3. Of \$6,400 for 213 days at 5%.
4. Of \$22,800 for 2 yr. 73 da. at 4%.
5. Of \$960 for 5 yr. 300 da. at $3\frac{1}{2}$%.
6. Of \$484.80 for 6 yr. 202 da. at $5\frac{1}{2}$%.
7. Of \$13,000 from Jan. 17, 1897, to Nov. 29, 1897, at $4\frac{1}{2}$%.
8. Of \$968.40 from Apr. 19, 1865, to July 1, 1896, at 3%.
9. Of \$1,234.60 from Dec. 23, 1888, to Mar. 17, 1890, at 6%.
10. Of \$43,000 from May 29, 1891, to Nov. 3, 1895, at 7%.

Answers.—(1) \$202.19+; (2) \$353.49; (3) \$186.74—; (4) \$2,006.40; (5) \$195.62—; (6) \$174.74; (7) \$506.47—; (8) \$906.41—; (9) \$91.12+; (10) \$13,342.96—.

ANNUAL INTEREST.

28. Unless otherwise specified, interest upon debts is understood to be payable annually. In case it is not so paid, it is permitted in some States to charge interest upon overdue interest. In some other States this practice is illegal. Where it is intended to charge "annual interest," the written obligation should contain the words "interest payable annually."

EXAMPLE.—What is the interest at 6% of \$2,400 for 6 years 6 months, interest payable annually, if no interest is paid until the end of the time?

SOLUTION.—

Interest for 1 year $= \$2{,}400 \times .06 =$ \$144.00

Interest for 6 yr. 6 mo. $= \$144 \times 6\frac{1}{2} =$ \$936.00

Interest of \$144 for $5\frac{1}{2} + 4\frac{1}{2} + 3\frac{1}{2} + 2\frac{1}{2} + 1\frac{1}{2} + \frac{1}{2}$

$=$ Interest for 18 yr. $= \$144 \times .06 \times 18 =$ 155.52

\$1,091.52 Ans.

EXPLANATION.—The first year's interest remains unpaid for $5\frac{1}{2}$ yr., the second for $4\frac{1}{2}$ yr., etc. The sum of these periods is 18 yr. One year's interest of the principal is \$144, and the interest of this for 18 yr. is \$155.52. The sum of \$936 and \$155.52 is the entire interest due.

29. Rule.—*Find the interest of the principal for one year, and for the entire time the debt runs. Find the sum of the several periods that the annual interest remains unpaid, and for this time find the interest of one year's interest of the debt. The sum of the interest of the main debt and that of the unpaid annual interest will be the result required.*

EXAMPLES FOR PRACTICE.

30. In the following examples, assume that the annual interest is payable but unpaid, and find the entire interest due at the end of the given time:

1. Of $240 for 2 yr. 6 mo. at 6%.
2. Of $380 for 3 yr. 8 mo. at 5%.
3. Of $1,000 for 4 yr. 9 mo. at 7%.
4. Of $1,200 for 3 yr. 3 mo. 15 da. at 6%.
5. Of $387.50 for 5 yr. 6 mo. 20 da. at 4%.
6. Of $7,625 for 8 yr. 7 mo. 18 da. at 5%.

Answers.—(1) $37.73– ; (2) $74.42– ; (3) $376.60; (4) $253.74; (5) $94.03+ ; (6) $3,921.79+.

PROBLEMS IN INTEREST.

31. Given the interest, rate, and time, to find the principal.

We know that $I =$ Prin. $\times$ rate $\times$ time (in years); or, more briefly,

$$\frac{Prt}{100} = I,$$

when r is the rate per cent. Hence, multiplying each side by 100, we have $Prt = 100I$. Now, if we divide each side by rt, we have $\frac{Prt}{rt} = \frac{100I}{rt}$. Canceling rt of the left member of this equation, we have

$$P = \frac{100I}{rt}.$$

That is, *the principal is equal to 100 times the interest divided by the product of the rate per cent. and the time.*

EXAMPLE.—What principal in 4 yr. 9 mo. at 5% will give $152 interest?

SOLUTION.—Applying the formula and noticing that 9 mo. $= \frac{3}{4}$ yr.,

$$P = \frac{152 \times 100}{5 \times 4\frac{3}{4}} = \frac{\overset{8}{\cancel{152}} \times \overset{20}{\cancel{100}} \times 4}{\cancel{5} \times \cancel{19}} = \$640. \quad \text{Ans.}$$

EXPLANATION.— $4\frac{3}{4}$ being in the divisor, the fraction must be reduced to the form of an improper fraction and inverted. The rate also appears in the inverted form of $\frac{100}{5}$, since .05 cannot be inverted.

EXAMPLE.—What principal at $3\frac{2}{5}\%$ will, in 2 yr. 3 mo. 15 da., give $374 interest?

SOLUTION.—Changing the time to $\frac{55}{24}$ years, writing $3\frac{2}{5}\%$ as $\frac{17}{5}$, and applying the formula, we have

$$P = \frac{374 \times 100}{\frac{17}{5} \times \frac{55}{24}} = \frac{\overset{2}{\cancel{374}} \times 100 \times \cancel{5} \times 24}{\cancel{17} \times \underset{\cancel{5}}{\cancel{55}}} = \$4{,}800. \quad \text{Ans.}$$

32. Rule.—*To find the principal when the interest, rate, and time are given, divide 100 times the interest by the product of the time in years and the rate per cent.*

EXAMPLES FOR PRACTICE.

33. Find the principal, when

1. Interest = $96, time = 2 years, rate = 4%.
2. Interest = $131.25, time = 2 yr. 6 mo., rate = 6%.
3. Interest = $62.25, time = 2 yr. 3 mo. 20 da., rate = $4\frac{1}{2}\%$.
4. Interest = $60, time = 180 days, rate = 5%.
5. Interest = $546, time = 3 yr. 5 mo. 18 da., rate = $3\frac{1}{2}\%$.
6. Interest = $23.75, time = 1 yr. 8 mo. 24 da., rate = $5\frac{1}{2}\%$.
7. Interest = $43.60, time = 2 yr. 11 mo. 12 da., rate = 7%.
8. Interest = $124.30, time = 3 mo. 20 da., rate = $3\frac{2}{5}\%$.

Answers.—(1) $1,200; (2) $875; (3) $600; (4) $2,400; (5) $4,500; (6) $249.13—; (7) $211.14—; (8) $11,094.55—.

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34. Given the principal, the interest, and the time, to find the rate per cent.

Resuming the equation,

$$Prt = 100\,I;$$

dividing each side by Pt, we have

$$\frac{Prt}{Pt} = \frac{100\,I}{Pt}, \text{ or } r = \frac{100\,I}{Pt}.$$

That is, rate per cent. = (interest $\times$ 100) $\div$ (principal $\times$ time).

EXAMPLE.—At what rate per cent. will \$480 in 3 years 10 months give \$92 interest?

SOLUTION.—Applying the formula,

$$r = \frac{100 \times 92}{480 \times 3\frac{5}{6}} = \frac{\overset{5}{\cancel{100}} \times \overset{4}{\cancel{92}} \times \cancel{6}}{\underset{120}{\cancel{480}} \times \underset{20}{\cancel{23}}} = 5\%. \quad \text{Ans.}$$

35. **Rule.**—*Change the time to years, and divide 100 times the interest by the product of the principal and the time.*

EXAMPLES FOR PRACTICE.

36. Find the rate per cent. when

1. Principal = \$2,875, time = 4 yr. 7 mo. 6 da., interest = \$529.
2. Principal = \$760, time = 3 yr. 9 mo. 18 da., interest = \$144.40.
3. Principal = \$1,260, time = 2 yr. 1 mo. 10 da., interest = \$119.70.
4. Principal = \$2,340, time = 2 yr. 6 mo. 20 da., interest = \$328.90.
5. Principal = \$4,870, time = 3 yr. 5 mo. 24 da., interest = \$1,017.83.
6. Principal = \$7,200, time = 123 days, interest = \$114.80.
7. Principal = \$1,500, time = 1 yr. 9 mo. 18 da., interest = \$99.
8. Principal = \$1,600, time = 5 yr. 7 mo. 6 da., interest = \$380.80.

Answers.—(1) 4%; (2) 5%; (3) $4\frac{1}{2}\%$; (4) $5\frac{1}{2}\%$; (5) 6%; (6) $4\frac{4}{9}\%$; (7) $3\frac{2}{3}\%$; (8) $4\frac{1}{4}\%$.

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37. **Given the principal, interest, and rate per cent., to find the time.**

If we divide the equation $Prt = 100\,I$ by Pr, we shall have

$$\frac{Prt}{Pr} = \frac{100\,I}{Pr}, \text{ or } t = \frac{100\,I}{Pr}.$$

Otherwise expressed, this means that

time (in years) = (100 × interest) ÷ (principal × rate per cent.).

EXAMPLE.—In what time will $4,480 at 6% give $871.36 interest?

SOLUTION.—Applying the formula,

$$\text{time} = \frac{100 \times 871.36}{4{,}480 \times 6} = \frac{\overset{389}{\cancel{87136}}}{\underset{20}{\cancel{4480}} \times 6} = \frac{389}{120} \text{ yr.}$$

$\frac{389}{120}$ yr. $= 3\frac{29}{120}$ yr.; $\frac{29}{120} \times 12 = \frac{29}{10}$ mo. $= 2\frac{9}{10}$ mo.; $\frac{9}{10} \times 30 = 27$ da.

EXPLANATION.—The result is obtained in years, and must be reduced to years, months, and days. $\frac{389}{120}$ years $= 3\frac{29}{120}$ years. Since there are 12 months in one year, $\frac{29}{120}$ of a year $= \frac{29 \times 12}{120}$ $= \frac{29}{10}$ months, or $2\frac{9}{10}$ months. Since there are 30 days in one month, $\frac{9}{10}$ of a month $= \frac{9 \times 30}{10} = 27$ days. Hence, the required time is 3 yr. 2 mo. 27 da. Ans.

38. Rule.—*To find the time, when the principal, interest, and rate are given, divide 100 times the interest by the product of the principal and the rate per cent., and reduce the result, which is years, to years, months, and days.*

EXAMPLES FOR PRACTICE.

39. Find the time, when

1. Principal = $4,800, interest = $652, rate = 6%.
2. Principal = $680, interest = $103.20, rate = 5%.
3. Principal = $360, interest = $26.325, rate = 4½%.
4. Principal = $338.75, interest = $35.23, rate = 4%.
5. Principal = $1,080, interest = $112.50, rate = 3%.
6. Principal = $1,800, interest = $63, rate = 3½%.
7. Principal = $1,050, interest = $45.50, rate = 5%.
8. Principal = $1,000, interest = $143, rate = 6½%.
9. Principal = $2,400, interest = $74.80, rate = 5½%.

Answers.—(1) 2 yr. 3 mo. 5 da.; (2) 4 yr. 9 mo. 18 da.; (3) 1 yr. 7 mo. 15 da.; (4) 2 yr. 7 mo. 6 da.; (5) 3 yr. 5 mo. 20 da.; (6) 1 yr.; (7) 10 mo. 12 da.; (8) 2 yr. 2 mo. 12 da.; (9) 6 mo. 24 da.

40. Given the amount, time, and rate, to find the principal.

Since at 6% the interest of \$1 for, say, 2 years, is 12 cents, the amount is \$1 + 12 cents = \$1.12. Each \$1 of the principal will, in like manner, amount to \$1.12. Hence, if we divide the amount of any principal in a given time at a specified rate by the amount of \$1 for that time and rate, it will give the number of times \$1 is contained in the principal.

Or using the rate per cent. instead of the rate, we may, by means of the following formula, express the process of finding the principal:

$$P = \frac{100\,A}{100 + r\,t}.$$

That is, (100 × amount) ÷ (100 + rate per cent. × time) = principal.

EXAMPLE.—What principal will, in 5 yr. 6 mo., at 4%, amount to \$591.70?

SOLUTION.—Applying the formula,

$$P = \frac{100 \times 591.70}{100 + 4 \times 5\frac{1}{2}} = \frac{59{,}170}{122} = \$485. \quad \text{Ans.}$$

41. Rule.—*To find the principal, when the amount, rate, and time are given, divide 100 times the amount by 100 increased by the product of the rate per cent. and the time.*

NOTE.—In using this rule, and the formula given in Art. **40**, care must be taken that the time be expressed in years and fractions of a year.

EXAMPLES FOR PRACTICE.

42. Find the principal that will amount

1. To \$1,005 in 5 yr. 8 mo. at 6%.
2. To \$3,459 in 3 yr. 4 mo. 24 da. at 4½%.
3. To \$2,985 in 2 yr. 1 mo. 10 da. at 5%.
4. To \$1,443.60 in 2 yr. 10 mo. 24 da. at 7%.
5. To \$2,353.50 in 1 yr. 6 mo. 12 da. at 3%.

Answers.—(1) \$750; (2) \$3,000; (3) \$2,700; (4) \$1,200; (5) \$2,250.

TRUE DISCOUNT.

43. The student has learned that any deduction made from a debt or other obligation is a *discount.* In making such deductions, the element of *time* may or may not be considered. When time is considered, we have one of the applications of interest. **True discount** is discount when time is considered and *no interest is allowed on the discount.* True discount corresponds exactly to the problems of Interest given in Arts. **40–42**—the case in which the amount, rate, and time are given, to find the principal. The terms employed, however, are different.

The *principal* is called the **present worth.**

The *rate* is called the **rate of discount.**

The *interest* is called the **true discount.**

The *amount* is called the **debt,** or **obligation.**

True discount is so called to distinguish it from *bank discount,* which will be treated later.

44. The *present worth* of an obligation is a sum such that, if it be put at interest at a specified rate for a given time, it will amount to the obligation.

Thus, if the specified rate is 5%, a debt of $105 due in one year is worth $100 *now,* since $100 placed at interest at 5% will in one year amount to $105.

45. *True discount* is the difference between a debt due at a future time and its present worth.

Thus, $5 in the illustration given above is the true discount of $105 due in one year, when the rate of discount is 5%.

46. Given the debt, rate of discount, and time, to find the present worth and the discount.

The present worth may be found by means of the formula of Art. **40,** or by the rule of Art. **41.** Thus,

$$P = \frac{100\,A}{100 + r\,t}, \text{ or present worth } = \frac{100 \times \text{debt}}{100 + r\,t}.$$

EXAMPLE.—A debt of $773, which has 1 yr. 5 mo. 20 da. yet to run, is discounted at 5%. What is (*a*) the present worth, and (*b*) the discount?

SOLUTION.—1 yr. 5 mo. 20 da. $= 1\frac{17}{36}$ years.

(*a*) Present worth $= \dfrac{100 \times 773}{100 + 1\frac{17}{36} \times 5}$

$= \dfrac{77{,}300 \times 36}{3{,}865} = \$720.$ Ans.

(*b*) Discount $= \$773 - \$720 = \$53.$ Ans.

47. If the debt bears interest, its amount for the time it has to run must first be found, and that sum discounted at the specified rate.

EXAMPLE.—A debt of $1,200 having 1 yr. 6 mo. to run bears interest at 6%. What is its present worth discounted at 5%?

SOLUTION.—Amount of $1,200 in 1 yr. 6 mo. = $1,308.

Applying the formula,

Present worth $= \dfrac{100 \times 1{,}308}{100 + 1\frac{1}{2} \times 5} = \dfrac{130{,}800}{107.5} = \$1{,}216.74+.$ Ans.

48. **Rule.—I.** *Divide 100 times the amount of the debt when it is due, by 100 increased by the product of the time and the rate of discount. The result will be the present worth.*

II. *Subtract the present worth from the debt, and the remainder will be the discount.*

EXAMPLES FOR PRACTICE.

49. What is the present worth and the discount

1. Of $900 due in 9 months, discounted at 4%?
2. Of $1,000 due in 1 yr. 6 mo., discounted at 6%?
3. Of $800 due in 5 mo., discounted at 5%?
4. Of $2,800 due in 1 yr. 3 mo. 12 da., discounted at 8%?
5. Of $625 due in 2 yr. 5 mo. 15 da., discounted at 7%?
6. Of a note for $600 bearing interest at 5%, having 2 yr. 10 mo. to run, and discounted at 6%?

Answers.—(1) $873.79—, $26.21+; (2) $917.43+, $82.57—; (3) $783.67+, $16.33—; (4) $2,539.30—, $260.70+; (5) $533.24—, $91.76+; (6) $585.47, $99.53.

COMPOUND INTEREST.

50. If the interest of a principal is added to the principal at regular intervals to form by each addition a new principal for the next interval, the resulting interest is called **compound interest.**

Thus, if $100 be placed at compound interest at 6%, with the understanding that the interest is to be compounded annually, the principal will be $100 for the first year, $106 for the second year, $112.36 for the third year, etc.

51. Most savings banks allow compound interest, although in most States its payment cannot be legally enforced, even though it be specified in a contract.

Unless otherwise stated, interest is understood to be compounded annually. If it be compounded semiannually, one-half the annual rate is taken as the rate; if quarterly, one-fourth the annual rate is taken; etc.

52. When the time is given in years, months, and days, the interest is compounded for the greatest number of entire periods included in the time, and the simple interest of the last principal is found for the remaining time.

EXAMPLE.—Find the compound interest of $800 for 1 yr. 9 mo. 20 da. at 6%, interest compounded semiannually.

SOLUTION.—

$800 = prin. 1st 6 mo.
24 = int. 1st 6 mo. = $800 × .03.
$824 = prin. 2d 6 mo.
24.72 = int. 2d 6 mo. = $824 × .03.
$848.72 = prin. 3d 6 mo.
25.46 = int. 3d 6 mo. = $848.72 × .03.
$874.18 = prin. for 3 mo. 20 da.
16.03 = int. for 3 mo. 20 da. = $874.18 × .06 × $\frac{11}{36}$.
$890.21 = amt. for 1 yr. 9 mo. 20 da.
800 = original prin.
$90.21 = comp. int. for 1 yr. 9 mo. 20 da. Ans.

EXPLANATION.—In 1 yr. 9 mo. 20 da. there are three complete periods of 6 mo. each, and 3 mo. 20 da. besides. Since the

annual rate is 6%, for 6 mo. the rate per cent. is 3%. Finding the interest at 3%, and adding the principal for these three periods, gives $874.18. The amount of this sum for the remaining 3 mo. 20 da. is $890.21, from which we subtract the original principal. The remainder, $90.21, is the required compound interest.

53. Compound interest is calculated in actual business by means of a table. The table shows the amount of $1 at all the different rates, and for all the different times that are likely to occur. Having the amount of $1 at any given rate and for any number of periods, we multiply it by the number of dollars in any given principal. The result will be the amount of that sum for the given time. If the original principal be subtracted from this amount, the remainder is the compound interest required.

The formula for compound interest is, therefore,

$$\text{Comp. Int.} = P(1+r)^n - P.$$

In this formula, r is the rate, and n the number of periods —it is an exponent expressing the power to which $1+r$ must be raised to give the amount of $1 for that number of periods.

EXAMPLE.—What is the compound interest of $600 for 4 years at 6%, interest compounded annually?

SOLUTION.—Applying the formula,

$$\text{Comp. int.} = \$600\,(1+.06)^4 - \$600.$$

$1.06^4 = 1.262477$; $\$600 \times 1.262477 = \$757.4862$;

$\$757.4862 - \$600 = \$157.49$. Ans.

54. In the foregoing formula, r is the rate for one of the equal intervals of time—it is not necessarily the rate per year. For instance, if the interest is compounded quarterly, r is one-fourth of the rate per year; if compounded semiannually, r is one-half the rate per year. In the following table, the numbers in the columns headed "$2\frac{1}{2}$ per cent.," "3 per cent.," etc., are the values obtained by raising $1+r$ to the power whose exponent is the number of complete periods during which the interest is compounded. Thus, in the column headed "4 per cent.,"

COMPOUND INTEREST TABLE.

Showing the amount of $1, at various rates, compound interest, from 1 to 20 years, interest compounded annually.

Yr.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	6 per cent.
1	1.025000	1.030000	1.035000	1.040000	1.050000	1.060000
2	1.050625	1.060900	1.071225	1.081600	1.102500	1.123600
3	1.076891	1.092727	1.108718	1.124864	1.157625	1.191016
4	1.103813	1.125509	1.147523	1.169859	1.215506	1.262477
5	1.131408	1.159274	1.187686	1.216653	1.276282	1.338226
6	1.159693	1.194052	1.229255	1.265319	1.340096	1.418519
7	1.188686	1.229874	1.272279	1.315932	1.407100	1.503630
8	1.218403	1.266770	1.316809	1.368569	1.477455	1.593848
9	1.248863	1.304773	1.362897	1.423312	1.551328	1.689479
10	1.280085	1.343916	1.410599	1.480244	1.628895	1.790848
11	1.312087	1.384234	1.459970	1.539454	1.710339	1.898299
12	1.344889	1.425761	1.511069	1.601032	1.795856	2.012197
13	1.378511	1.468534	1.563956	1.665074	1.885649	2.132928
14	1.412974	1.512590	1.618695	1.731676	1.979932	2.260904
15	1.448298	1.557967	1.675349	1.800944	2.078928	2.396558
16	1.484506	1.604706	1.733986	1.872981	2.182875	2.540352
17	1.521618	1.652848	1.794676	1.947901	2.292018	2.692773
18	1.559659	1.702433	1.857489	2.025817	2.406619	2.854339
19	1.598650	1.753506	1.922501	2.106849	2.526950	3.025600
20	1.638616	1.806111	1.989789	2.191123	2.653298	3.207136

Yr.	7 per cent.	8 per cent.	9 per cent.	10 per cent.	11 per cent.	12 per cent.
1	1.070000	1.080000	1.090000	1.100000	1.110000	1.120000
2	1.144900	1.166400	1.188100	1.210000	1.232100	1.254400
3	1.225043	1.259712	1.295029	1.331000	1.367631	1.404908
4	1.310796	1.360489	1.411582	1.464100	1.518070	1.573519
5	1.402552	1.469328	1.538624	1.610510	1.685058	1.762342
6	1.500730	1.586874	1.677100	1.771561	1.870414	1.973822
7	1.605781	1.713824	1.828039	1.948717	2.076160	2.210681
8	1.718186	1.850930	1.992563	2.143589	2.304537	2.475963
9	1.838459	1.999005	2.171893	2.357948	2.558036	2.773078
10	1.967151	2.158925	2.367364	2.593742	2.839420	3.105848
11	2.104852	2.331639	2.580426	2.853117	3.151757	3.478549
12	2.252192	2.518170	2.812665	3.138428	3.498450	3.895975
13	2.409845	2.719624	3.065805	3.452271	3.883279	4.363492
14	2.578534	2.937194	3.341727	3.797498	4.310440	4.887111
15	2.759031	3.172169	3.642482	4.177248	4.784588	5.473565
16	2.952164	3.425943	3.970306	4.594973	5.310893	6.130392
17	3.158815	3.700018	4.327633	5.054470	5.895091	6.866040
18	3.379932	3.996019	4.717120	5.559917	6.543551	7.689964
19	3.616527	4.315701	5.141661	6.115909	7.263342	8.612760
20	3.869684	4.660957	5.604411	6.727500	8.062309	9.646291

1.872981 is the value of 1.04^{16} to 6 places of decimals; it is also the amount at compound interest of $1 for 16 years at 4%,

compounded annually, of $1 for 8 years at 8%, compounded semiannually, and of $1 for 4 years at 16%, compounded quarterly, since in each case the number of periods is 16 and the rate is .04. Hence, if it was required to find the amount at compound interest of $1 compounded annually, it may be obtained for any rate per cent. from 1 to 20 years from the table; if compounded semiannually, find the number of periods, divide the annual rate per cent. of interest by 2, and treat these results as "years" and "rate per cent.," respectively, when using the table; if compounded quarterly, divide the annual rate per cent. of interest by 4.

EXAMPLE.—Find the amount of $1 at compound interest for 3 yr. 6 mo. at 7% when compounded (*a*) annually; (*b*) semiannually; (*c*) quarterly.

SOLUTION.—(*a*) Referring to the table, the amount for 3 yr. at 7% is $1.225043. The simple interest for 6 mo. on $1.225043 at 7% is $1.225043 $\times .035 =$ $.042877—. The total amount is $1.225043 + $.042877— = $1.267920—. Ans.

(*b*) When the interest is compounded semiannually, there are $3\frac{1}{2} \times 2 = 7$ periods in 3 yr. 6 mo.; $7\% \div 2 = 3\frac{1}{2}\%$. Referring to the table, we find in the column headed "$3\frac{1}{2}$ per cent.," opposite 7, in column of years, 1.272279; hence, when the interest is compounded semiannually, the amount of $1 for 3 yr. 6 mo. at 7% is $1.272279. Ans.

(*c*) When the interest is compounded quarterly, there are $3\frac{1}{2} \times 4 = 14$ periods; $7\% \div 4 = 1\frac{3}{4}\%$. Since there is no column in the table headed "$1\frac{3}{4}$ per cent.," the amount must be calculated directly. The amount of $1 is, then, $\$1\,(1 + .0175)^{14} = \$1 \times 1.0175^{14} = \$1.274913$. Ans.

55. Rule.—*Multiply the amount of $1 for the given time and rate by the principal, and from the product subtract the principal. The remainder is the compound interest.*

If there is a partial period, find the amount of the last principal for this partial period, and then subtract the original principal.

EXAMPLES FOR PRACTICE.

56. Find the compound interest of the following amounts for the times mentioned, and prove the correctness of your result by using the table. Unless otherwise stated the interest is to be compounded annually.

1. Of $1,600 for 2 yr. at 8%.

2. Of $960 for 3 yr. 8 mo. at 6%.

3. Of $1,280 for 5 yr. 6 mo. at 4%.

4. Of $2,400 for 3 yr. 10 mo. 20 da. at 5%.

5. Of $360 for 4 yr. 4 mo. 12 da. at 6%.

6. Of $480 for 3 yr. 9 mo. 24 da. at 8%, interest compounded semi-annually.

7. Of $237.50 for 1 yr. 11 mo. 15 da. at 12%, interest compounded quarterly.

8. Of $3,875 for 2 yr. 11 mo. 12 da. at 6%, interest compounded semi-annually.

Answers.—(1) $266.24; (2) $229.11; (3) $308.46; (4) $501.78; (5) $104.49; (6) $167.65—; (7) $61.90—; (8) $738.48—.

PARTIAL PAYMENTS.

57. A debt or obligation may be discharged at one payment; or, from time to time, payments *in part* may be made, and finally at a time of *settlement* the remainder of the debt may be paid. Now, it is obvious that interest should be allowed upon such payments as are made, since interest is charged upon the obligation itself. But, if a payment should be less than the interest upon the debt since a previous payment had been made, to subtract such payment from the debt with accrued interest would result in increasing the principal. This would be a species of compound interest which, in many States, is illegal.

58. When a partial payment of a note is made, the date of payment and its amount are written upon the back of the note, and this record of it is called an **indorsement.**

The following rule for partial payments has been formulated by the Supreme Court of the United States, and has been adopted by most of the States:

59. United States Rule.—I. *Find the amount of the principal to the time when the payment, or the sum of the*

payments, is greater than the interest then due. From the amount subtract the payment or the sum of the payments, and treat the remainder as a new principal.

II. *Proceed in this manner to the date of settlement, and the last amount will be the sum still due.*

60. To compute partial payments by the United States rule.

EXAMPLE.—

$1,200. *New York, Sept. 16, 1895.*

On demand I promise to pay John Crawford, or order, Twelve Hundred Dollars, with interest at 6%, value received.

Edward G. Carson.

Indorsements: Jan. 1, 1896, $120; May 7, 1896, $300; Dec. 22, 1896, $16; Sept. 19, 1897, $400. What was due Jan. 1, 1898?

SOLUTION.—

Principal	$1200
Interest from Sept. 16, '95, to Jan. 1, '96 (3 mo. 15 da.)	21
Amount	1221
First payment	120
New principal	1101
Interest from Jan. 1, '96, to May 7, '96 (4 mo. 6 da.)	23.12
Amount	1124.12
Second payment	300
New principal	824.12
Interest from May 7, '96, to Sept. 19, '97 (1 yr. 4 mo. 12 da.)	67.58
Amount	891.70
Sum of third and fourth payments	416.00
New principal	475.70
Interest from Sept. 19, '97, to Jan. 1, '98 (3 mo. 12 da.)	8.09
Amount due at time of settlement	$ 483.79

Ans.

In this example, 360 days are considered as 1 year. The third payment of $16 is less than the interest due at the time it was made; hence, according to the rule, it is added to the next payment of $400 and the interest is computed to the time of the fourth payment.

EXAMPLES FOR PRACTICE.

61. Find the amount due at the time of settlement on each of the following:

1. *$2,000.* *Philadelphia, July 1, 1896.*

One year after date, for value received, I promise to pay Wm. Gray, or order, Two Thousand Dollars, with interest at 6%.

Henry G. Brown.

Indorsements: Dec. 16, 1896, $350; Mar. 1, 1897, $25; Oct. 25, 1897, $400; June 14, 1898, $275. Time of settlement, July 1, 1899.

2. A note for $3,000 at 6% dated Mar. 5, 1892, bore the following indorsements: Dec. 20, 1892, $400; Mar. 14, 1893, $60; Nov. 30, 1893, $360; July 15, 1894, $600. Time of settlement, Jan. 1, 1895.

3. A note for $4,000 at 6% dated Sept. 1, 1886, is indorsed as follows: Jan. 1, 1887, $500; July 1, 1887, $450; Jan. 1, 1888, $90; Sept. 1, 1888, $800. Time of settlement, Jan. 1, 1889.

4. Face of note, $3,600, rate 8%, dated May 1, 1890. Indorsements: Dec. 1, 1890, $600; Mar. 1, 1891, $100; Dec. 1, 1891, $800; July 1, 1892, $1,000. Time of settlement, Dec. 1, 1892.

Answers.—(1) $1,216.80; (2) $2,024.38; (3) $2,625.50; (4) $1,691.36.

When the time from the date of a note or other obligation is less than a year, settlement is usually made by a method called the *merchants' rule.*

62. The Merchants' Rule.—I. *By the method of exact interest, find the amount of each of the several payments from the time each is made to the date of settlement.*

II. *Subtract the sum of these amounts from the amount of the obligation from its date to the time of settlement. The remainder will be the amount still due.*

EXAMPLE.—Face of note, $2,000; rate, 6%; date of note, Dec. 31, 1888; time of settlement, Nov. 15, 1889. Indorsements: Mar. 10, 1889, $200; June 1, 1889, $300; Aug. 20, 1889, $400; Oct. 1, 1889, $500. What was due at time of settlement?

SOLUTION.—

Principal	$2000
Interest of $2,000 for *319 da	104.88
Amount	$2104.88

*Table XX, § 4, will be found very useful for finding the number of days between two dates.

Amount brought forward		$2104.88
Amount of $200 for 250 da	$208.22	
Amount of $300 for 167 da	$308.24	
Amount of $400 for 87 da	$405.72	
Amount of $500 for 45 da	$503.70	
Sum of payments, with interest		$1425.88
Amount due at time of settlement		$679.00

Ans.

EXAMPLES FOR PRACTICE.

63. Solve the following examples by the merchants' rule:

1. Debt, $5,000; rate, 5%; date of note, July 1, 1894; date of settlement, June 15, 1895. Indorsements: Sept. 10, 1894, $1,000; Dec. 12, 1894, $800; Mar. 12, 1895, $1,200; May 1, 1895, $1,200. What is due at the time of settlement?

2. Face of note, $8,400; rate, 4%; date of note, Jan. 1, 1897; time of settlement, Oct. 20, 1897. Indorsements: Mar. 30, 1897, $1,600; June 1, 1897, $800; Aug. 10, 1897, $2,000; Sept. 15, 1897, $3,000. What is due on the note at time of settlement?

3. Debt, $5,000; rate, 6%; date of note, Nov. 1, 1897; time of settlement, May 20, 1898. Indorsements: Dec. 20, 1897, $900; Jan. 1, 1898, $1,200; Mar. 24, 1898, $1,200; Apr. 20, 1898, $400. What is due at time of settlement?

Answers.—(1) $957.67; (2) $1,193.60; (3) $1,401.41.

PROMISSORY NOTES.

64. A **promissory note** is a written promise to pay a certain sum at a certain time.

65. The **maker** or **drawer** of a note is the person that promises to pay; the **payee** is the person to whom the note is payable; and the **holder** is the person that owns it.

66. The **face** of a note is the sum promised to be paid. This sum should be written both in figures and in words.

67. Notes are of two kinds—notes *bearing interest*, and notes *not bearing interest*. When no rate of interest is specified, the legal rate in the state or county where the note is made is to be understood. If a note not bearing

interest is not paid when due, it bears interest at the legal rate after that time until paid.

68. The following table is given for reference:

INTEREST LAWS.

Laws of each state and territory regarding rates of interest and penalties for usury, with the law or custom as to days of grace on notes and drafts. Compiled from the latest state and territorial statutes.

States and Territories.	Legal Rate of Interest. Per Cent.	Rate Allowed by Contract. Per Cent.	Penalties for Usury.	Grace or No Grace.
Alabama	8	8	Forfeiture of entire interest.	Grace.
Arizona	6	Any rate.	None.	No grace.
Arkansas	6	10	Forfeiture of principal and int.	Grace.
California	7	Any rate.	None.	No grace.
Colorado	8	Any rate.	None.	No grace.
Connecticut	6	6	None.	No grace.
Delaware	6	6	Forfeiture of principal and int.	No grace.
Dist. of Columbia	6	10	Forfeiture of entire interest.	No grace.
Florida	8	10	Forfeiture of entire interest.	No grace.
Georgia	7	8	Forfeiture of excess of interest.	No grace.
Idaho	7	12	None.	No grace.
Illinois	5	7	Forfeiture of entire interest.	No grace.
Indiana	6	8	Forfeiture of excess of interest.	No grace.
Indian Territory	6	10	Forfeiture of principal and int.	Grace
Iowa	6	8	Forfeiture of entire interest.	No grace.
Kansas	6	10	Forfeiture of excess of interest.	No grace.
Kentucky	6	6	Forfeiture of excess of interest.	No grace.
Louisiana	5	8	Forfeiture of entire interest.	No grace.
Maine	6	Any rate.	None.	Grace.
Maryland	6	6	Forfeiture of excess of interest.	No grace.
Massachusetts	6	Any rate.	None.	No grace.
Michigan	5	7	Forfeiture of entire interest.	No grace.
Minnesota	6	10	Forfeiture of principal and int.	No grace.
Mississippi	6	10	Forfeiture of entire interest.	Grace.
Missouri	6	8	Forfeiture of entire interest.	No grace.
Montana	8	Any rate.	None.	No grace.
Nebraska	7	10	Forfeiture of entire interest.	No grace.
Nevada	7	Any rate.	None.	Grace.
New Hampshire	6	6	Forfeiture of thrice the excess.	No grace.
New Jersey	6	6	Forfeiture of entire interest.	No grace.
New Mexico	6	12	Forfeiture of entire interest.	Grace.
New York*	6	6	Forfeiture of principal and int.	No grace.
North Carolina	6	6	Forfeiture of entire interest.	No grace.
North Dakota	7	12	Forfeiture of entire interest.	No grace.
Ohio	6	8	Forfeiture of excess of interest.	No grace.
Oklahoma	7	12	Forfeiture of entire interest.	Grace.
Oregon	6	10	Forfeiture of principal and int.	No grace.
Pennsylvania	6	6	Forfeiture of excess of interest.	No grace.
Rhode Island	6	Any rate.	None.	No grace.
South Carolina	7	8	Forfeiture of entire interest.	Grace.
South Dakota	7	12	Forfeiture of entire interest.	Grace.
Tennessee	6	6	Forfeiture of excess of interest.	No grace.
Texas	6	10	Forfeiture of entire interest.	Grace.
Utah	8	Any rate.	None.	No grace.
Vermont	6	6	Forfeiture of excess of interest.	No grace.
Virginia	6	6	Forfeiture of entire interest.	No grace.
Washington	6	12	None.	No grace.
West Virginia	6	6	Forfeiture of excess of interest.	No grace.
Wisconsin	6	10	Forfeiture of entire interest.	No grace.
Wyoming	8	12	Forfeiture of entire interest.	No grace.

* Upon call loans of $5,000 or upwards, on collateral security, any rate of interest is legal.

69. A note should be so written as to show where it was made and when, the sum promised to be paid, whether it does or does not bear interest, and the words "value received." The law assumes that no one is to be compelled to pay unless he has received what he deems an equivalent. If "value received" is omitted, the holder may have to prove that the maker of the note did actually receive value for the money promised in it.

70. A note usually specifies where it is to be paid—usually at a bank. If no place is designated, the holder's place of business is understood.

71. If a note contains the words "or order," it is a **negotiable note,** and may pass like a bank note from one person to another. If the holder of a negotiable note wishes to dispose of it, he is generally required to guarantee its payment by *indorsing* it—that is, by writing his name across the back of the note. There are several kinds of indorsements. Thus, if the holder is John Smith, he may, on the back of the note, write *John Smith.* This is an indorsement *in blank*, and makes John Smith responsible for the payment of the note.

He may write, *Pay to William Jones.* The note is then payable only to William Jones.

If he indorses it, *Pay to William Jones, or order*, it is payable to William Jones, or to any one to whom William Jones may order it to be paid.

He may indorse it, *Pay to William Jones, or bearer*, and it is payable to any person that presents it.

If it be indorsed, *Pay to bearer*, it is payable to the person that presents it for payment.

72. A **joint and several** note is a note signed by two or more persons, who become collectively and individually responsible for its payment.

A note is **protested** when the holder of it notifies the indorsers in legal form and within the time prescribed by law that the note is unpaid. Unless such protest is legally

made, the indorsers are not responsible for its payment. This protest must reach the indorser not later than the day when the note is payable.

73. Some forms of notes used in actual business are given below.

$250. *New York, Sept. 17, 1896.*

On demand I promise to pay George Camp, or order, Two Hundred and Fifty Dollars, value received.

Howard Gray.

$1,000. *Scranton, July 5, 1898.*

Three months after date, for value received, I promise to pay Stephen Girard, or order, One Thousand Dollars, with interest at 5%. *Charles Goldwin.*

$3,000. *Philadelphia, July 5, 1898.*

Six months after date, we, or either of us, will pay to George Owen, Three Thousand Dollars, value received, with interest at 6%.

George Kirwin.
Henry Potter.
Erastus Kirby.

Payable at the First National Bank.

74. In most states, *three days of grace* are allowed before a note must be paid. If a bank discounts a note, interest is charged for days of grace in States where days of grace are legal.

75. If a note falls due on a Sunday, or on a legal holiday, it is usually payable during banking hours on the business day preceding its maturity. Interest, however, is charged for three days of grace in such cases. In some States, a note falling due on a Sunday or a legal holiday is payable on the first business day thereafter.

EXAMPLES FOR PRACTICE.

76. Solve the following examples:

1. Write a negotiable demand note for $600, with interest at 6%, and make Brown, Jones & Co. the payee.

2. Write a non-negotiable note for $4,000, with interest at 5%, payable in 30 days, yourself being the maker and Howard Crosby the payee.

3. How much will pay the following note when due without grace?

$575\frac{50}{100}$. *New York, Sept. 19, 1898.*

Sixty days after date, for value received, I promise to pay Ralph Newton, or order, Five Hundred Seventy-Five and $\frac{50}{100}$ Dollars, with interest at $4\frac{1}{2}\%$.

Henry Miles.

Ans. $579.82.

BANK DISCOUNT.

77. **Bank discount** is the charge made by a bank for paying a note or other obligation before it is due. This charge is the interest on the amount of the obligation from the time it is discounted until its maturity. This interest is subtracted from the face of the obligation, and its holder receives for it the remainder, which is called the **proceeds.** Hence, bank discount is inequitable, since interest is charged, not only upon the sum actually paid for the obligation, but also upon the discount.

78. In states where days of grace are allowed, bank discount is calculated for 3 days more than the time specified in the note.

Thus, if a 60-day note for $1,000 is discounted at a bank, the interest of $1,000 is found for 63 days, and is subtracted from $1,000. If the rate of discount is 6%, the holder will receive as proceeds $1,000 − $10.50 = $989.50.

79. It is evident that the owner of the note should receive for it the *true present worth* of $1,000 payable in 63 days, or $989.61. The bank gives him only $989.50, or 11 cents less than he should get. When it is considered that the sums annually discounted by most banks mount up into millions, it will be seen how much of their gain is unearned.

80. The **maturity** of a note is on the last day of grace. The time of maturity is generally indorsed on the note, thus, Mar. 7/10, which means that it matures nominally on Mar. 7, and legally on Mar. 10.

The **term of discount** is the time from the discounting of the note to its maturity.

81. In the case of an interest-bearing note, the sum discounted is the amount of the note at maturity.

82. Banks usually require that a discounted note shall be payable at the bank that discounted it, and they rarely discount notes having more than 90 days to run.

83. To find the time when a note matures, the term of discount, the discount, and the proceeds.

EXAMPLE.—Find (*a*) the discount, and (*b*) the proceeds of the following note:

$484\frac{60}{100}$. *Newark, N. J., Oct. 4, 1897.*

Sixty days after date, for value received, I promise to pay William Hall, or order, Four Hundred Eighty-Four and $\frac{60}{100}$ Dollars, at the Ninth National Bank. *Henry Parshall.*

Discounted, Oct. 20, 1897, at 6%.

SOLUTION.—

(*a*) Maturity, Dec. 3/6, 1897.
Term of discount, 47 days.
Discount, $3.80. Ans.

(*b*) Proceeds, $484.60 − $3.80 = $480.80. Ans.

EXPLANATION.—Sixty days after Oct. 4 is Dec. 3, and three days of grace make the date of legal maturity Dec. 6. From the time of discount, Oct. 20, to Dec. 6 is 47 days, for which the interest at 6% is $3.80. Subtracting the discount, $3.80, from the face of the note, $484.60, gives $480.80, the proceeds.

EXAMPLE.—Find (*a*) the discount and (*b*) the proceeds of the following note:

$1,060. *Chicago, Ill., August 6, 1898.*

For value received, I promise to pay, three months after date, to Ernest Hazard, or order, One Thousand Sixty Dollars, with interest at 5%. *Emil Reeves.*

Discounted, Sept. 1, 1898, at 6%.

SOLUTION.—

(*a*) Maturity, Nov. 6/9, 1898.

Amount of note at maturity	$1073.69	
Term of discount, 69 days		
Discount	12.35	Ans.

(*b*) Proceeds ..... $1061.34 Ans.

84. Rule.—I. *If the note bears interest, find its amount at the time of maturity.*

II. *Find the interest on the face of the note, or, if it is an interest-bearing note, on the amount of the note at maturity at the given rate of discount for 3 days more than the time it has to run until its nominal maturity, and the result will be the bank discount.*

III. *Subtract the bank discount from the face of the note, or from its amount at maturity, and the remainder will be the proceeds.*

EXAMPLES FOR PRACTICE.

85. Solve the following examples:

1. Find (*a*) the bank discount and (*b*) the proceeds of a note for $5,000, due in 60 days, discounted at 6%. Ans. (*a*) $52.50. (*b*) $4,947.50.

2. Find (*a*) the bank discount and (*b*) the proceeds of a note for $4,000, due in 90 days, discounted at 5%. Ans. (*a*) $51.67. (*b*) $3,948.33.

3. Required, (*a*) the bank discount and (*b*) the proceeds of a note for $7,600, due in 30 days discounted at 8%. Ans. (*a*) $55.73. (*b*) $7,544.27.

4. A note for $8,000 dated July 5, 1898, is discounted at 5% on Sept. 7, 1898. If it is a 90-day note, what are the proceeds? Ans. $7,967.78.

5. A note for $2,800 bearing interest at 6%, and due in 60 days, is discounted at 5%. What are the proceeds? Ans. $2,804.64.

6. $8,476$\frac{00}{100}$. *St. Louis, Mo., Jan. 8, 1898.*

Six months after date I promise to pay to Charles Brown, or order, Eight Thousand Four Hundred Seventy-Six Dollars, value received, with interest at 6%. *Howard Bristow.*

Find the proceeds of the foregoing note if discounted April 25, 1898, at 5%. Ans. $8,641.11.

7. A note for $2,800, due in 3 months, is dated June 5, 1899, and bears interest at 7%. It is discounted July 20, at 6%. Find the proceeds. Ans. $2,826.87.

8. A note for $96,000, with interest at 8%, is dated Nov. 16, 1897, and is due in 90 days. It is discounted Dec. 20, at 6%. Find how much the proceeds differ from the true present worth. Ans. $9.38.

86. To find the face of a note when the proceeds, time, and rate are given.

EXAMPLE.—The proceeds of a note discounted at a bank for 45 days at 6% were $1,488. What was the face of the note?

SOLUTION.—

Proceeds of $1 for 45 + 3 days = $.992.

Face of the note = $1,488 ÷ $.992 = $1,500. Ans.

87. Rule.—*Divide the proceeds by the proceeds of $1 for 3 days more than the given time.*

EXAMPLES FOR PRACTICE.

88. Solve the following examples:

1. A 60-day note discounted at a bank at 6% yields $1,246.77. What is its face? Ans. $1,260.

2. The proceeds of a note for 2 mo. 12 da., discounted at a bank at 7%, are $4,079.625. Find the face of the note. Ans. $4,140.

3. Given, rate of discount, 8%; time, 90 days; proceeds, $3,613.74. Find the face of the note. Ans. $3,690.

4. Given, rate of discount, 7%; time, 30 days; proceeds, $86,382.135. Find the face of the note. Ans. $86,940.

5. Given, time, 51 days; rate, 4%; proceeds, $484.575. What is the face of the note? Ans. $487.50.

6. A note for 60 days discounted at 4½% yields $81,815.335. What is its face? Ans. $82,464.75.

7. Write in proper form a 60-day note payable at the Chemical Bank of New York, which, when discounted when the note is made, will yield at 5%, $7,850 proceeds. Ans. Face, $7,915.97.

DUTIES.

89. Duties, or **customs,** are taxes levied by governments on imported goods for the purpose of producing revenue and for the protection of home industries.

90. There are two kinds of duties: *ad valorem* and *specific*. An **ad valorem** duty is estimated at a certain per cent. of the market value of the goods in the country from

which they are imported; as, silks, 50%, musical instruments 15%, etc. The market value of the goods is the invoice value after deducting discounts and before extra charges, such as commission, freight, boxing, etc., are added.

91. A **specific** duty is a duty levied on imported goods according to the weight, measurement, or number of the articles, without reference to their value; as, wheat 15 cents per bushel, coal 75 cents per ton, etc. Some kinds of merchandise are subject to both ad valorem and specific duties. In computing specific duties, the long ton of 2,240 pounds and the hundredweight of 112 pounds are used.

92. An **invoice** is an itemized statement of the merchandise shipped. It contains the names of purchaser and seller, a description of the quality and quantity of the goods, the price and incidental charges. Invoices are made out in the weights and measures and the currency of the country from which the goods are imported. Thus, the price and cost of goods imported from Germany would be given in *marks;* from France, in *francs;* from England, in £ *s. d.*

93. Before computing duties the following allowances are made: **Tare,** a deduction for the weight of boxes or crates; **leakage,** an allowance for loss of liquids imported in barrels or casks; and **breakage,** an allowance for loss of liquids imported in bottles. The **net quantity** is what remains after deducting tare, leakage, or breakage.

94. Ad valorem duties are computed by the rules of percentage; the net invoice price is regarded as the *base*, the ad valorem duty as the *percentage*, and the rate of duty as the *rate.*

Duties are not computed on fractions of a dollar; if the cents are less than 50 they are rejected; if more, they are counted as a dollar.

95. **Rule.**—*To find the ad valorem duty, reduce the net invoice price to U. S. money, if necessary, deduct allowances, and multiply the remainder (expressed in even dollars) by the rate of ad valorem duty.*

To find the specific duty, multiply the net quantity by the rate of specific duty per unit of quantity.

EXAMPLE 1.—What is the duty on an invoice of silks valued at 24,360 francs, the ad valorem rate being 60%?

SOLUTION.—Referring to Art. **98**, 24,360 francs = 24,360 × .193 = $4,701.48. Duty = $4,701 × .60 = $2,820.60. Ans.
The 48 ct. is rejected, being less than 50 ct. (Art. **94.**)

EXAMPLE 2.—What is the duty on 820 gallons of brandy at $1.50 per gallon, leakage 3%?

SOLUTION.— Leakage = 820 × .03 = 24.6 gal.
Net quantity = 820 − 24.6 = 795.4 gal.
Duty = 795.4 × 1.50 = $1,193.10. Ans.

EXAMPLES FOR PRACTICE.

96. What is the ad valorem duty on an importation invoiced at
(*a*) £430 12 s. 4 d., allowing 5% breakage, rate of duty 40%?
(*b*) 36,750 lira, allowing 2% for tare, rate of duty 24%?
(*c*) 9,264 marks, rate of duty 85%?
(*d*) 4,700 yen, rate of duty 14%?

Ans. (*a*) $796.40. (*b*) $1,668.24. (*c*) $1,874.25. (*d*) $327.74.

What is the specific duty on an importation of
(*e*) 3,200 bushels potatoes at 15 cents per bushel?
(*f*) 60 dozen bottles of wine at $3.00 per dozen, breakage 10%?
(*g*) 125 gross of empty bottles, breakage 6%, duty 10 cents per dozen?
(*h*) 3 tons 6 hundredweight of iron castings at $\frac{3}{4}$ cent per pound?

Ans. (*e*) $480. (*f*) $162. (*g*) $141. (*h*) $55.44.

1. What is the duty on 25,670 pounds of pig iron at $5.00 per ton? Ans. $57.30.

2. What is the duty on five blocks of marble each 12 feet long, 4 feet wide, and $2\frac{1}{2}$ feet thick, at 65 cents per cubic foot? Ans. $390.

3. What is the duty on an importation of 2,650 yards of woolen goods weighing 620 pounds net and valued at 72 cents per yard, the rates of duty being 60 cents per pound and 30% ad valorem? Ans. $944.40.

4. An importation of musical instruments from Germany is valued at 13.670 marks; what is the duty at 17½% ad valorem? Ans. $569.28.

5. An importer buys French silks at $1.80 per yard and pays a duty of 35% ad valorem, and $.60 per yard specific; at what price per yard must the silk be sold to yield a profit of 25%? Ans. $3.79.

6. What is the duty at 65%, upon a consignment of 1,350 dozen kid gloves invoiced at 115 francs per dozen? Ans. $19,475.95.

CONVERSION OF CURRENCY.

97. For commercial transactions between importers and exporters, each country has established a monetary system different from the systems of other countries; it is important, therefore, to know the equivalents of these different systems. For every country, either gold or silver has been chosen as the standard. A certain fixed weight of that standard has a name, and is called the **monetary unit.** In most countries, this unit is not coined; thus, the pound sterling is coined, but the gold dollar, the gold franc, the gold mark, etc. are not.

The gold valuations of these units are proclaimed each year by the Secretary of the Treasury of the United States, and these valuations are used in custom-house computations of the value of all importations of merchandise.

98. The following table gives a list of the countries having fixed currencies, their standards, monetary units, United States gold equivalents, and their principal coins.

COUNTRIES WITH FIXED CURRENCIES.

NOTE.—The following official (United States Treasury) valuations of foreign coins do not include "rates of exchange."

Countries.	Standard.	Monetary Unit.	Value in U. S. Gold.	Coins.
Argentine Republic .	Gold and silver	Peso	$0.965	Gold—argentine ($4.824) and ½ argentine; silver—peso and divisions.
Austria-Hungary . .	Gold	Crown	.203	Gold—20 crowns ($4.052) and 10 crowns.
Belgium	Gold and silver	Franc	.193	Gold—10 and 20 franc pieces; silver—5 francs.
Brazil	Gold	Milreis	.546	Gold—5, 10, and 20 milreis; silver—½, 1, and 2 milreis.
British North America (except Newfoundland)	Gold	Dollar	1.00	
British Honduras . .	Gold	Dollar	1.00	
Chile	Gold	Peso	.365	Gold—escudo ($1.25), doubloon ($3.65), and condor ($7.30); silver—peso and divisions.
Costa Rica	Gold	Colon	.465	Gold—2, 5, 10, and 20 colons; silver—5, 10, 25, and 50 centisimos.
Cuba	Gold and silver	Peso	.926	Gold—doubloon ($5.01,7); silver—peso (60 cents).
Denmark	Gold	Crown	.268	Gold—10 and 20 crowns.
Ecuador	Gold	Sucre	.487	Gold—10 sucres ($4.8665); silver—sucre and divisions.
Egypt	Gold	Pound (100 piasters)	4.943	Gold—10, 20, 50, and 100 piasters; silver—1, 2, 10, and 20 piasters.
Finland	Gold	Mark	.193	Gold—10 and 20 marks ($1.93 and $3.85,9).
France	Gold and silver	Franc	.193	Gold—5, 10, 20, 50, and 100 francs; silver—5 francs.
Germany	Gold	Mark	.238	Gold—5, 10, and 20 marks.
Great Britain	Gold	Pound sterling	4.86⅔	Gold—sovereign (pound sterling) and half sovereign.
Greece	Gold and silver	Drachma	.193	Gold—5, 10, 20, 50, and 100 drachmas; silver—5 drachmas.
Haiti	Gold and silver	Gourde	.965	Silver—gourde.
India	Gold	Rupee	.324	Gold—sovereign ($4.8665); silver—rupee and divisions.
Italy	Gold and silver	Lira	.193	Gold—5, 10, 20, 50, and 100 lire; silver—5 lire.
Japan	Gold	Yen	.498	{ Gold—1, 2, 5, 10, and 20 yen.
Liberia	Gold	Dollar	1.00	{ 100 sen = 1 yen.
Netherlands	Gold and silver	Florin	.402	Gold—10 florins; silver—½, 1, and 2½ florins.
Newfoundland . . .	Gold	Dollar	1.014	Gold—$2 ($2.02,7).
Peru	Gold	Sol	.487	Gold—libra ($4.8665); silver—sol and divisions.
Portugal	Gold	Milreis	1.08	Gold—1, 2, 5, and 10 milreis.
Russia	Gold	Ruble	.515	Gold—imperial ($7.718) and ½ imperial ($3.80); silver—¼, ½, and 1 ruble.
Spain	Gold and silver	Peseta	.193	Gold—25 pesetas; silver—5 pesetas.
Sweden and Norway	Gold	Crown	.268	Gold—10 and 20 crowns.
Switzerland	Gold and silver	Franc	.193	Gold—5, 10, 20, 50, and 100 francs; silver—5 francs.
Turkey	Gold	Piaster	.044	Gold—25, 50, 100, 200, and 500 piasters.
Uruguay	Gold	Peso	1.034	Gold—peso; silver—peso and divisions.
Venezuela	Gold and silver	Bolivar	.193	Gold—5, 10, 20, 50, and 100 bolivars; silver—5 bolivars.

99. To convert foreign money into United States gold valuations.

Rule.—*Multiply the total amount in foreign money by the United States equivalent of the foreign unit.*

EXAMPLE.—What is the value, in United States money, of the following French invoice?

Goods.	French Money. Francs.	United States Money.	
		Dollars.	Cents.
28½ gross bottles of brandy, at 74 francs per dozen			
80 pieces silk, each 32 meters by 75 centimeters, at 4 francs 95 centimes per square meter			
Total, in United States money			

SOLUTION.—

28½ gr. = 28½ × 12 = 342 doz.
342 × 74 fr. = 25,308 fr.
25,308 × $.193 = $4,884.44.
75 cm. = .75 m.
32 m. × .75 = 24 sq. m.
24 sq. m. × 80 = 1,920 sq. m.
1,920 × 4.95 fr. = 9,504 fr.
9,504 × $.193 = $1,834.27.
$4,884.44 + $1,834.27 = $6,718.71.

Goods.	French Money. Francs.	United States Money.	
		Dollars.	Cents.
28½ gr. bottles of brandy, at 74 fr. per doz.	25,308	4,884	44
80 pieces silk, each 32 m. by 75 cm., at 4 francs 95 centimes per sq. m.	9,504	1,834	27
Total, in United States money		6,718	71

Ans. $6,718.71.

100. To convert United States money into foreign money.

Rule.—*Divide the total amount, in United States money, by the United States equivalent of the foreign unit.*

EXAMPLE.—What is the value, in marks, of the following United States invoice?

Goods.	United States Money.		German Money. Marks.
	Dollars.	Cents.	
20,720 pounds cement at $11.42 per ton, less 5%			
190 plates of plate glass 27 in. × 48 in. at $2.14 per square foot			
Total, in German money			

SOLUTION.—

20,720 lb. ÷ 2,240 = 9.25 T.
9.25 × $11.42 = $105.64.
$105.64 × .05 = $5.28. $105.64 − 5.28 = $100.36.
$100.36 ÷ .238 = 421.68 marks.
27 in. × 48 in. = 1,296 sq. in.
1,296 sq. in. × 190 = 246,240 sq. in.
1 sq. ft. = 144 sq. in.
246,240 sq. in. ÷ 144 = 1,710 sq. ft.
1,710 × $2.14 = $3,659.40.
$3,659.40 ÷ .238 = 15,375.63 marks.
421.68 + 15,375.63 = 15,797.31 marks.

Goods.	United States Money.		German Money. Marks.
	Dollars.	Cents.	
20,720 lb. cement at $11.42 per T., less 5%	100	36	421.68
190 plates of plate glass 27 in. × 48 in., at $2.14 per sq. ft.	3,659	40	15,375.63
Total, in German money			15,797.31

Ans. 15,797.31 marks.

EXAMPLES FOR PRACTICE.

101. Solve the following examples:

1. Find the value, in marks, of the following United States invoice:

Goods.	United States Money.		Marks.
	Dollars.	Cents.	
144 dozen bottles of wine at $12 per gross . . .			
125 tons coal at $3.75 per ton			
Total, in German money			

Ans. 2,574.58 marks.

2. What is the duty on cigars and cigarettes valued at 24,352 pesetas and weighing 239.5 kilos, ad valorem duty 25% and specific duty $4.50 per pound? Ans. $3,551.01.

3. What is the duty on an invoice of 10,000 pairs French hosiery valued at 9,000 francs, ad valorem duty 15% and specific duty 67½ cents per dozen pairs? Ans. $823.05.

4. Find the value, in United States money, of the following Japanese invoice:

Goods.	Japanese Money.	United States Money.	
	Yens.	Dollars.	Cents.
2,000 pieces earthenware valued at 6,000 yens, allowing a discount of 4½%			
600 rolls matting 40 yd. to the roll at 50 sens per yard			
Total, in United States money			

Ans. $8,829.54

ARITHMETIC.

(PART 9.)

STOCKS AND BONDS.

1. If any work involving a large expenditure of money is to be undertaken, it is usual to organize a company, and procure a charter under the laws of some state. The chartered company then issues shares, which are sold to any persons having money to invest, and willing to incur the chances of loss. The advantage of having a charter for the company is that the shareholders, in case the business is unprofitable, are liable for the debts of the company only to the amount of their shares. Otherwise, any member of the company can be compelled to pay all of its debts. Moreover, a chartered company can do business just as an individual—that is, it may sue or be sued for debts, enter into contracts, etc.

2. The **par value** of shares is the amount—usually \$25, \$50, or \$100—specified in the certificates issued to its subscribers.

When the business of a company is profitable, its shares sell *above par*, or for more than their face value; if it is unprofitable, the shares sell at a *discount*, or *below par*.

3. When a company gains in its business, it pays its shareholders, from time to time, part of its profits, called **dividends.**

4. Dividends are *declared* quarterly, semiannually, or annually, and are usually paid at the general office of the company.

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5. If the business is at a loss, the shareholders may be required to make good the loss. Such payments are called **assessments.**

6. If a man buys stocks of a company and makes partial payments on them, such payments are called **instalments.**

7. A **bond** is a written obligation under seal to pay a certain sum at a specified time.

8. Bonds to furnish money for the national or any state government, or for a city, county, town, village, or for an incorporated company, are prepared and sold in the open market. The money accruing from such sales may be used for current expenses, or for such improvements as may be desired. The bonds are secured by the property of those who issue them, and bear interest payable quarterly, semi-annually, or annually.

9. **Registered bonds** are numbered, and the names of their purchasers are recorded. To sell registered bonds, the transfer must be recorded on the books of the company that issued them. Sometimes bonds have **coupons** attached, stating the amount of interest due at certain times. These coupons may be detached, and exchanged for money at the general office of the company, or at a bank acting for the company.

10. Government or State bonds are usually designated by the interest they bear, or by the time when they are payable. Thus, "U. S. $3\frac{1}{2}$'s, 1907" are bonds of the United States Government bearing interest at $3\frac{1}{2}\%$, and payable in 1907.

11. A **stock broker** is a person whose business consists in buying and selling bonds or stocks for others. His compensation is a certain per cent. of the *par value* of the stocks bought or sold. The compensation of a broker is called **brokerage.**

12. The *par value* of stocks is to be understood as 100, unless some other value is given. Whatever may be the market price of stocks and bonds, brokerage is calculated on their par value.

EXAMPLE 1.—Find the cost of 480 shares of Canadian Pacific stock bought at $123\frac{1}{2}$, if the brokerage is $\frac{1}{8}\%$.

SOLUTION.— $(123\frac{1}{2} + \frac{1}{8}) \times 480 = \$59{,}340$. Ans.

EXAMPLE 2.—How many shares of bank stock selling at 112 can be bought for \$89,700, if the brokerage is $\frac{1}{8}\%$? The par value of the shares is \$50.00.

SOLUTION.—The cost of 1 share at the market price is $1.12 \times \$50 = \$56$; the brokerage per share is $.00\frac{1}{8} \times \$50 = \$0.06\frac{1}{4}$. Therefore, total cost of 1 share $= \$56 + \$0.06\frac{1}{4} = \$56.06\frac{1}{4}$, and the number of shares bought $= \$89{,}700 \div \$56.06\frac{1}{4} = 1{,}600$ shares. Ans.

13. NOTE.—Unless otherwise stated, the par value of each share will always be assumed to be \$100.00. Then, in example 1 above, the cost of 1 share is \$123.50. But the broker gets $\frac{1}{8}\%$, or $\frac{1}{8}$ of \$1.00 per share for selling them. Therefore, the total cost to the purchaser of 480 shares is $(123\frac{1}{2} + \frac{1}{8}) \times 480 = \$59{,}340$.

14. Rule.—I. *To find the cost of any number of shares of stock, multiply the sum of the market price per share and the brokerage by the number of shares, and the product will be the cost.*

II. *To find the number of shares that can be bought for a given sum, divide the given sum by the cost of one share, including the brokerage, and the quotient will be the number of shares.*

15. EXAMPLE.—How much must be invested in railroad stock that pays a quarterly dividend of $2\frac{1}{2}\%$, in order to have an income of \$4,000, if they are bought at $104\frac{1}{2}$, brokerage being $\frac{1}{8}\%$?

SOLUTION.—The expression, "a dividend of $2\frac{1}{2}\%$," means $2\frac{1}{2}$ per cent. on the *par* value of the stock. Hence, since the *quarterly* dividend is $2\frac{1}{2}\%$, the *annual* income per share of \$100 will be \$10. Consequently, to obtain an annual income of \$4,000, there must be bought $4{,}000 \div 10 = 400$ shares; then each share will cost $\$104\frac{1}{2} + \$\frac{1}{8}$, and their total cost will be 400 times as much, or $\$104\frac{5}{8} \times 400 = \$41{,}850$. Ans.

EXAMPLE.—What per cent. is realized by buying 4% bonds at $89\frac{7}{8}$, brokerage being $\frac{1}{8}\%$?

SOLUTION.—Since each share costs $\$89\frac{7}{8} + \$\frac{1}{8} = \$90$, and each share yields \$4 annual income, the per cent. realized will be found by dividing the gain, \$4, by the entire cost of one share. Hence,

$$\$4 \div (\$89\tfrac{7}{8} + \$\tfrac{1}{8}) = .04\frac{4}{9} = 4\frac{4}{9}\%. \quad \text{Ans.}$$

16. Rule.—I. *To find the investment that will yield a given income, divide the income by the gain from one share,*

and the quotient will be the number of shares that must be bought; then multiply the cost of one share by the number of shares, and the product will be the investment.

II. *To find the rate per cent. of income from money invested in stocks or bonds, divide the gain yielded by one share by the cost of a share, and multiply the quotient by 100.*

EXAMPLES FOR PRACTICE.

17. 1. What must be paid for 128 shares of Standard Oil stock at $128\frac{1}{4}$, brokerage $\frac{1}{8}\%$? Ans. $16,432.

2. How many shares of Union Gas Co. stock at $98\frac{3}{4}$ can be bought for $39,550, the brokerage being $\frac{1}{8}\%$? Ans. 400 shares.

3. How much will 68 U. S. 4% bonds of 1907 cost at $116\frac{1}{2}$, brokerage being $\frac{1}{8}\%$? Ans. $7,930.50.

4. The cost of some railroad stock was $18,150, for which a man paid $137\frac{3}{8}$ per share, and $\frac{1}{8}\%$ brokerage. How many shares did he buy? Ans. 132 shares.

5. Find the cost of 240 shares of mining stock at $98\frac{3}{8}$, brokerage being $\frac{1}{4}\%$. Ans. $23,670.

6. How much must be paid for 5% city bonds to yield an annual income of $1,250, if they cost $104\frac{7}{8}$, brokerage $\frac{1}{8}\%$? Ans. $26,250.

7. What is the rate per cent. received on the foregoing investment? Ans. $4\frac{16}{21}\%$.

8. Bank stock that pays an annual dividend of 10% is bought for $109\frac{7}{8}$, brokerage $\frac{1}{8}\%$. What per cent. is realized by investing in it? Ans. $9\frac{1}{11}\%$.

9. How much must be paid for the bank stock mentioned above, to endow a college professorship with an annual income of $5,000? Ans. $55,000.

10. How much more is the rate per cent. of income on 8% stock bought at $119\frac{7}{8}$ than on 6% stock bought at $107\frac{7}{8}$, brokerage in each case being $\frac{1}{8}\%$? Ans. $1\frac{1}{6}\%$.

11. How much per cent. better is an investment in 8% securities bought at 80 than 7% stock at 84, not regarding brokerage? Ans. $1\frac{2}{3}\%$.

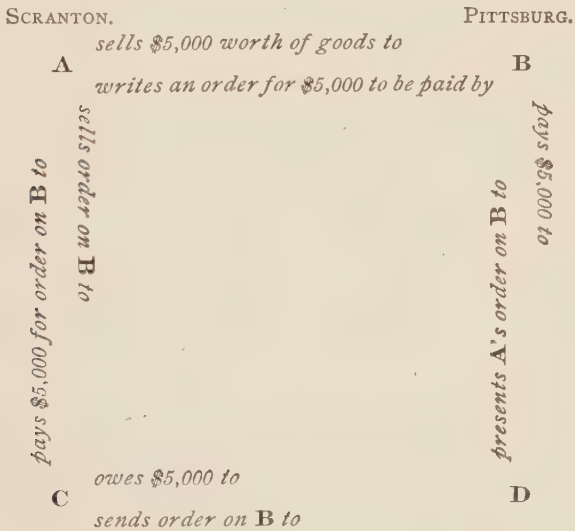
12. How much better is a gain of 20% on an investment at 80 than a gain of 18% on an investment at 90? Ans. 5%.

13. A man left his wife an annual income of $8,000 by an investment in 5% bonds bought at $111\frac{1}{4}$, brokerage being $\frac{1}{8}\%$. What did they cost? Ans. $178,200.

EXCHANGE.

18. **Exchange** is a method of paying debts in a distant place without transmitting money. It is done by means of *drafts*, or *bills of exchange*.

19. Thus, suppose that A, who does business in Scranton, *sells* $5,000 worth of goods to B, who does business in Pittsburg; also, that C in Scranton *owes* $5,000 to D in Pittsburg. Then, B in Pittsburg owes A in Scranton, and C in Scranton owes D in Pittsburg. A writes an order directing B to pay to C $5,000, and sells this order to C, who pays A $5,000 for it. C endorses the order, and makes it payable to D; he sends the order to D, who presents it to B, and B pays D $5,000. A and D thus have the money that was due them, B and C have the goods, and no money had to be sent outside of Scranton or outside of Pittsburg. The diagram below will serve to make the operation clearer. The order directing payment to be made is called a **draft,** or a **bill of exchange.**



The foregoing explanation explains the purpose and convenience of a draft. Usually, however, the transaction is

conducted through a bank or broker. Thus, suppose that B in Pittsburg owes A in Scranton $5,000, and that B wishes to pay A without incurring the danger and risk of actually sending the money. He goes to a bank in Pittsburg and buys a draft, say, on New York, paying $5,000 and as much more as the bank charges him for its trouble. The Pittsburg bank then makes out a draft directing a New York bank to pay to A's order $5,000. B sends draft to A, who endorses it and presents it either at the New York bank, or, more conveniently, at a Scranton bank, where it is cashed. The Scranton bank sends the draft to some other bank (preferably a New York bank) with which the Scranton bank has dealings, and this bank in turn presents it at the bank drawn on, which credits the bank presenting the draft, and debits the Pittsburg bank. At stated intervals the various banks balance their accounts, at which time the debtor banks forward to the creditor banks the amounts due them.

Or, again, suppose that B in Pittsburg owes A in Scranton $5,000, and that A desires to collect this amount from B. A draws * a draft, or order, on B, directing him to pay the cashier of A's bank the $5,000 due A. A then leaves this draft at his bank, and the bank sends it to some bank in Pittsburg, which bank presents the draft to B, who pays that bank the money. This bank then notifies the Scranton bank that it has collected the draft and has placed the money to the Scranton bank's credit. The Scranton bank then notifies A that the draft has been collected and that the money has been placed to A's credit.

20. Exchange between different parts of the *same* country is **domestic exchange,** and between *different* countries is **foreign exchange.**

21. A **sight draft** is payable when presented for payment; a **time draft** is payable after a specified time. Time drafts are usually made to run 30, 60, or 90 days, and in some places days of grace are allowed on them.

* To **draw** a draft is to request the payment of a certain sum, at a certain time, to a certain person, his order, or to bearer.

When a time draft is presented to the drawee, if he desires to pay it, he writes across its face the word "Accepted," the date of acceptance, and his name. The draft then becomes due, the number of days stated in the draft, from the date of acceptance, instead of from the date of the draft. A time draft, thus accepted, in reality becomes a promissory note. Sight drafts do not need to be accepted, as they are usually paid when presented.

22. The **drawer** of a draft is the person that requests payment to be made; the **drawee** is the person requested to make such payment; and the **payee** is the person to whom the money is to be paid.

23. A draft is **accepted** by the drawee if he writes on its face the word "Accepted" and the date of acceptance, together with his name. He thus becomes liable for its payment.

24. **Discount** is allowed on time drafts, and is computed on the amount or on the face of the draft.

FORM OF A DRAFT.

$465\frac{25}{100}$. *Scranton, Pa., Jan. 3, 1898.*

At sight, pay to the order of Henry Hudson, Four Hundred Sixty-five $\frac{25}{100}$ *Dollars, value received, and charge to the account of* *The Colliery Engineer Company.*

To Brown & Bird,
Chicago, Ill.

EXPLANATION.—The Colliery Engineer Co. corresponds to A in Art. **19,** Brown & Bird to B, and Henry Hudson to C. Henry Hudson endorses the draft, "Pay to the order of J. R. Robinson," and signs his name; he forwards the draft to J. R. Robinson, of Chicago (who corresponds to D), and he presents it to Brown & Bird for acceptance.

DOMESTIC EXCHANGE.

25. Exchange may be at *par*, *above par*, or *below par*. For example, consider two cities, as New York and New Orleans. If New Orleans owes more money in New York than New York owes in New Orleans, there will be more persons in New Orleans who wish to buy drafts on New York than there are sellers; hence, a buyer in New Orleans will be willing to pay a seller more than the face value for a draft on New York. Therefore, in New Orleans, exchange on New York will be above par, or at a *premium*. In New York, however, there will be more sellers than buyers, and the seller will be willing to take less than the face of a draft on New Orleans. Therefore, in New York, exchange on New Orleans will be below par, or at a *discount*. If there is an equality of debts between New York and New Orleans, exchange will be at par. The premium or discount is only so much as will suffice to cover the cost of safely transferring the money from the debtor city to the creditor city.

26. To find the cost of a sight draft.

EXAMPLE 1.—Find the cost of a sight draft on Baltimore for $2,800, exchange being at $\frac{1}{2}\%$ premium.

SOLUTION.— $\$2{,}800 \times .005 = \$14$; $\$2{,}800 + \$14 = \$2{,}814$. Ans.
Or, $\$2{,}800 \times 1.005 = \$2{,}814$. Ans.

EXAMPLE 2.—What must be paid for a sight draft on New York for $3,675, at $\frac{3}{4}\%$ discount?

SOLUTION.— $\$3{,}675 \times .0075 = \$27.5625$;
$\$3{,}675 - \$27.56 = \$3{,}647.44$. Ans.
Or, $\$3{,}675 \times (1 - .0075) = \$3{,}647.44$. Ans.

27. Rule.—*Find the premium or the discount. The sum of the face of the draft and the premium, or the difference between the face of the draft and the discount, will be the cost of the draft.*

EXAMPLES FOR PRACTICE.

28. Solve the following examples:

1. Find the cost of a sight draft for $1,876 at (*a*) $1\frac{1}{4}\%$ premium; (*b*) $\frac{1}{2}\%$ discount.

Ans. { (*a*) $1,899.45. (*b*) $1,866.62.

2. The face of a sight draft is $7,875.56, and the premium is $\frac{1}{2}\%$. Find its cost. Ans. $7,938.56.

3. How much will it cost to pay, by a sight draft on San Francisco, a bill of $7,528, when exchange is at $1\frac{1}{2}\%$ discount? Ans. $7,415.08.

29. To find the cost of a time draft.

EXAMPLE.—Find the cost of the following draft at $\frac{3}{4}\%$ premium, when money is worth 5% interest.

$4,800$\frac{00}{100}$. *New York, July 1, 1898.*

Ninety days after sight, pay to William Wood, or order, Four Thousand Eight Hundred Dollars, value received, and charge to my account.

John Steinway.

To Henry Brothers,
New Orleans, La.

SOLUTION.—In states where days of grace are allowed, 3 days must be added to the number of days specified in the draft. Hence, since the draft is *payable* in Louisiana, a state where days of grace are allowed (see *Arithmetic*, Part 8), 90 days + 3 days of grace = 93 days.

$4,800 \times .0129\frac{1}{2} = \$62$ = int. of $4,800 at 5% for 93 da.
$4,800 − \$62 = \$4,738$ = proceeds of $4,800.
$4,800 \times .0075 = \$36$; $\$4,738 + \$36 = \$4,774$. Ans.

EXPLANATION.—The bank discount at 5% for 93 days on $4,800 is $62; hence, the present worth, or proceeds, of $4,800, payable in 93 days, is $4,738. The premium on the face of a draft is $36. This, with the proceeds, is the cost of the draft.

30. Rule.—*Find the proceeds of the face of the draft for three days more than the time the draft has to run. Find, also, the premium or the discount, on the face of the draft. The sum of the proceeds and the premium, or the difference between the proceeds and the discount, will be the cost of the draft.*

If days of grace are not allowed in the state where the draft is payable, find the proceeds for the time the draft has to run, and then proceed as before.

EXAMPLES FOR PRACTICE.

31. Solve the following examples:

1. The face of a draft is \$5,000; the discount is $\frac{1}{2}\%$; the time to elapse before it is payable is 60 days. Find the cost of the draft when money is worth 6%. Ans. \$4,922.50.

2. How much will it cost to pay 30 days after sight a bill of \$3,250 in Charleston, S. C., at a premium of $1\frac{1}{2}\%$, money being worth 6%? Ans. \$3,280.87.

3. When exchange is at a discount of $\frac{3}{4}\%$, and money is worth 4%, what must be paid for a 60-day time draft for \$6,000? Ans. \$5,913.

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32. To find the face of a sight or a time draft.

EXAMPLE.—What is the face of a sight draft bought for \$3,000, exchange being $\frac{1}{2}\%$ discount?

SOLUTION.— \$1.00 − \$.005 = \$.995.
\$3,000 ÷ .995 = \$3,015.08. Ans.

EXPLANATION.—One dollar of the face will cost \$0.995; hence, there must be as many times one dollar of the face as .995 is contained times in the entire cost, or 3,015.08.

EXAMPLE.—Find the face of a 60-day draft costing \$2,343.51 when money is worth 7% and exchange is at $1\frac{3}{8}\%$ premium.

SOLUTION.—When the place where the draft is to be paid is not stated, days of grace should be allowed. Hence, the time is 60 + 3 = 63 days.

Interest of \$1 for 63 days at 7% = \$0.01225.
Proceeds of \$1 = \$1 − \$0.01225 = \$0.98775.
Premium on \$1 = \$0.01375.
Proceeds + premium = \$0.98775 + \$0.01375 = \$1.0015.
Face of draft = \$2,343.51 ÷ 1.0015 = \$2,340. Ans.

33. Rule.—*Divide the amount paid for the draft by the amount that will pay for \$1 of its face.*

EXAMPLES FOR PRACTICE.

34. Solve the following examples:

1. Find the face of a 90-day draft costing \$6,000, when discount is $1\frac{1}{4}\%$ and money is worth 5%. Ans. \$6,156.48.

2. A man paid $484.72 for a 60-day draft, premium being 1%, and money worth 6% interest. What was the face of the draft?

Ans. $484.96.

3. If a draft that is payable 30 days after sight costs $2,800 when discount is $\frac{3}{4}$% and money worth 6%, what is its face? Ans. $2,836.88.

4. Find the face of a sight draft costing $1,200, when exchange is at $1\frac{1}{8}$% discount. Ans. $1,216.73.

FOREIGN EXCHANGE.

35. Foreign bills of exchange are drawn in sets of two, called a **set of exchange.** These are numbered *1* and *2*, and are sent by different mails; only the first presented for payment has any value. Formerly, foreign bills of exchange were drawn in sets of three, and are frequently so drawn now.

36. Exchange is at a premium or at a discount according to the balance of trade, or to the time that must elapse before payment is to be made.

Thus, let A and B denote two countries engaged with each other in commerce. Suppose that the balance of trade is in B's favor. By this is meant that A owes B more than B owes A. Now, it is clear that A must send money at some risk and expense to B to equalize matters. If one were to get in A a draft payable in B, it would put A more deeply in debt to B. This fact would put a premium upon the draft, and a discount upon a draft drawn in B upon A.

Again, a sight draft upon either country would cost more than a time draft.

37. The Secretary of the Treasury of the United States issues, on the first of January of each year, a statement showing, in terms of its own gold monetary unit, the value of the monetary unit of each other country, that is, in dollars and cents.

38. The daily papers of our commercial cities give quotations showing the rates of exchange from day to day. One of these follows:

Sterling exchange* was again weak and lower. Continental exchange was also lower. Rates are: Long bills, $4.82\frac{1}{2}$ @ $4.82\frac{3}{4}$; sight drafts, $4.84\frac{3}{4}$ @ $4.85, and cable transfers, $4.85\frac{1}{4}$ @ $4.85\frac{1}{2}$. Francs are quoted at $5.21\frac{7}{8}$ for long and 5.20 for short; reichsmarks, $94\frac{1}{2}$ @ $94\frac{9}{16}$ for long and $95\frac{1}{8}$ @ $95\frac{3}{16}$ for short; guilders, $39\frac{7}{8}$ @ $39\frac{15}{16}$ for long and 40 @ $40\frac{1}{16}$ for short.

NOTE.—A reichsmark (mark of the empire) is the same as a mark, about $23\frac{3}{4}$ cents. The exchange value of 4 marks, or reichsmarks, is given in commercial quotations in the daily newspapers.

39. A person going from New York to England carries with him, instead of money, a draft like the following:

New York, Oct. 1, 1897.

Exchange for £820-12-6 sterling.

At sight pay this First of Exchange, second of same tenor and date unpaid, to Edward Howe, or order, the sum of Eight Hundred Twenty Pounds £820-12-6 sterling.

Value received, and charge to the account of

Smith, Jones & Co.

To Baring Bros. & Co.,
London, England.

EXAMPLE 1.—Find the cost of the foregoing draft in New York when exchange on London is $4.84\frac{3}{4}$.

SOLUTION.— £820-12-6 = £820 12s. 6d. = £820.625.
$4.8475 × 820.625 = $3,977.98. Ans.

EXAMPLE 2.—What must be paid for a draft on Paris of 8,000 francs, when $1 is quoted at $5.21\frac{1}{2}$?

SOLUTION.— 1 franc = $1 ÷ 5.215;
($1 ÷ 5.215) × 8,000 = $1,534.04. Ans.

40. **Rule.**—*To find the cost of a draft upon a foreign country, multiply the quoted value of a foreign monetary unit by the given number of such units.*

*Sterling exchange is Bills of Exchange payable in English money called Pounds Sterling. Long bills are those payable 30, 60, 90 or more days after being received. Short bills are those payable from sight to 30 days after being received. Sight drafts are payable at sight, that is, as soon as received.

EXAMPLES FOR PRACTICE.

41. Solve the following examples:

1. Find the cost of a draft on London, at 60 days' sight, for £987 16s., exchange being $\$4.82\frac{3}{4}$. Ans. $4,768.60.

2. When exchange on Paris is quoted at 5.23, what must be paid for a sight draft for 2,800 francs? Ans. $535.37.

3. I bought a long draft on Berlin for 8,425 reichsmarks when exchange was quoted at $94\frac{3}{4}$ per 4 reichsmarks. What did it cost? Ans. $1,995.67.

4. What must be paid for a draft on Amsterdam for 8,000 guilders, exchange being $40\frac{1}{16}$? Ans. $3,205.

ARBITRATION OF EXCHANGE.

42. **Arbitration of exchange** is the process of finding the cost of a draft on one place through one or more intermediate places.

Thus, the quoted rates between New York and Vienna may be high, while those between New York and London, and between London and Vienna may be low. It may be cheaper for a man in New York to purchase a draft on Vienna through London than to purchase directly on Vienna.

In this roundabout or circuitous method, the intermediate brokers charge *brokerage* for their services, usually $\frac{1}{8}\%$.

43. EXAMPLE 1.—When exchange between New York and Paris is 5.22, between New York and London is $4.83, and between London and Paris 24.84 francs to the pound, which is cheaper—direct or circuitous exchange from New York upon Paris for 10,000 francs, London brokerage being $\frac{1}{8}\%$?

SOLUTION.—By direct exchange, the cost of a draft for 10,000 francs is

$$10{,}000 \div 5.22 = \$1{,}915.71.$$

By circuitous exchange, the cost of the draft in pounds is

$$10{,}000 \div 24.84 = £402.5765.$$

$$\text{Cost of draft} + \text{brokerage} = £402.5765 \times 1.00\tfrac{1}{8} = £403.0797.$$

$$\text{Cost of draft in dollars} = \$4.83 \times 403.0797 = \$1{,}946.87.$$

Therefore, direct exchange is cheaper by

$$\$1{,}946.87 - \$1{,}915.71 = \$31.16. \quad \text{Ans.}$$

EXAMPLE 2.—A merchant sends 12,000 reichsmarks from New York to Berlin through London and Amsterdam. Exchange on London is \$4.85, between London and Amsterdam 11.86 guilders to the pound, and between Amsterdam and Berlin 1.72 reichsmarks to the guilder. What is the cost of the draft in dollars, if brokerage at each place is $\frac{1}{8}\%$?

SOLUTION 1.—First change the reichsmarks to guilders, then the guilders to pounds, and finally the pounds to dollars, adding each time the commission.

$12{,}000 \times 1.00\frac{1}{8} = 12{,}015$, the number of reichsmarks required to pay the debt and the broker's commission in Berlin.

$(12{,}015 \div 1.72) \times 1.00\frac{1}{8} = 6{,}994.197$, the number of guilders required in Amsterdam.

$(6{,}994.197 \div 11.86) \times 1.00\frac{1}{8} = 590.467+$, the number of pounds required in London.

$590.467 \times 4.85 \times 1.00\frac{1}{8} = 2{,}867.34$, the number of dollars required to purchase the bill of exchange in New York. \$2,867.34. Ans.

SOLUTION 2.—Another solution is the following, in which the vertical line is one of the signs of division, and indicates that the product of the numbers on the left of the line is to be divided by the product of the numbers on the right.

$$\begin{array}{rl} \$4.85 \times 1.00\frac{1}{8} & \mid \text{£1.} \\ \text{£1} \times 1.00\frac{1}{8} & \mid \text{11.86 guilders.} \\ \text{1 guilder} \times 1.00\frac{1}{8} & \mid \text{1.72 marks.} \\ \text{12,000 marks} \times 1.00\frac{1}{8} & \mid \$\,? \end{array}$$

$$\frac{\$4.85 \times 1.00125^4 \times 12{,}000}{11.86 \times 1.72} = \$2{,}867.34. \quad \text{Ans.}$$

To avoid the use of decimals having such an inconveniently large number of figures, it is better to use common fractions. Thus, $1.00\frac{1}{8} = \frac{8.01}{8} = \frac{801}{800}$; hence, the last expression above becomes

$$\frac{\$4.85 \times (\frac{801}{800})^4 \times 12{,}000}{11.86 \times 1.72} = \frac{\$4.85 \times 801^4 \times 12{,}000}{11.86 \times 800^4 \times 1.72} = \$2{,}867.34.$$

By using common fractions instead of the decimals, the results will be more accurate and the principle of cancelation can be more readily employed.

44. It will be noticed that in the above arrangement the various monetary units appear once on each side of the vertical line of division. The same method may be employed with other than monetary units.

EXAMPLE.—If 4 apples are worth 3 peaches, and 9 peaches are worth 5 oranges, how many apples must be given for 60 oranges?

SOLUTION.—

12 60 oranges	x apples.
3 9 peaches	5 oranges.
4 apples	3 peaches.

$$x = 12 \times 3 \times 4 = 144 \text{ apples.} \quad \text{Ans.}$$

EXPLANATION.—The unknown quantity x should always be placed on the right-hand side of the vertical line of division, and the given quantity to which x is equivalent on the left-hand side. Then arrange the other quantities so that each quantity of the same kind appears on each side of the line. In the present example, 60 oranges are equivalent to a certain number of apples; hence, we place 60 oranges on the left and x on the right. Now, since 5 oranges are equivalent to 9 peaches, and we already have oranges on the left-hand side, we place the 5 oranges on the right-hand side and the 9 peaches on the left-hand side. For the same reason, we place 4 apples on the left-hand side and 3 peaches on the right-hand side. Canceling, we find that $x = 144$ apples.

EXAMPLES FOR PRACTICE.

45. Solve the following examples:

1. I sent from New York to Christiania, Norway, 4,740 crowns through London and Amsterdam. If 1 crown = .66⅔ guilder, 11.85 guilders = £1, and $4.85 = £1, what does the draft cost, brokerage at London on Amsterdam, and at Amsterdam on Christiania, being ⅛%? Ans. $1,300.62 +.

2. If 5 cords of oak wood are worth 3 cords of hickory, and 4 cords of hickory are worth 10 cords of pine, what should be paid for oak when pine is $1.25 per cord? Ans. $1.87½.

3. If 9 bushels of wheat are worth 14 bushels of rye, 12 bushels of rye are worth 17 bushels of corn, and 3 bushels of corn are worth 5 bushels of oats, how many bushels of oats should be given for 324 bushels of wheat? Ans. 1,190 bu.

AVERAGE OR EQUATION OF PAYMENTS.

46. Suppose that A owes B any sum, say \$100, due in 10 days, a second \$100, due in 20 days, and a third \$100, due in 30 days, there is evidently a time when he may pay B the entire \$300 without loss of interest to either party. Clearly, this time is 20 days after A incurs the debts. The conditions are that A may retain and use the first \$100 for 10 days, the second \$100 for 20 days, and the third \$100 for 30 days. But, so far as the interest is concerned, the use of \$100 for 10 days is equivalent to the use of \$100 × 10, or \$1,000, for 1 day; \$100 for 20 days equals \$2,000 for 1 day; \$100 for 30 days equals \$3,000 for 1 day. A's privilege, therefore, equals \$1,000 + \$2,000 + \$3,000, or \$6,000 for 1 day. But \$6,000 for 1 day is the same as \$300 for 20 days. Expressing this argument more briefly, we have

\$ 1 0 0 for 10 days = \$ 1,0 0 0 for 1 day.
\$ 1 0 0 for 20 days = \$ 2,0 0 0 for 1 day.
\$ 1 0 0 for 30 days = \$ 3,0 0 0 for 1 day.

\$ 3 0 0) \$ 6,0 0 0

2 0 days.

47. Average, or equation, of payments is the process of finding the equitable time when payment of several sums, due at different times, may be made in one payment.

48. The equated, or average, time of payment is the time when several debts with different *terms of credit* may be equitably made in one payment.

49. Example 1.—At a certain time A agrees to pay \$1,000 as follows: \$300 in 30 days, \$200 in 60 days, and \$500 in 90 days. Find the equated time of payment; that is, the time at which the entire debt may be paid without interest and still be fair to both parties.

Solution.—

\$ 3 0 0 × 30 = \$ 9 0 0 0
\$ 2 0 0 × 60 = \$ 1 2 0 0 0
\$ 5 0 0 × 90 = \$ 4 5 0 0 0

\$ 1 0 0 0) \$ 6 6 0 0 0

6 6 days.

The whole sum may equitably be paid 66 days after the obligation is incurred. Ans.

EXAMPLE 2.—A man owes $250 due Mar. 1, $300 due Apr. 20, $450 due May 5, and $500 due June 25. When is the equated time of payment?

SOLUTION.—

$250	× 0 =	0
$300	× 50 =	$15000
$450	× 65 =	$29250
$500	× 116 =	$58000
$1500		)$102250
		68+

68 days after Mar. 1, or May 8. Ans.

EXPLANATION.—Taking Mar. 1, the time when the first debt is due, as the *date of reference*, or the time from which to determine the terms of credit, the term of credit for $250 is 0 days. The term of credit for $300 is from Mar. 1 to Apr. 20, or 50 days; for $450 the term of credit is from Mar. 1 to May 5, or 65 days; and for $500 the term of credit is from Mar. 1 to June 25, or 116 days. We multiply each debt by its term of credit, and divide the sum of the products by the sum of the debts. The quotient, 68 days, is the number of days after Mar. 1 when one payment of the whole indebtedness may equitably be made, or May 8.

50. In the example just given, the length of time between the date when the equated time was computed and Mar. 1 was not stated. This does not matter, since the only effect that would be produced by introducing the number of days between this date and Mar. 1 would be to increase the numbers on the right of the signs of equality, and, since the total debt remains the same, the number of days obtained for the equated time will be increased by an amount just equal to the number of days between this date and Mar. 1, which, of course, does not change the date of settlement. For example, suppose that the equated time was computed 30 days preceding Mar. 1, that is on Jan. 29. Then, the first debt falls due in 30 days; the second, in $30 + 50 = 80$ days; the third, in $30 + 65 = 95$ days; and the fourth, in $30 + 116 = 146$ days.

In order, therefore, to find the equated time, we proceed as follows:

$250	× 30 =	$7500
300	× 80 =	24000
450	× 95 =	42750
500	× 146 =	73000
$1500		) $147250

98.1+ or 98 days after Jan. 29.

But 98 days after Jan. 29 is the same as 68 days after Mar. 1.

The number of days as computed above is always taken to the nearest integer; that is, if the fractional part is .5 or greater, 1 day is added. For instance, had the above result been 98.6, the number of days would have been taken as 99.

51. Rule.—*Taking as the date of reference the date when the first debt is due, find the term of credit for each debt.*

Multiply each debt by its term of credit, and divide the sum of the products by the sum of the debts. The quotient to the nearest integer will be the number of days from the date of reference to the equated time.

EXAMPLES FOR PRACTICE.

52. Solve the following examples:

1. A owes B $500 due in 8 months, and $900 due in 4 months. When may he equitably pay B both debts in one payment?

Ans. 5 mo. 13 da.

2. Find the equated time for paying $400 due May 10, $500 due June 20, $900 due July 30, and $1,000 due Aug. 15. Ans. July 17.

3. Tefft, Weller & Co. sold to E. King & Co. goods as follows: on June 15, $2,500 on 30 days' credit, and, on June 30, $3,600 on 20 days' credit. Find the equated date of payment. Ans. July 18.

4. What is the equated time for the payment of three notes: one for $600, dated Aug. 9, 1897, for 3 months; the second for $800, dated Oct. 1, 1897, for 2 months; the third for $1,200, dated Dec. 21, 1897, for 6 months? Ans. Feb. 27, 1898.

5. On Jan. 1, 1898, a merchant sold a bill of goods amounting to $3,600, payable as follows: $\frac{1}{2}$ in 30 days, $\frac{1}{3}$ in 60 days, and the remainder in 90 days. Find the equated time of payment. Ans. Feb. 20, 1898.

AVERAGE OR EQUATION OF ACCOUNTS.

53. Goods are usually sold on credit, the term of credit being commonly 30 days, 60 days, or 90 days. The prices of the goods are fixed for the time on which they are sold, interest being charged if payment is not made at the end of the time specified, and a discount being given if the debt is paid before the end of the term of credit. Now, if one or more payments are made on the bill before it is due, it is evident that a rebate ought to be given the purchaser, or else his term of credit on the remainder of the bill ought to be extended. For example, suppose that Wm. Marshall buys a bill of goods amounting to $1,525.86 on Jan. 5, on 90 days' credit. The price was so fixed that the seller would not lose anything by selling the goods on 90 days' credit. Suppose that, on Jan. 27, Mr. Marshall pays $425.40 on account, and on Feb. 24, $506.62. There still remains unpaid $1,525.86 − ($425.40 + $506.62) = $593.84. Mr. Marshall ought either to receive a rebate, or his term of credit on the $593.84 still unpaid ought to be extended, in order to compensate him for having paid a part of the bill before it was due. To determine how long the term of credit should be extended, we reason as follows: The first payment was made 22 days, and the second payment 50 days, after the goods were bought. Hence, Mr. Marshall lost the use of $425.40 for 90 − 22 = 68 days, and of $506.62 for 90 − 50 = 40 days; or of $425.40 × 68 = $28,927.20 for 1 day, and of $506.62 × 40 = $20,264.80 for 1 day. Consequently, altogether, he lost the use of $28,927.20 + $20,264.80 = $49,192 for 1 day. Therefore, to make things equal all around, the term of credit on the $593.84 still unpaid should be extended 49,192 ÷ 593.84 = 83 days, and the date of settlement should be 90 + 83 = 173 days after Jan. 5, or June 27.

In equation of payments, only one side of the account is considered, the items being either all debits or all credits; but, when both sides of the account are considered, the process of finding the equated time is called **equation of accounts.** In averaging accounts, the method of finding

the equated time is nearly the same as in equation of payments; the method is shown in the following examples:

54. Example.—Find the equated time for the settlement of the following account:

Henry Wardell.

1889.				1889.			
Jan.	20	Mdse., 30 days,	800	Mar.	1	Cash,	400
Feb.	18	" 90 "	600		20	"	600
Mar.	14	" 60 "	1,000	Apr.	1	"	1,000
Apr.	10	" 30 "	1,200		20	"	500

Solution.—The date of reference is Feb. 19, since 30 days after Jan. 20 is Feb. 19.

Feb. 19,	800 × 0 =	0		Mar. 1,	400 × 10 =	4000	
May 19,	600 × 89 =	53400		Mar. 20,	600 × 29 =	17400	
May 13,	1000 × 83 =	83000		Apr. 1,	1000 × 41 =	41000	
May 10,	1200 × 80 =	96000		Apr. 20,	500 × 60 =	30000	
	3600	232400			2500	92400	
	2500	92400					
	1100	)140000					
		127					

Average term of credit, 127 days.
Equated time, 127 days after Feb. 19, or June 26. Ans.

Explanation.—As in equation of payments, the debts are multiplied by the number of days from the date of reference to the dates when they respectively become due. It is most convenient to take as the date of reference (usually called the **focal date**) the date when the first debt becomes due; or, if a payment is made before the first debt becomes due, take the date of the first payment as the focal date. The same operation is performed on the payments, *using the same focal date.* The sum of the credits is then subtracted from the sum of the debits, the sum of the products on the credit side from the sum of the products on the debit side, and the second remainder is divided by the first remainder.

Example.—In New York, what will be the cash balance of the following ledger account Jan. 10, 1898, interest at 6%?

WM. BONNER.

1897.				1897.			
Sept.	1	Mdse., 90 days,	800	Sept.	12	Cash,	600
	20	" 60 "	900	Oct.	20	Draft, 30 days,	500
Oct.	25	" 60 "	1,000	Nov.	15	Cash,	800
Nov.	1	" 30 "	2,000	Dec.	12	"	1,000

SOLUTION.—In this case it will be more convenient to take as the focal date Sept. 1. It will be noticed that one of the payments, that of $500 on Oct. 20, is a 30-day draft. Since this draft can be cashed for its face value only after 30 days, i. e., on Nov. 19, this payment must be considered as having been made on Nov. 19; and, as days of grace are not allowed in New York, they are not added to the time the draft has to run. Proceeding as follows to find the equated time of settlement,

800 × 90 =	72000	600 × 11 =	6600
900 × 79 =	71100	500 × 79 =	39500
1000 × 114 =	114000	800 × 75 =	60000
2000 × 91 =	182000	1000 × 102 =	102000
4700	439100	2900	208100
2900	208100		
1800	)231000		

$128\frac{1}{3}$ days.

we find it to be 128 days after Sept. 1, or Jan. 7, 1898. If the bill is not settled until Jan. 10, it is clear that in equity 3 days' interest should be charged on the $1,800 yet unpaid. The interest of $1,800 for 3 days at 6% is $.90; hence, the total amount that should be paid is $1,800.90.

Ans.

EXAMPLE.—When should interest begin on the balance of the following account?

HENRY WELLINGTON.

1902.				1902.			
June	16	Mdse.,	1,200	July	21	Cash,	800
July	21	"	1,000	Aug.	11	"	1,200
Aug.	13	"	2,000		30	Draft, 30 days,	1,600
Sept.	15	"	3,600	Sept.	20	" 10 "	2,400

SOLUTION.—We take, as the focal date, June 1st. The reason for taking the first day of the month as the focal date is that it is easier to reckon the number of days between the first day of the month and some later date than it is to reckon the number of days between some other

day of the month and a later date. Days of grace are allowed on the two time drafts, because no state is mentioned in the example.

1200 ×	15 =	18000	800 ×	50 =	40000
1000 ×	50 =	50000	1200 ×	71 =	85200
2000 ×	73 =	146000	1600 ×	123 =	196800
3600 ×	106 =	381600	2400 ×	124 =	297600
7800		595600	6000		619600
6000					595600
1800				1800)	24000

13+ days.

We find the products of the items and number of days as in the two previous examples. It will be noticed that the sum of the products on the credit side is greater than the sum of the products on the debit side, while the sum of the payments is less than the sum of the debts. We divide the difference between the sums of the products by the difference between the sum of the debts and the sum of the payments, obtaining for our result 13 days. Now, instead of *adding* this 13 days to the focal date, we subtract. The reason for subtracting will be evident when we consider that the last three payments were made a comparatively long time after the merchandise was bought. In other words, the merchant lost the use of the money due him in payment of the goods for a time equivalent to the difference between 13 days preceding June 1st (or May 19th) and the date of settlement, and he should receive interest on $1,800 for this time. If, however, the sum of the products on the credit side had exceeded the sum of the products on the debit side, and the sum of the payments had exceeded the sum of the debts, the number of days obtained would have been *added* to the focal date, as in the two preceding examples. Hence, the interest on the balance of the above account should begin on May 19th. Ans.

55. Rule.—*Find the date on which each item of the account matures, and take the first day of the month in which the earliest of these dates occurs, as the focal date. Find the number of days between the focal date and the date of maturity of the different items, and multiply each item by the number of days so found. Divide the difference between the sums of the products by the difference between the sum of the debts and the sum of the payments, and the quotient will be the equated time. If the greater sum of the items and the greater sum of the products are both on the same side of the account,* **add** *the equated time to the focal date. But, if the greatest sum of the items and the greatest sum of the products*

are on opposite sides of the account, **subtract** *the equated time from the focal date. The result obtained by adding or subtracting the equated time from the focal date will be the date when the balance of the account is equitably due.*

EXAMPLES FOR PRACTICE.

56. Solve the following examples, allowing 3 days of grace on time drafts:

1. Find the cash balance of the following account, July 1, 1903, interest at 6%; also, the equated time of settlement.

GEORGE GRIFFIN.

1903.				1903.			
Jan.	10	Mdse.,	2,500	Feb.	20	Cash,	4,000
Feb.	10	"	4,800	April	24	Mdse.,	1,800
Mar.	10	"	2,000	May	7	Draft, 10 days,	2,800
May	20	"	6,800	June	12	Cash,	5,000

2. Find the equated time of settlement of the following account:

WALTER ROBERTS.

1901.				1901.			
Jan.	1	Mdse., 90 days,	360	Feb.	4	Cash,	300
Mar.	4	" 30 "	480	April	6	Draft, 30 days,	900
May	13	" 60 "	640	May	7	" 60 "	600
June	12	" 30 "	960				

3. Find the equated time of settlement of the following account:

WILLARD SMITH.

1896.				1896.			
Jan.	21	Mdse., 60 days,	4,000	Jan.	31	Cash,	2,500
Feb.	3	" 60 "	5,000	Feb.	14	Real Estate,	3,500
May	13	" 90 "	8,000	April	15	Draft, 60 days,	6,000
June	15	" 30 "	7,800	May	1	Cash,	7,500
				July	1	"	2,000

Answers.—(1) $2,626.67; Aug. 31, 1902; (2) Aug. 16; (3) Apr. 27, 1897.

57. NOTE.—The student will find the table given in *Arithmetic*, Part 4, of great assistance to him in working examples in equation of accounts.

STATEMENT OF SIMPLE ACCOUNTS.

58. A set of books is kept in some manner by every person doing business. This is done so as to enable one party to keep an account of his dealings with others, so that he may know how much they owe him and how much he owes them.

59. The two principles governing such accounts are expressed by the terms debit and credit. If the following directions for debit and credit are strictly followed, a mistake will never be made.

Rule I.—When to debit.

The person: When you trust any one.
The person: When you pay any one.

Rule II.—When to credit.

The person: When he trusts you.
The person: When he pays you.

60. A **balance** is the result of adding together all the debits and all the credits and then subtracting the lesser amount from the greater. The remainder is the balance due one or the other.

If the debit footing is the larger, there is a debit balance; and if the credit footing is the larger, there is a credit balance.

The amount of the balance is entered on the lesser side of the account.

61. Single lines are ruled across the money columns on the line immediately under the lowest entry.

The footings of both sides are placed below the single lines so ruled.

Double lines are ruled across the date and money columns on both sides of the account under the footings.

The balancing entry and rulings are usually made in red ink. In the following example the balancing entry appears in *italic*. The amount of the balance is brought down, in black ink, below the double ruling on the side opposite to the red-ink balancing entry.

EXAMPLE.—On April 1, 1904, Amos Ward owed Graves & Coon $68.90 on account. April 4, he sold them 68 barrels potatoes at $2.75 per barrel. April 6, he gave them a draft on San Francisco for $1,860, which they accepted at $\frac{1}{4}\%$ discount. April 9, they sold Ward 894 bushels corn at $38\frac{1}{2}$ cents per bushel. April 16, they bought of him 2,960 feet lumber at $1.25 per hundred feet. April 19, they sold him $34\frac{1}{2}$ dozen chairs at 90 cents each. April 21, Ward bought of them 1,260 eggs at 14 cents per dozen. April 28 he gave them a note for $1,820 due in 60 days. April 29, he bought of them 2,980 pounds hay at $15 a ton. Make an itemized statement of the above account as it should appear taken from the books of Ward; make a proper heading, close the account, and bring down the balance as it should appear May 1, 1904.

SOLUTION.—In making entries in an account of this kind, you should first determine with whom the account is kept, and in applying the rules for debit and credit, Art. **59**, reason out the debits and credits from the standpoint of the party on whose books the account appears.

Dr. GRAVES & COON IN ACCOUNT WITH AMOS WARD Cr.

1904					1904				
Apr.	4	To 68 bbl. potatoes at $2.75 per bbl.	187	00	Apr.	1	By balance	68	90
	6	To draft	1,860	00		6	" discount on draft	4	65
	16	To 2,960 ft. lumber at $1.25 per 100 ft.	37	00		9	" 894 bu. corn at $38\frac{1}{2}$c. per bu.	344	19
	28	To note	1,820	00		19	" $34\frac{1}{2}$ doz. chairs at 90c. each	372	60
						21	" 1,260 eggs at 14c. per doz.	14	70
						29	" 2,980 lb. hay at $15 per ton	22	35
							Balance	*3,076*	*61*
			3,904	00				3,904	00
May	1	To balance	3,076	61					

April 1.—Amos Ward owes Graves & Coon. Ward credits Graves & Coon *because they trusted him.*

April 4.—Ward sells Graves & Coon. He debits them *because he trusts them.*

April 6.—Ward gives Graves & Coon a draft. He debits them for the amount of the draft *because he pays them.* He credits them with the $\frac{1}{4}\%$ discount because they receive only the face of the draft less the amount of the discount.

April 9.—Graves & Coon sell Ward. He credits them *because they trust him.*

April 16.—Graves & Coon buy from Ward. He debits them *because he trusts them.*

April 19.—Graves & Coon sell Ward. He credits them *because they trust him.*

April 21.—Ward buys from Graves & Coon. He credits them *because they trust him.*

April 28.—Ward gives Graves & Coon a note. He debits them *because he pays them.*

April 29.—Ward buys from Graves & Coon. He credits them *because they trust him.*

The debit and credit footings are then made in pencil on a separate slip. The lesser footing is $827.39 and the greater footing $3,904.00. The difference, $3,904.00 − $827.39 = $3,076.61, is the balance due Amos Ward by Graves & Coon, and is entered, with red ink, on the credit side of the account, because this is the lesser side. A single red line is ruled across the money column on the debit and credit sides immediately below the lowest entry, which in this case is the balancing entry. The footings of both money columns are then placed below the single red lines, and double red lines are ruled immediately below the footings, as well as across the date columns on both sides of the account, as shown above. The balance due by Graves & Coon is then entered, with black ink, below the double lines so ruled, on the side opposite to that on which the red ink balancing entry is made.

EXAMPLES FOR PRACTICE.

62. Solve the following problems:

1. On December 1, 1902, William Martin owed George Drake $250 on account. December 3, he sold him 75 gallons of vinegar at 20 cents per gallon; December 5, he bought of Drake 25 dozen eggs at 16 cents per dozen; December 6, he gave him a draft for $1,250, which he accepted at a discount of $\frac{3}{4}\%$; December 8, he sold Drake 300 barrels of apples at $2 per barrel; December 11, he bought of Drake 90 bushels of potatoes at 75 cents per bushel; December 15, he gave him a note for $125; December 18, he bought of Drake 50 barrels of flour at $2.50 per barrel. Make an itemized statement of the above account as it should appear taken from the books of Drake; make a proper heading, close the account, and bring down the balance as it should appear January 1, 1903. Ans. $1,534.12.

2. On November 1, 1901, H. W. Walker owed Smith & Wilson $1,000 on account. November 3, he sold them 7,850 feet of lumber at $14 per thousand feet; November 4, they sold him 50 horses at $250 per head; November 6, he gave them a draft for $500; November 7, they sold him 800 bags of lime at 60 cents per bag; November 10, he

sold them 500 pounds of nails at 4 cents per pound; November 12, they sold him 3 hogs weighing 1,370 pounds at $8\frac{1}{4}$ cents per pound; November 16, he sold them 110 bundles laths at 90 cents per bundle; November 18, they bought of him 9,780 pounds crushed stone at $1.55 per ton; November 21, he gave them a note for $1,280 due in 60 days. Make an itemized statement of the above account as it should appear taken from the books of Walker; make a proper heading, close the account, and bring down the balance as it should appear December 1, 1901. Ans. $12,076.55.

3. On June 1, 1904, James Farrell, a commission merchant, owed John Miller $185.72. On June 2, Miller shipped Farrell 7,000 pounds of sugar, which he sold at $5\frac{1}{4}$ cents per pound, charging $\frac{1}{4}\%$ commission; June 15, Farrell sold to John Miller 9 carloads of grain, 600 bushels each, at 58 cents per bushel; June 17, Farrell sold for Miller 1,008 pounds of butter at $22\frac{1}{2}$ cents per pound, charging $16.50 for cold storage and $\frac{3}{4}\%$ commission; June 20, Miller bought of Farrell. 4,800 pounds of cement at 90 cents per hundred; June 24, Miller paid draft $2,610 drawn on him by Farrell; June 27, Miller received Farrell's note for $600, due in 90 days. Make an itemized statement of the above account as it should appear taken from the books of Miller.

Ans. $404.30.

4. On April 1, 1904, James Bowen owed Price & Sons $75 on account. April 3, he sold them 70 barrels of apples at $2.50 per barrel; April 5, they bought of him 3 crates of eggs, each containing 24 dozen at 18 cents per dozen; April 7, they sold him 65 bushels of potatoes at 75 cents per bushel; April 9, they paid a draft drawn on them for $250; April 11, they sold him 4,480 pounds of hay at $12 per ton; April 13, he gave them a note for $1,000. Make an itemized statement of the above account as it should appear taken from the books of Bowen; make a proper heading, close the account, and bring down the balance as it should appear June 1, 1904. Ans. $787.33.

SPELLING

(PART 1)

RULES FOR SPELLING

1. The plural of nouns is generally formed by adding *s* to the singular; as field, fields; friend, friends; temple, temples.

2. Nouns ending in *s*, *sh*, *ch* (soft), *x*, or *z*, add *es* for the plural; as, class, classes; brush, brushes; watch, watches; tax, taxes; buzz, buzzes.

3. Nouns ending in *y*, with a vowel* before the *y*, add *s* to form the plural; as, monkey, monkeys; day, days; valley, valleys.

4. Nouns ending in *y*, with a consonant before the *y*, change *y* to *i* and add *es* to form the plural; as, story, stories; army, armies; colony, colonies.

5. Nouns ending in *f* and *fe* generally change *f* or *fe* into *ves* to form the plural; as, calf, calves; knife, knives; thief, thieves.

6. Nouns ending in *o* with a consonant before the *o* add *es* to form the plural; as, calico, calicoes; cargo, cargoes; mango, mangoes; echo, echoes.

7. There are, however, some exceptions to this rule, and they are here given as follows:

albinos	embryos	mestizos	sextos
armadillos	gauchos	octavos	siroccos
cantos	halos	octodecimos	solos
centos	inamoratos	pianos	tobaccos
didos	Juntos	provisos	twos
dominos	lassos	quartos	tyros
duodecimos	mementos	salvos	virtuosos
dynamos	merinos	sextodecimos	zeros

*The *vowels* are *a*, *e*, *i*, *o*, *u*, and sometimes *w* and *y*. The other letters of the alphabet are called *consonants*.

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8. Nouns ending in *o* with a vowel before the *o* add *s* to form the plural; as, studio, studios; portfolio, portfolios; nuncio, nuncios; oratorio, oratorios.

The following words are used to designate a few of the lessons. To aid the student the definitions are here given:

Homonym, a word agreeing in sound but different in meaning from another.

Opposite, in opposition or marked contrast.

Similar, bearing resemblance to one another.

Abbreviation, a word shortened so that a part stands for the whole.

Synonym, a word having the same or almost the same meaning as some other.

9. It is found difficult to recollect the relative position of *e* and *i* in such words as *receive*, *believe*, etc. The following rule will generally determine this point:

When the derivative noun ends in *tion* the verb is spelled with *ei*; thus,

conception	conceive
deception	deceive
reception	receive

But when the derivative noun does not end in *tion* the verb is spelled with *ie*; as,

belief	believe
relief	relieve

FAMILIAR WORDS

1

clothes	cloak	clean	a'pron	words
shoe	your	slate	col'lar	spell
stock'ing	sharp	jack'et	rib'bon	les'son
coat	write	mit'ten	here	pen'cil

2

peel	stem	mouth	tree	teeth
core	fore'head	chin	pulp	check
seeds	eye	flow'er	rind	ear
juice	nose	fruit	tongue	hair

3

leaves	board	masts	e nough′	skip′ping
aunt	green	fence	through	eve′ning
their	mean	howls	dread′ful	cheer′ful
claws	learn	ap′ples	ven′ture	sum′mer

4—THE CLOCK

face	key	case	fig′ures
hour	glass	i′ron	min′ute
hands	brass	di′al	ham′mer
wheel	weights	wood	pen′du lum

5—RELATIVES

pa′pa	son	aunt	wife
fa′ther	sis′ter	niece	kin′dred
moth′er	broth′er	un′cle	hus′band
par′ents	chil′dren	cous′in	grand′fa ther
mam′ma	daugh′ter	neph′ew	grand′daugh ter

6

earn	wrong	guide	weak	were
blue	knee	cough	col′or	worse
lamb	piece	debt	read′y	man′y
bus′y	ov′en	lose	broad	strong

7

lunch	break′fast	but′ter	poul′try	fruit
bread	beef	oat′meal	sau′sage	buns
din′ner	sug′ar	pork	ven′i son	pud′ding
sup′per	ba′con	game	pies	pre serves′

8

sand	drank	ham′mock	tramp	wag′on
hash	sad′dle	hand′some	stamp	scant′y
back	glad′ly	gas	scratch	ram′ble
latch	stag′ger	crab	rat′tle	prac′tice
catch	nar′row	plank	trav′el	pal′ace

9—THE HOUSE

floor	nurs'er y	chim'ney	man'tel	en'try
porch	clos'et	cup'board	fur'nace	pan'try
par'lor	gar'ret	cel'lar	door'step	li'bra ry
kitch'en	fire'place	hearth	at'tic	stair'case

10—IN THE SCHOOLROOM

ru'ler	wall	satch'el	pens
cray'on	class	tran'som	globe
pic'ture	shelf	reg'is ter	pa'per
teach'er	seats	pro'gram	shut'ters
schol'ars	chart	pen'-wi per	neat'ness
black'board	chairs	waste'-bas ket	at ten'tion

11

cit'y	switch	kit'ten	bring	click
quit	skip	glimpse	width	wink
script	milk	still	fringe	cling
kissed	whist	miss	stiff	print
bridge	which	prim	knit	a bout'

12

edge	wedge	rem'e dy	dense	when
fence	smell	wheth'er	meant	necks
hedge	shel'ter	cent	health	sweat
tempt	red'dish	deaf	next	depth

13

cof'fin	con'cert	dodge	trot'ted	con vey'
for'est	shop	knock	hol'low	of fense'
bot'tle	knob	mock	pock'et	be yond'
gloss'y	notch	rock'et	fond	con verse'
bon'net	lodge	pop'py	bod'y	coun'tries

14—BIRDS

gull	grouse	swal'low	vul'ture	crane
swan	ra'ven	par'tridge	bob'o link	ea'gle
wren	par'rot	lark	night'in gale	cuck'oo
quail	os'trich	her'on	owl	spar'row
stork	pig'eon	o'ri ole	crow	con'dor
hawk	pea'cock	blue'jay	dove	lin'net

15—WHAT BIRDS DO AND HAVE

coo	car'ol	whis'tle	poise	tal'on
caw	hov'er	mi'grate	perch	wings
chirp	war'ble	soar	pin'ion	feath'ers
cheep	twit'ter	whir	beak	plu'mage

16

crust	bunch	crutch	buzz	luck
ug'ly	jump	rub'ber	sung	mush
drum	brush	just	crumb	shrub
dumb	struck	dust	hut	pump

17

live	space	moves	called	sur'face
ball	earth	through	breast	beau'ti ful
light	might	great	curled	won'der ful
gives	world	round	dressed	beau'ti ful ly

18

po'ny	mus'tard	char'coal	tor'ment	hat'ter
med'al	par'ing	floun'der	scrib'ble	stick'y
cor'ner	mor'tar	hoe'ing	pres'ence	crip'ple
med'dle	pea'nuts	peel'ing	din'gy	grate'ful

19—GEOGRAPHY

sea	o'cean	sound	rap'ids	ca nal'
bay	pond	chan'nel	riv'u let	glac'i er
gulf	riv'er	lake	cat'a ract	cas cade'
strait	brook	creek	pool	foun'tain

20—LAND

cape	isth'mus	prai'rie	plain
is'land	head'land	pla teau'	o'a sis
con'ti nent	moun'tain	vol ca'no	val'ley
pen in'su la	prom'on to ry	ta'ble-land	des'ert

21

be fore'	pu'pil	whose	sweet
ver'y	do'ing	eve'ry	dust
it self'	man'y	till	du'ly
young	pub'lic	e rect'	please
proof	tru'ly	speech	al'ways
much	town	vow'el	doz'en

22

chief	sup′port	smo′ky	of fense′
u′sual	al′most	bronze	ea′ger
ow′ing	there′fore	warm′ly	un til′
canned	loud′ly	great′ly	love′ly
troub′le	wher ev′er	growl	cease
com′mon	cheap′er	ex press′	kind′ly

23

fir′ing	doubt	men′tal	be side′
set′tled	re′al ly	mere′ly	pawn
se vere′	safe′ty	zeal	mud′dy
ful fil′	snow′y	got′ten	du′ties
car′fare	re deem′	bar′gain	rain′y
use′ful	fig′ure	read′ing	false
should	there by′	for′ty	mapped

24

lean	mid′dle	mis take′	rum′ple	south
lame	on′ly	naugh′ty	quar′rel	sor′ry
lamp	noise	ripe	pres′ent	sev′en
met′al	of′ten	or′der	soul	shad′ow
mer′ry	my self′	re fuse′	rust	shov′els

25

taste	spo′ken	tum′ble	whale	worth
teach	thief	thought	up set′	wreck
stood	thirst	wake	up′per	whose
sprang	un less′	un til′	year	young
sto′ries	try′ing	waste	wool	bathe

26

sure	talk′a tive	groan	shiv′er
quite	re la′tion	kite	whisk′ers
reach	mil′i ta ry	strength	gal′lon
speech	in ter rupt′	though	sal′low

KITCHEN—27—DINING ROOM

tongs	sieve	urn	fork
ba'sin	stove	chi'na	spoon
pok'er	broom	plates	ta'ble
shov'el	ket'tle	sau'cer	cru'et
dip'per	scut'tle	gob'let	cast'ers
buck'et	dust'pan	tea'cup	nap'kin
coal'-hod	skim'mer	tu reen'	tum'bler
grid'i ron	sauce'pan	plat'ter	side'board

BEDROOM—28—PARLOR

soap	sheet	vase	so'fa
bowl	lounge	car'pet	stool
tow'el	pil'low	mir'ror	scarf
bol'ster	bu'reau	cur'tain	mu'sic
pitch'er	blank'et	has'sock	pi an'o
scis'sors	mat'tress	book'case	por'trait
nee'dles	bed'stead	arm'chair	cab'i net
thim'ble	cov'er let	ot'to man	cush'ions

29—HOMONYMS

Write the following sentences, using the right word:

Better alone than in (bad, bade) company. Don't give (to, two, too) much for the whistle. Everything comes in (thyme, time) to (hymn, him) who can (wait, weight). He who follows (two, too, to) (hares, hairs) is sure (two, too, to) catch neither. It never (reins, rains, reigns) but it (pores, pours). Men speak of the (fair, fare) and of the things they see (there, their). Out of (site, sight, cite), out of (mind, mined). Time and (tied, tide) (wait, weight) for (know, no) man. Where (there's, theirs) a will there's always a (weigh, way). (Faint, feint) (hart, heart) ne'er (won, one) (fare, fair) lady.

30

balk	al'mond	tomb	dumb'ly
calk	salm'on	jamb	plumb'er
balm	be calm'	plumb	numb'ness
alms	em balm'	doubt	climb'er
salve	balm'y	debt'or	comb'ing
halves	psalm	crumbed	doubt'ful

31

friend	like	sha'dy	sev'er al
glad	brook	cro quet'	ber'ries
birds	woods	ten'nis	ev'er y
fish	re ceive'	gained	moun'tains

32

seal	er'mine	floe	moss'es
whale	rein'deer	Lapp	li'chens
sa'ble	po'lar bear	au ro'ra	red'snow
wal'rus	ei'der duck	snow hut	ice'bergs

33—TREES

pine	ash	al'der	birch
larch	elm	ma'ple	lo'cust
hol'ly	oak	eb'o ny	lin'den
ce'dar	beech	wil'low	wal'nut
spruce	ol'ive	hick'o ry	rose'wood
cy'press	pop'lar	chest'nut	pal met'to

34—TOOLS

file	gouge	lev'er	chis'el
adz	square	au'ger	lev'el
rake	lathe	gim'let	trow'el
spade	wrench	mal'let	hatch'et
plane	shears	pin'cers	cork'screw

35

juice	loaf	liq'uid	sev'er al
cane	re fine'	hogs'head	plan ta'tion
mill	joints	mo las'ses	blos'som
scum	leaves	boiled	stom'ach

36—HOMONYMS

Write the following sentences, selecting the right word:

The past is not (holy, wholly) (vane, vain, vein) if rising on its (wrecks, recks) to something nobler (we, wee) attain.—*Longfellow.* In the morning (sow, sew) thy (seed, cede).—*Bible.* Then he said, "Good (knight, night)," and with muffled (o'er, oar, ore) silently (road, rowed, rode) to the

Charlestown shore.—*Longfellow.* The heaviest (dews, dues) fall on clear (nights, knights). The (pale, pail) light of the moon is the reflection of the (son's, sun's) light. "(There, Their) graves are green; they may be (scene, seen)," the little (made, maid) replied.—*Wordsworth.*

37—WORDS OF SIMILAR MEANING

arch	pris'on	ti'dy	soul
curve	dun'geon	neat	spir'it
bluff	tar'dy	strange	plun'der
cliff	late	quaint	rob
bench	co'zy	queer	rich
set tee'	snug	droll	fer'tile
blithe	ha'sty	twain	e rect'
joy'ous	rash	coup'le	up'right
fleet	grim	tenth	loose
nim'ble	sur'ly	tithe	un bound'
gruff	hurt	van	com'fort
glum	in'jure	front	con sole'
gang	sin'gle	wont	kill
crew	sole	hab'it	mur'der
giv'er	hank	peo'ple	rest
do'nor	skein	per'sons	peace

38

know	va'por	er'rand	ques'tion
bruise	bal loon'	nei'ther	scare'crow
sneeze	no'tice	thir'teen	of'fi cer
rinse	e lev'en	be cause'	an oth'er
can'dy	e nough'	fair'y	nec'es sa ry

39

On your way home you may see:

men	roads	gates	wom'en
stores	streets	parks	av'e nues
hous'es	gar'dens	hors'es	lamp'-posts
church'es	chil'dren	sta'bles	car'riag es

40—CLOTHING

hose	mack'in tosh	veil	muff
gloves	gai'ters	scarf	boots
cra vat'	col'lar	hood	tip'pet
skirt	trou'sers	ruf'fle	par'a sol
shawl	hand'ker chief	neck'tie	o'ver alls

41—HOMONYMS

1. ewe—a female sheep
   you—person addressed
   yew—an evergreen tree
2. cru'el—unkind
   crew'el—soft yarn
3. choir—a band of singers
   quire—24 sheets of paper
4. bough—a branch
   bow—to bend, front of ship
5. knead—to work dough
   need—want
6. stake—a post, a wager
   steak—a slice of meat
7. gait—manner of walking
   gate—a kind of door
8. main—chief
   mane—hair on horse's neck
   Maine—one of the U. S.

Write the following sentences, putting the right word in the right place:

I had most (5) of blessing.—*Shakespeare.*—May I join the (3) invisible?—*George Eliot.* (1)s and bleating lambs.—*Milton.* Added woes may (4) me to the ground.—*Pope.* (2)s are used in embroidery. (1) must (5) the dough to make good bread. The man who walked through the (7) had a peculiar (7). Have a care for the (8) chance.—*Butler.* Twenty (3)s make a ream. He bought a slice of sirloin (6). Me. is the abbreviation of (8). The Mistletoe (4) is a poem. Like a dew-drop from the lion's (8). Joan of Arc was burned at the (6). The wood of the (1) was used for making bows.

42—REVIEW

beech	squash	sen'tence	pal'ace
sieve	fields	car'riage	lil'y
those	knife	pic'ture	dai'sy
wren	through	ea'gle	bu'reau
strait	tongs	chest'nut	tu reen'
pear	peach	on'ion	Eu'rope
sleeve	pen'cil	ba'sin	po'ny
fruit	tongue	re fuse'	os'trich
birch	Wednes'day	mi'grate	mo'ment

43—REVIEW

rob'in	let'tuce	meant	scis'sors
pig'eon	rai'sin	wrote	pump'kin
wal'nut	cur'rant	please	thim'ble
pop'lar	for'ward	ev'er y	pitch'er
for'est	ber'ries	qui'et	ba na'na
gob'let	rhu'barb	ache	a'pri cot
mat'tress	spin'ach	Christ'mas	beau'ti ful
cru'et	Eng'land	pil'low	Feb'ru a ry
min'ute	chis'el	mead'ow	po ta'toes

44—IN THE COUNTRY

farm'house	ditch	sheep	swing
vines	knoll	lambs	in'sects
gar'den	ar'bor	cat'tle	or'chard
flow'ers	hay'-loft	fields	ber'ries
barn	mead'ow	wheat	but'ter flies

45—VEGETABLES

kale	tur'nip	car'rot	cu'cum ber
beet	rhu'barb	let'tuce	as par'a gus
bean	pars'nip	pump'kin	cau'li flow er
cress	cab'bage	spin'ach	egg'plant
gar'lic	mush'room	to ma'to	rad'ish

46—HOMONYMS

1. leaf—part of a tree or book
   lief—willingly
2. great—large, noble
   grate—to rub, iron frame for fire
3. heard—did hear
   herd—of cattle
4. lain—reclined
   lane—a narrow road
5. base—mean, foundation
   bass—a part in music
6. aught—anything
   ought—bound by duty
7. right—opposite of wrong
   rite—a form
   wright—a workman
   write—act of writing

Write the following sentences, putting the right word in the right place:

I see the (7), and I approve it too.—*Ovid.* It is a long (4) that has no turning.—*Prov.* We all do fade as a (1).—

Bible. Do what you (6), come what may. Baptism is a religious (7). The (2) fishes eat up the little ones.—*Shakespeare.* I had as (1) sing (5). A ship (7) works in a shipyard. Have you (3) (6) against him? It is a (5) thing to betray a man who has trusted you.—*Prov.* A bright fire glowed in the (2). The lowing (3) winds slowly o'er the lea.—*Gray.* I never dare to (7) as funny as I can.—*Holmes.* The book has (4) on my table several days.

47

glass	smooth	brit'tle	trans par'ent
win'dow	sash	panes	weights
hemmed	gath'ered	stitched	seam'stress
sleeves	a'pron	emp'ty	rode
learned	eas'i ly	rap'id ly	per'fect ly

48

palms	ca ca'o	li'on	el'e phant
cof'fee	cam'phor	jag u ar'	ser'pent
cot'ton	in'di go	gi raffe'	go ril'la
sa'go	In'di a-rub'ber	cam'el	croc'o dile
spi'ces	sug'ar-cane	mon'key	leop'ard

49

We should be:

good	cor'dial	help'ful	civ'il
frank	sin cere'	thought'ful	o blig'ing
prompt	lov'ing	ge'ni al	gen'er ous
hon'est	truth'ful	stu'di ous	o be'di ent
no'ble	care'ful	pa'tient	tem'per ate
po'lite	hope'ful	court'e ous	in dus'tri ous

50

We should not be:

mean	stin'gy	rude	im po lite'
curt	cru'el	sur'ly	dis hon'est
proud	self'ish	care'less	cow'ard ly
la'zy	un kind'	haugh'ty	quar'rel some
sulk'y	fret'ful	de ceit'ful	dis hon'or a ble
sau'cy	sul'len	tat'tling	dis o be'di ent

51—ABBREVIATIONS

Jan'u a ry	Jan.	Ju ly'	July
Feb'ru a ry	Feb.	Au'gust	Aug.
March	Mar.	Sep tem'ber	Sept.
A'pril	Apr.	Oc to'ber	Oct.
May	May	No vem'ber	Nov.
June	June	De cem'ber	Dec.

WINDS—52—BOATS

gale	cy'clone	barge	ca noe'
gust	si moom'	yawl	cut'ter
breeze	ty phoon'	sloop	schoon'er
squall	tor na'do	yacht	ves'sel
zeph'yr	whirl'wind	do'ry	frig'ate
tem'pest	hur'ri cane	gon'do la	steam'er

53

goat	ot'ter	oats	flax
deer	pan'ther	corn	hemp
wolf	squir'rel	maize	to bac'co
moose	buf'fa lo	bar'ley	tim'o thy
rab'bit	an'te lope	ce're al	mul'ber ry

54

stealth	vague	whiff	ooze	wrought
trough	voice	trait	cruise	squeeze
thrift	vogue	grate	browse	grudge
twilled	vault	fierce	shrimp	sword

55—FRUITS

fig	peach	cher'ry	cran'ber ry
prune	grape	cit'ron	blue'ber ry
date	cur'rant	dam'son	straw'ber ry
plum	lem'on	pine'ap ple	rasp'ber ry
quince	mel'on	a'pri cot	goose'ber ry
pear	rai'sin	ba na'na	huck'le ber ry

56—FLOWERS

tu'lips	lark'spurs	pan'sies	sun'flow ers
as'ters	lil'ies	sweet peas	hol'ly hocks
li'lacs	dai'sies	hy'a cinths	vi'o lets
clo'vers	mar'i golds	dan'de li ons	ge ra'ni ums

57

fell	cause	stum'bled	troub'le
lack	killed	horse'man	neg'li gence
steed	caught	in'ju ry	un fast'ened
foe	slight	loos'ened	at ten'tion

58

co here'	e'qual	need'y	wear'y
se crete'	se vere'	speed'y	drear'y
im pede'	de'cent	feed'ing	trea'son
con vene'	fe'male	heed'ful	cheap'ly
ex treme'	pre'cept	free'dom	year'ling
su preme'	de scribe'	cheer'ful	mean'ing

59

toil	gov'ern	pawn	scrub	trick
moat	heat	pledge	scour	trench
dirt	jest	quoth	di rect'	yawn
frown	joke	ruse	sell	wan'der
filth	stray	scheme	seize	be fore'
gape	pile	la'bor	scowl	col'umn

GEOGRAPHY—60—ARITHMETIC

source	po'lar	add	mul'ti plier
ax'is	cli'mate	plus	prod'uct
or'bit	sav'age	sum	di vide'
cir'cle	e qua'tor	sub tract'	div'i dend
mo'tion	ho ri'zon	min'u end	di vi'sor
dai'ly	ze'nith	sub'tra hend	quo'tient
year'ly	par'al lel	re main'der	proof
trop'ic	hem'i sphere	mul'ti ply	a rith'me tic
pole	ge og'ra phy	mul ti pli cand'	prob'lem

61—USED IN COOKING

rice	so'da	all'spice	va nil'la
sage	gin'ger	gel'a tine	vin'e gar
mace	pars'ley	choc'o late	hom'i ny
yeast	pep'per	sal e ra'tus	tap i o'ca
cloves	nut'meg	buck'wheat	cin'na mon

62

Write the plurals of the words in this exercise:

skein	loaf	con'cert	man'sion
cit'y	dai'sy	sheaf	buf'fa lo
o'cean	tur'key	of'fi cer	jour'ney
wretch	suit'or	sur'face	os'trich
box	scratch	sand'wich	at tor'ney

63

se rene'	rus'tic	waste'ful	si'lent
com'ic	tac'it	fear'ful	gloom'y
pu'trid	tur'bid	ru'ral	mud'dy
ea'ger	stur'dy	plac'id	flor'id
rud'dy	som'ber	ar'dent	rot'ten
lav'ish	tim'id	hard'y	mirth'ful

64

gift	geese	gib'bous	gey'ser	guard
girt	ga'ble	gew'gaw	gal'lant	gid'dy
gild	gir'dle	tar'get	gaunt'let	gig'gle
gear	giz'zard	gal'lop	guest	guin'ea
gimp	gher'kin	gar'gle	gauze	gor'geous

65—OPPOSITES

right	dry	find
wrong	moist	lose
near	strong	sweet
dis'tant	weak	sour
gay	quick'ly	rare
sad	slow'ly	com'mon
a like'	ac cept'	a'ged
un like'	de cline'	youth'ful
in'door	a part'	cease
out'door	to geth'er	con tin'ue
im prop'er	o bey'	no'where
prop'er	dis o bey'	some'where

66—FOODS

toast	gru'el	broth	dump'ling
tarts	crack'er	sir'loin	sand'wich
veal	cook'y	om'e let	dough'nut
bis'cuit	waf'fle	muf'fin	sauer'kraut
sal'ad	hon'ey	por'ridge	cus'tard
mut'ton	catch'up	chow'der	suc'co tash

67

ag'ile	rig'id	im'age	fidg'et
viv'id	ac'tive	jus'tice	res'pite
doc'ile	fes'tive	den'tist	cor'nice
hos'tile	rep'tile	ser'vice	des'tine
mis'sile	crev'ice	prom'ise	doc'trine

68

cowl	crowd	rouse	de vour'
prowl	bow'er	shout	foun'dry
clown	dow'er	crouch	scoun'drel
crown	cow'ard	mound	com'pound
drown	drow'sy	flounce	pro nounce'

69

cleft	flare	flaw	liege	rouge
pyre	dealt	goad	lapse	rogue
brief	freak	farce	weird	shone
copse	fraud	guile	grease	prism
corpse	fledge	gloat	mold	pierce
clench	chasm	gorge	mourn	plague
cleanse	dredge	gourd	league	scourge

70—OCCUPATIONS

tai'lor	flo'rist	farm'er	mil'li ner
ba'ker	join'er	weav'er	min'is ter
doc'tor	gro'cer	build'er	gar'den er
law'yer	bank'er	butch'er	car'pen ter
cob'bler	mer'chant	drug'gist	black'smith

71—OUTDOOR SPORTS

fish'ing	ri'ding	dri'ving	ska'ting
sail'ing	ten'nis	play'ing	mar'bles
row'ing	leap'ing	bowl'ing	coast'ing
boat'ing	cro quet'	jump'ing	base'ball
ba'thing	nut'ting	swing'ing	sleigh'ing

72

so'lar	nec'tar	dro'ver	dif'fer	lar'der
lu'nar	lat'ter	can'ker	hin'der	blis'ter
tar'tar	an'ger	la'ter	gan'der	blub'ber
stel'lar	bet'ter	cof'fer	fil'ter	mem'ber

73

loi'ter	coil	an noy'	in'voice
toi'let	broil	de coy'	re joice'
poi'son	spoil	boy'ish	pur loin'
coin'age	hoist	em ploy'	oint'ment
ap point'	choice	boy'cott	em broid'er

74

cen'ser	cir'cus	ac'id	ceil'ing
cen'sus	ci'pher	cir'cuit	cym'bal
cen'tral	cin'der	cyl'in der	cen'tu ry
ce ment'	cer'tain	cel'e brate	ce les'tial
cen'taur	cen'sure	cem'e ter y	cen ten'ni al

75—HOMONYMS

1. cent—a coin
   scent—an odor
   sent—did send
2. plum—a fruit
   plumb—perpendicular
3. stare—to look earnestly
   stair—a step
4. fore—in front
   four—a number
5. to—as in "Give it to me"
   too—as in "too cold"
   two—a number
6. threw—did throw
   through—as in "through the air"
7. fir—a tree
   fur—fine, soft hair
8. earn—to get or merit by labor
   urn—a vase

Write the following sentences, putting the right word in the right place. Underline the words inserted.

It is not what we (8), but what we save that makes us rich. —*Prov.* A bird in the hand is worth (5) in the bush.—*Prov.* We (1) the silver (8) (5) them. I remember the (7) trees, dark and high.—*Hood.* A (1) is one-hundredth part of a dollar. Prunes are dried (2)s. A dog has (4) feet, but the (4) feet are the two front feet. And all the world would (3). —*Cowper.* The (1) of the roses will hang round it still.—*Moore.* Who (6) the stone (6) the window? Russian sable is a costly (7). (5) many cooks spoil the broth. Masons test a wall with a (2)-line. White marble (3)s lead to the Capitol.

76

nev'er	neth'er	fel'on	men'tal
tem'per	skep'tic	zeal'ot	plen'ty
per'ish	cher'ub	stead'y	pet'rel
cher'ish	her'ald	peas'ant	jeal'ous
wed'lock	blem'ish	threat'en	weath'er
ped'dler	thread'bare	pleas'ure	meas'ure

77

com'pass	cac'tus	cha'os	chord
cul'prit	com'rade	chrome	chyle
cur'ry	cab'in	chro'mo	chyme
cur'few	ca'ble	chron'ic	chol'er a
com'et	com pute'	cho'ral	chron'i cle
com plete'	cu'pola	chem'ist	char'ac ter
col'umn	com plex'ion	Chris'tian	cha me'le on

78—ARITHMETIC

rate	cu'bic	in'te ger	a mount'
terms	fac'tor	dec'i mal	prin'ci pal
prime	frac'tion	mul'ti ple	di vis'i ble
dig'it	al'i quot	in'ter est	in sur'ance
ze'ro	dis'count	com pos'ite	bro'ker age
a'cre	ex am'ple	nu'mer a tor	per cent'age
naught	hun'dredth	de nom'i na tor	av oir du pois'

79—ANIMAL SOUNDS

purr	yelp	bleat	snort	squeak
hum	howl	cluck	cack'le	roar
low	quack	neigh	whin'ny	scream
grunt	growl	croak	bel'low	buzz
squeal	mew	gob'ble	chir'rup	screech

80—INSECTS

bee	gnat	ant	drag'on-fly
wasp	moth	wee'vil	bum'ble bee
flea	roach	mos qui'to	but'ter fly
lo'cust	bee'tle	glow'worm	ka'ty did
hor'net	crick'et	silk'worm	grass'hop per

81

a'li as	fa'cial	ef face'	fa'tal
a'gen cy	pa'tron	va'cant	ha'zy
ma'ni ac	sta'tion	en gage'	ba'bel
brace'let	an'cient	be came'	az'ure
fa'vor ite	pa rade'	pro fane'	ha'tred
va'por ize	dra'per y	dis place'	man'ger

82

gym'nast	gib'lets	hom'age	gen'u ine
gyp'sy	gen'ius	gib'bet	mag'is trate
en'gine	gen teel'	gen'e sis	gym nas'tics
mar'gin	herb'age	gest'ure	gym na'si um

83—REVIEW

ache	bruise	bu'reau	could
ac cept'	Arc'tic	care'ful	dai'sy
a fraid'	bis'cuit	car'riage	cur'tain
a gain'	au'tumn	cel'er y	col'ored
al'mond	ba na'na	chim'ney	con'ti nent
an'i mal	break'fast	Christ'mas	cran'ber ry

84

dif'fer ent	friend	health'y	juice
dough'nut	gi raffe'	help'ful	knife
ear'nest	gru'el	hom'i ny	knuck'le
ei'ther	e qua'tor	ho ri'zon	laugh
el'e phant	er'rand	hy'a cinth	learn
e nough'	Feb'ru a ry	in'ter es ting	isth'mus

85—REVIEW

maize	mo las'ses	o'a sis	rai'sin
li'chen	mos qui'to	o blige'	pic'ture
liq'uid	moun'tain	om'e let	pitch'er
mat'tress	nei'ther	os'trich	pleas'ant
mi'grate	neph'ew	oys'ter	prai'rie
min'u end	niece	par'al lel	pump'kin

86

rhu'barb	skein	tongue	zone
rough	sleigh	tor'rid	whose
salm'on	sev'er al	un til'	would
schol'ar	skel'e ton	tem'per ate	wrong
scis'sors	spin'ach	thous'and	ze'nith
sen'tence	squir'rel	vol ca'no	writ'ten

87—SYNONYMS

ef'fort	de cide'	lack'ing	a tone'
en deav'or	de ter'mine	de fi'cient	ex'pi ate
re past'	ven'er ate	out'ward	con fuse'
col la'tion	re vere'	ex ter'nal	be wil'der
en close'	spring'y	down'cast	de ride'
en vel'op	e las'tic	de ject'ed	rid'i cule

88

fool	priest	dra'ma	han'dle	lad'der
lock	del'ta	fel'low	hel'met	lob'ster
feast	an'gel	mask	freck'le	mar'ket
hook	se'cret	fos'sil	liq'uor	ma'tron

89

rul'er	an'cient	tow'er ing	for'tress es
sto'ry	ver'dant	ex ten'sive	at tract'ive
roams	fruit'ful	sep'ul cher	ap pa ri'tion
stream	slum'bers	be stow'ing	pic tur esque'
sta'ted	en tombed'	re main'der	ben e dic'tion

90—MASCULINE AND FEMININE

he'ro	her'o ine	beau	belle
host	host'ess	wiz'ard	witch
act'or	ac'tress	sir	mad'am
god	god'dess	bach'e lor	maid, spin'ster
heir	heir'ess	wid'ow er	wid'ow
jan'i tor	jan'i tress	man serv'ant	maid serv'ant
proph'et	proph'et ess	land'lord	land'la dy

91—HOMONYMS

1. break—to part by force
   brake—for stopping wheels, a fern
2. week—seven days
   weak—feeble
3. waist—part of the body
   waste—a desert, to squander
4. piece—a part, a composition
   peace—quiet
5. hail—frozen rain, to salute
   hale—healthy
6. wait—to stay
   weight—heaviness
7. heel—part of the foot
   heal—cure
8. peal—a loud sound
   peel—to strip off the skin

In the following sentences put the right word in the right place. Underline the words inserted.

Achilles was slain by being wounded in the (7). A (4) of banana (8) should not be thrown on the pavement. The (1)ing waves dashed high.—*Hemans.* (5), holy light.—*Milton.* The engine whistled "Down (1)s." What is your (6)? I lay me down in (4) to sleep.—*Willard.* Physician, (7) thyself.—*Bible.* If you are (5), you cannot be (2). The deep thunder, (8) on (8), afar.—*Byron.* There was a belt about her (3). Sunday is the first day of the (2). Learn to labor and to (6).—*Longfellow.* (3) not, want not.—*Prov.*

92

jui'cy	tip'sy	po'sy	ma'zy
spi'cy	fuss'y	pal'sy	diz'zy
fan'cy	moss'y	pro'sy	cra'zy
flee'cy	mass'y	flim'sy	breez'y
mer'cy	drop'sy	glass'y	driz'zly

93

daunt	twelfth	come'ly	bel'lows
brooch	a dult'	com mand'	as sume'
nymph	de sist'	a're a	cask'et
crea'ture	ad vance'	dis arm'	cit'i zen
con sume'	di gest'	cha grin'	duc'tile

94—HOMONYMS

1. air—what we breathe
   e'er—ever
   ere—before
   heir—one who inherits
2. sail—of a ship
   sale—a selling
3. ho'ly—sacred
   whol'ly—completely
4. plain—level ground, clear
   plane—flat surface, tree, tool
5. rain—water from clouds
   reign—rule
   rein—for a horse
6. coarse—rough
   course—way
7. col'lar—band for the neck
   chol'er—anger
8. dy'ing—ceasing to live
   dye'ing—coloring

Write the following sentences, putting the right word in the right place:

Westward the (6) of empire takes its way.—*Berkely.* O, there is sweetness in the morning (1)—*Byron.* (3) angels guard thy bed—*Watts.* How gladly would we buy time were it for (2). How beautiful is the (5) after the dust and heat—*Longfellow.* The top of my desk is a (4) surface. What! drunk with (7)—*Shakespeare.* The (7) was made of cloth. We thought her (8) when she slept. The Prince of Wales is (1) to the English throne. Russia is almost (3) a vast (4). Write home (1) the ship (2)s. The (5) guides the horse. The cochineal insects furnish a red color for (8). Shakespeare lived during the (5) of Queen Elizabeth.

95—CLOTH

jean	sat'in	si le'sia	cal'i co
baize	vel'vet	me ri'no	al pac'a
serge	flan'nel	cam'bric	dam'ask
plush	mus'lin	bro cade'	cor'du roy
lin'en	mo'hair	chev'i ot	cas'si mere
tweed	de laine'	ging'ham	vel vet een'
chintz	broad'cloth	cash'mere	seer'suck er

96

monk	piv'ot	pub'lic	tas'sel
mur'mur	part'ner	rum'mage	trel'lis
mys'ter y	phys'ic	skir'mish	tus'sle
par'a ble	pi'ous	sol'emn	weap'on
par'cel	pi'ra cy	spe'cie	wel'fare
mes'sage	pit'i ful	stam'mer	syr'inge

97

tu'bu lar	bear'er	vic'tor	ju'ror
tab'u lar	lodg'er	val'or	fla'vor
pop'u lar	cor'o ner	tu'tor	ru'mor
cir'cu lar	mourn'er	tre'mor	or'a tor
cal'en dar	strag'gler	trai'tor	stu'por
sec'u lar	vend'er	tor'por	splen'dor
mus'cu lar	in tru'der	suit'or	sur vey'or

98—ON THE WRITING DESK

ream	ru'ler	let'ter	pa'per-weight
quill	tab'let	e ra'ser	port fo'li o
quire	blot'ter	ink'stand	mu'ci lage
stamps	wa'fer	fools'cap	en'vel ope
pen'knife	cal'en dar	dic'tion a ry	seal'ing-wax

99—FISH

eel	trout	dol'phin	her'ring
cod	pike	sar'dine	mack'er el
carp	shark	had'dock	pick'er el
perch	shad	suck'er	stur'geon
bass	smelt	min'now	hal'i but

100

un just′	un just′ly	re′al	re′al ly
lan′guid	lan′guid ly	meek	meek′ly
se cure′	se cure′ly	court	court′ly
se′ri ous	se′ri ous ly	an′nu al	an′nu al ly
spite′ful	spite′ful ly	in tent′	in tent′ly
un u′su al	un u′su al ly	for′mer	for′mer ly
dread′ful	dread′ful ly	fre′quent	fre′quent ly

SPELLING

(PART 2)

1—SYNONYMS

a base′	ac quit′	ad′age	a dorn′
de grade′	ab solve′	max′im	dec′or ate
ab hor′	ac cede′	ac cost′	ad vice′
de test′	com ply′	sa lute′	coun′sel
a bide′	for sake′	a dieu′	ac quaint′
so′journ	a ban′don	good by′	in form′

2—OPPOSITES

ab′sent	free	guilt′y	stub′born
pres′ent	cap′tive	in′no cent	yield′ing
be stow′	dis perse′	in te′ri or	fail′ure
re ceive′	as sem′ble	ex te′ri or	suc cess′
de stroy′	e merge′	de crease′	em′i grate
con struct′	im merge′	aug ment′	im′mi grate

3

sa li′va	i′ci cle	con fide′	pi′rate
en vi′ron	pli′a ble	com bine′	bri′ny
en ti′tle	di′a ry	re cite′	mi′nus
vi′o late	li′a ble	sur mise′	li′bel
ri′val ry	si′phon	re quire′	fi′nal
pi′e ty	bi′ped	com prise′	vi′per

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4

braid	ex am'ine	coax	ra'zor
beard	ex cur'sion	skull	ven'ture
bloom	de light'ful	hoarse	whith'er
bleach	in struc'tion	a piece'	smoth'er

5—MINING

zinc	mi'ca	gran'ite	lode
lead	sul'phur	gyp'sum	shaft
quartz	car'bon	plat'i num	mi'ner
cop'per	salt"pe'ter	min'er al	tun'nel
ni'ter	mer'cu ry	me tal'lic	der'rick

6—FARMING

plow	har'row	ster'ile	fer'tile
reap'er	thresh'er	clay'ey	bog'gy
mow'er	fal'low	bar'ren	ar'a ble
scythe	swamp'y	loam'y	al lu'vi al
sick'le	fruit'ful	marsh'y	gua'no

7

mu'ti ny	flu'id	dis pute'	se clude'
du'ti ful	mu'sic	pro cure'	de lude'
pu'ri fy	glu'ten	in sure'	as sure'
cru'el ty	tu'mor	di lute'	a buse'
mu'tu al	fu'ture	en dure'	a muse'
lu'di crous	cu'ri ous	pre sume'	al lure'

8

doub'le	jew'el er	su pe'ri or	sleet
har'ness	tomor'row	va ca'tion	tacks
mat'ting	which ev'er	twi'light	sphere
val'u a ble	scen'er y	moun'tains	ce'dar

9

blouse	cis'tern	knuck'le	steppes
breathe	clean'ly	col'an der	tru'ly
el lipse'	dah'lia	fa mil'iar	vil'lain
de spair'	cru sade'	des'o late	cut'ler y
um'pire	cou'pon	de pos'it	hap'pened
jun'ior	com'merce	tran'som	oc curred'
a'corn	cour'te sy	ven'ti late	prec'i pice

10

po si'tion pat'ron ize live'li hood ac cu'mu late
gen'ius sys'tem in'tel lect ad van'tage
stud'ies tres'pass es fa'vor it ism in"dis pen'sa ble
pro fes'sion ex celled' op"por tu'ni ties com"men da'tion
suf'frage a bil'i ties in duce'ment op'er ate
ad mit'ted pos sess' ac cept'a ble dis course'

11

gen'tle man cour'te ous friend'ly dil'i gent
hu'man re spect'ful ly ge'nial ca'pa ble
mor'al ly re lig'ious sen'si ble com'pe tent
sac'ri fice grate'ful a gree'a ble con"sci en'tious
in"di vid'u al lov'a ble char'ac ter for giv'a ble
sin cere' hap'pi ness ed'u cate re fined'

12

pic"tur esque' lab'y rinth in cal'cu la bly
fash'ion rev"e la'tion gran'deur
or"na men'tal in"de scri'ba ble charm'ing
va'ri e ga"ted pro dig'ious mag nif'i cent
gro tesque' ra vine' bril'liant
u nique' at tract'ive ef"fer ves'cence
beau'ti ful ex"traor'di na ry sym met'ric al
ex'cel len cy con sum'mate

13

strict ear'li est thread peo'ple
is'su ing com pels' for'feit beg'gar
di vulge' im mense' im peach' mar'ket
in quir'y mod'ern fierce vil'lage
in cense' dis pel' con'sul bulk
get'ting bulge yield por'tion
mag'net re ly' heav'y con tain'
phase de nied' failed stroll

14—OCCUPATIONS

la'bor er man'a ger e lec"tri'cian com pos'i tor
in vent'or po lice'man ar'chi tect sur vey'or
phy si'cian fire'man en"gi neer' auc"tion eer'
su"per vi'sor ca'ter er book'keep"er ga'ger
sur'geon ma chin'ist pro fess'or pho tog'ra pher

15

grief	griev'ous	pore	po'rous
joy	joy'ous	ri'ot	ri'ot ous
vice	vi'cious	glo'ry	glo'ri ous
stud'y	stu'di ous	dan'ger	dan'ger ous
la'bor	la bo'ri ous	mur'der	mur'der ous
in'dus try	in dus'tri ous	ma la'ri a	ma la'ri ous

16—HOMONYMS

1. pair—two
   pare—to cut off
   pear—a fruit
2. their—belonging to them
   there—in that place
3. cap'i tal—chief town, stock
   cap'i tol—building
4. hear—to listen
   here—in this place
5. meat—animal food
   meet—proper, to come together
   mete—to measure
6. as cent'—a rising
   as sent'—agreement
7. pane—a plate of glass
   pain—an ache
8. rye—a grain
   wry—crooked

Write the following sentences, putting the right word in the right place:

(2) is nothing new under the sun.—*Bible.* (8) grows in cold countries. When shall we three (5) again?—*Shakespeare.* A (7) of glass. (4) rests his head upon the lap of earth.—*Gray.* Can you (1) a (1) with a (1) of scissors? The (3) is a white marble building. The (6) of Mont Blanc is full of danger. Be silent that you may (4).—*Shakespeare.* Sweet is pleasure after (7).—*Dryden.* Birds in (2) little nests agree.—*Watts.* Washington is the (3) of the United States. Is not the life more than (5)?—*Bible.* He gave his (6) to the proposal. There is a bird called the (8) neck. With what measure ye (5), it shall be measured to you again.—*Bible.*

17—KNOWN BY

Seeing	*Smelling*	*Tasting*
squal'id	ran'cid	lus'cious
un couth'	fra'grant	pun'gent
col'ored	per'fumed	bit'ter
o paque'	o'dor ous	in sip'id
ra'di ant	ar″o mat'ic	de li'cious

Touching		*Hearing*	
warm	smooth	clear	ech'o ing
rough	tep'id	loud	muf'fled
sleek	un e'ven	in dis tinct'	noi'sy

18

height	count'er	la'zi ly	ach'ing
re sult'	awk'ward	hes'i tate	sur prise'
con fess'	bus'i ly	spe'cial	quan'ti ty
wrig'gle	re mark'a ble	mus'cles	syl'la ble
fir'kin	gen'tle man	vil'lage	scold'ed
anx i'e ty	u'su al ly	pro tect'	gos'sa mer

19—REVIEW

e ra'ser	dic'tion a ry	scythe	com'mon
bar'y tone	mack'er el	feign	tac'it
or'ches tra	jew'el ry	zeph'yr	gyp'sy
cam paign'	veg'e ta ble	liq'uor	ges'ture
glac'i er	cas'si mere	giz'zard	salm'on
moun'tain	sar"sa pa ril'la	gey'ser	wool'en

20

al pac'a	par'lia ment	thief	dai'sy
sal'a ry	cyl'in der	toi'let	doc'ile
boy'cott	cem'e ter y	au'tumn	rep'tile
ox'y gen	am'e thyst	ac'id	im'age
em'er y	sep'a rate	pum'ice	cor'nice
tur quoise'	hun'dredth	ber'yl	ci'pher

21—REVIEW

skep'tic	sol'emn	weird	can'cel
squal'id	ver'ti cal	myrrh	con cede'
squir'rel	de li'cious	which	pleas'ure
schol'ar	syl'la ble	straight	ped'dler
myr'i ad	ker'o sene	weap'on	vil'lage
wiz'ard	med'i cine	syr'inge	griz'zly

22

hand'ker chief	cu'li na ry	lem"on ade'	o"leo mar'ga rine
rai'ment	fur'ni ture	cham pagne'	slam'ming
pan"ta loons'	ve ran'da	ap par'el	gel'a tine
chan"de lier'	u ten'sil	cor'ri dor	clock'work"
in te'ri or	gas'o line	bric'-a-brac"	heath

23

i'ci cle	ver mil'ion	a dieu'	tus'sle
rum'mage	Ni ag'a ra	sep'a rate	cer'tain
lunch'eon	nec'es sa ry	twelfth	cha grin'
sur'name	pen i ten'tia ry	be lieve'	cot'tage
car'a mel	re ceived'	the'a ter	sup pose'
span'iel	ex hi bi'tion	res'er voir"	de vel'op

24—HISTORY

char'ter	squaw	scalped	wil'der ness
spear	pap poose'	ship'wreck	suf'fer ing
wig'wam	war'rior	ex plore'	set'tle ment
wam'pum	cal'u met	col'o ny	trea'ty
moc'ca sin	dis cov'er y	per'ma nent	per'se cute
tom'a hawk	pi o neers'	nav'i ga"tor	mas'sa cre

25

cu'ti cle	lyr'ic al	cler'ic al	pay'a ble
mir'a cle	spher'ic al	crit'ic al	bla'ma ble
ob'sta cle	sur'gi cal	cyn'ic al	ta'ma ble
par'ti cle	trag'ic al	whim'si cal	teach'a ble
spec'ta cle	op'tic al	prac'ti cal	ten'a ble
trea'cle	clas'sic al	phys'i cal	ca'pa ble
man'a cle	com'ic al	med'ic al	af'fa ble

26

sing'er	preach'er	deaf'ness	soft'ness
wait'er	la'bor er	firm'ness	sweet'ness
start'er	lect'ur er	swift'ness	round'ness
catch'er	ex am'in er	fierce'ness	prompt'ness
think'er	com mand'er	hard'ness	wretch'ed ness

27

ac crue'	a thwart'	con'quer	ath'lete
a chieve'	vict'uals	a'lien	fau'cet
a ghast'	cro chet'	al'oes	cal'lous
greas'y	bou quet'	anx'ious	bur'glar
ca det'	bru nette'	brig'and	dul'cet
con geal'	car toon'	drug'get	daunt'less

28—WORDS USED IN ARITHMETIC

nu'mer ous	length	per"pen dic'u lar
di vis'i ble	ounce	bal'ance
re du'cing	par ti'tion	hor"i zon'tal
log'a rithm	in nu'mer a ble	math"e mat'i cal
summed	sub"di vi'sion	ad di'tion
cal"cu la'tions	com par' i son	cal'cu late
par'tial ly	ad di'tion al	per cent'
un e'qual ly	quan'ti ties	coin'age

29

su"per vise'	cler'ic al	fa ce'tious	in ten'tion
en deav'or	cer'ti fy	civ'il ly	dis crep'ancy
pre ce'dence	per mis'sion	cred'u lous	re nun"ci a'tion
pro fi'cient	prom'is so"ry	ex'pe dite	ag'gra vate
pri or'i ty	in'ter est	knowl'edge	im pos'si ble
ver'i fied	jus'ti fied	dow'ery	traf'fick ing
a bey'ance	cen'sure	no"to ri'e ty	fal'la cy

30

ab'sence	fuzz	go'pher	la pel'
at'om	cleat	griz'zly	loi'ter
bar'gain	clev'er	haz'ard	lag'gard
but'ton	e clipse'	hu'mor	loz'enge
cat'kin	en cir'cle	jock'ey	ma chine'
glut'ton	fa tigue'	graph'ic al	med'i cine

31

change	change'a ble	man'age	man'age a ble
charge	charge'a ble	mar'riage	mar'riage a ble
trace	trace'a ble	cour'age	cou ra'geous
no'tice	no'tice a ble	out'rage	out ra'geous
ser'vice	ser'vice a ble	ad vant'age	ad"van ta'geous

32

ey'ry	la'va	ma rine'	dis dain'
eye'let	leis'ure	mar'tyr	glob'ule
frag'ile	jour'nal	naph'tha	hos'tage
frag'ment	lan'guage	nui'sance	cres'cent
ghast'ly	laun'dry	nos'trils	tor'toise
gor'geous	mal treat'	cha rade'	tor'ture

33—CHRISTMAS DINNER

soup	tur'key	on'ions	ice cream'
rolls	squash	sauce	cake
pick'les	po ta'toes	mince pie	fruit
cel'er y	gra'vy	plum pud'ding	con'serve

34

bier	ag grieve'	ceil	rein
tier	re lief'	de ceit'	reign
mien	shield	de ceive'	weigh
wield	re trieve'	con ceive'	skein
lien	re prieve'	con ceit'	hei'nous
niece	mis'chief	re ceipt'	o bei'sance
siege	sor'tie	re ceive'	in veigh'
frieze	ker'chief	per ceive'	neigh'bor

35

chas tise'	bap tize'	a pol'o gize	or'gan ize
crit'i cize	cap size'	har'mo nize	mag'net ize
cat'e chize	re'al ize	gal'va nize	sym'pa thize
ad'ver tise	i'dol ize	fer'til ize	sol'em nize
ex'er cise	civ'i lize	col'o nize	rec'og nize
mer'chan dise	cen'tral ize	tan'ta lize	pat'ron ize
en'ter prise	le'gal ize	dram'a tize	mem'o rize

36

val'leys	pur'chased	sal'a ry	ce'dar
cur'rent	a cross'	spruce	scen'er y
cap'i tal	glac'i ers	in hab'it ants	val'u a ble
cli'mate	va ri'e ty	re sem'ble	de spair'

37

de clare'	can'cel	ap pease'	con sent'	af fair'
col lect'	a ver'	a bol'ish	re voke'	a gree'
al lay'	a mass'	as sert'	dis cuss'	an nul'
ap pal'	as suage'	con cede'	af firm'	ar'gue
at tach'	af fright'	zeal'ous	al low'	anx'ious
pac'i fy	ap pend'	con cern'	ar'dent	an'cient

38

sluice	thyme	whoop	suite	spouse
shrift	type	wreathe	realm	manse
scribe	theme	writhe	myrrh	phrase

39

nerve	un nerve′	ech′o	re ech′o
spell	mis spell′	e lect′	re″e lect′
spend	mis spend′	le′gal	il le′gal
step	mis step′	sev′er	dis sev′er
en gage′	re″en gage′	mor′tal	im mor′tal
sat′is fy	dis sat′is fy	sim′i lar	dis sim′i lar

40

leg′a cy	ur′gen cy	in′ti ma cy	em′bas sy
in′fan cy	cur′ren cy	ex′i gen cy	jeal′ous y
de′cen cy	con′stan cy	cel′i ba cy	min′strel sy
se′cre cy	fal′la cy	con spir′a cy	lep′ro sy
re′gen cy	clem′en cy	ac′cu ra cy	her′e sy
flu′en cy	bril′lian cy	e mer′gen cy	ec′sta sy
va′can cy	buoy′an cy	com′pe ten cy	hy poc′ri sy

41

a bun′dance	tem′per ance	au′di ence	ig′no rant
venge′ance	ac quaint′ance	dil′i gence	as sist′ant
ig′no rance	an noy′ance	ab′sti nence	de pend′ent
sus′te nance	ve′he mence	rev′er ence	el′e gant
vig′i lance	el′o quence	dif′fi dence	dil′i gent

42—LAW

cli′ent	i den′ti ty	ac quit′tal	as sail′ant
de fend′ant	crim′i nal	at tor′ney	de tect′ive
pe ti′tion	in sid′i ous	in′no cence	mis″de mean′or
pro ce′dure	in″for ma′tion	ad journ′	dis charge′a ble
mur′der er	con″fi den′tial	jus′tice	judg′ment
coun′sel	fraud′u lence	con demn′	as sas′sin

43—WORDS USED AT ELECTIONS

in au″gu ra′tion	prom′i nent	par tic′i pants
del′e gates	or′di nance	ri′val ry
pol″i ti′cian	as sem′blage	of′fi cers
af firm′a tive	tax′a ble	ex clu′sive ly
con′gress man	treas′ur y	cit′i fied
mu nic′i pal	sher′iff	can′di date
pre lim′i na ry	pro tect′or	ap point′ment

44

dis'cord	vac'il late	in va'ri a bly	da guerre'o type
friv'o lous	re'cent ly	et'i quette"	ca pac'i tate
e ter'nal	crowd'ed	in"can des'cent	ci ta'tion
mil len'ni um	ec cen'tric	re frig'er a"tor	like'li hood
ab'sti nence	fruit'ful	clam'or ing	par'a gon
con'science	sev'er al	pro du'cing	pre su'ma ble

45

il lu'mi nate	sur prised'	hun'dred	a cid'i ty
de crease'	aus tere'ly	tres'tle	han'dled
hol'i day	in fat'u ate	scarce'ly	as sim'i late
fore tell'	hin'dered	un known'	be liev'ing
va ri'et y	un but'toned	e'ven ly	sup pose'
prob'a ble	u biq'ui tous	more o'ver	har'bin ger

46

now'a days"	com mit'tee	e con'o mist	gov ern or
per sim'mon	coun'cil	stran'ger	guard'i an
vic'es	south'ern er	a'er o naut	cit'i zen
nat'u ral	com'pa nies	serv'ant	bach'e lor
char'i ty	heir'ess	ci vil'ian	for'eign er
sci'en tist	fam'i lies	cou'ri er	am"a teur'

47—WORDS LIABLE TO BE CONFOUNDED

curt'sy—a downward movement of the body by bending the knees, an act of respect

cour'te sy—politeness originating in kindness and exercised habitually

cro chet'—to knit worsted, silk, or other thread into a fabric with a single needle

crotch'et—a peculiar opinion

car'et—a sign (∧) placed below a line, indicating where omitted words, etc. should be inserted

car'at—a unit of weight for precious stones

car'rot—a vegetable

as sess'a ble—capable of being assessed

ac ces'si ble—capable of being reached
mar'shal—an officer
mar'tial—pertaining to military operations
bred—brought up
bread—an article of food
dis crete'—distinct or separate
dis creet'—wise in avoiding errors
co los'sal—of immense size
co los'sus—a gigantic statue
aisle—a passageway, as in a church
isle—an island
prin'ci pal—first or highest in rank
prin'ci ple—a source or cause from which a thing proceeds
core—the central or innermost part of a thing
corps—a number of persons acting together
corpse—a dead body
i'dyl—a short poem
i'dle—not occupied: doing nothing
i'dol—image of a heathen god
de vi'sor—one who gives by will
de vi' ser—one who devises or contrives
di vi' sor—that by which a number is divided
can'vas—a heavy, strong fabric, such as a sail
can'vass—to solicit votes, orders, subscriptions, etc.
ei'ther—one or the other of two
e'ther—the upper air
mus'cle—an organ composed of fibers the contraction of which causes bodily movement
mus'sel—a small fish
hu'me rus—the bone of the upper part of the arm
hu'mor ous—adapted to excite laughter

48

Ec″ua dor	A″si at'ic	Tal″la has'see	Geor'gi a
Vir gin'i a	Des Moines	Bor″deaux'	A'la mo
Mem'phis	Ven″ez ue'la	Ter're Haute	Fah'ren heit
Ches'a peake	Rap″pa han'nock	Lou″i si an'a	Ghent

49

trag'e dy	ir″re triev'a ble	bar'ba rous	ag'i tate
un for'tu nate	ghast'ly	be reave'ment	hor'ri ble
tor'ture	des troy'er	dep're cate	ca rou'sal
curse	short'-lived″	tol'er ate	numb'ness
mis for'tune	marred	dis″ap point' ment	ac'ci dent
col lapse'	griev'ance	scan'dal	suc cumb'
tire'some	hin'drance	hid'e ous	mourn'ers

50—GRAMMATICAL WORDS

par'a phrase	syn op'sis	lan'guage
su per'la tive	rhyth'mic	smooth
an'a lyze	mon'o syl la ble	glos'sa ry
ab bre″vi a'tion	sum'ma ry	neu'ter
id″i o syn'cra sy	gram'mar	re quire'ment
gram mat'ic al	il lit'er ate	a nal'y sis
in″con sist'ent	col lo'qui al	am big'u ous
phrase	cor rect'	at'trib ute
di plo'ma	an″te ce'dent	sum'ma rize
ap plied'		

51

cou ra'geous	ex pe'ri ence	ex hib'it	ex er'tion
com pet'i tor	vig'or ous	a breast'	non″pa reil'
de fen'sive	en cour'age	ex ci'ting	vir'tue
con'quer or	pre par'a to ry	ex'er cise	pre ce'dence
ath'lete	skilled	out stripped'	vin'di cate
sports'man	tus'sle	hand'i capped	ful fil'ment
keen'ness	dis'tance	ar range'ment	pro ce'dure

52—DISEASES

gout	ca tarrh'	jaun'dice	hem'or rhage
croup	asth'ma	quin'sy	rheu'ma tism
a'gue	mea'sles	scur'vy	an'eu rism
mumps	ul'cer	ty'phoid	pneu mo'ni a
fe'ver	ab'scess	scrof'u la	dys pep'si a
grippe	in flu en'za	nau'se a	pa ral'y sis
can'cer	chil'blain	pleu'ri sy	diph the'ri a
col'ic	ver'ti go	de lir'i um	hy dro pho'bi a

53—HISTORY

writs bul'let ef'fi gy in de pend'ent
re sist' pow'der al li'ance dec la ra'tion
mobbed ram'part as sem'bly ca lam'i ty
war'rant car'tridge pro hib'it vol un teer'
un'ion pa'tri ot tax a'tion e vac'u ate
suf'frage re pealed' tyr'an ny rev"o lu'tion
of fi'cial mi li'tia op pres'sion u ni ver'sal
re doubt' lib'er ty gov'ern ment rep re sent'

54—ARMY WORDS

sol'dier pla toon' sen'try siege
com'pa ny bat tal'ion pick'et sut'ler
reg'i ment sword u'ni form re cruit'
bri gade' sa'ber knap'sack hos'pi tal
di vi'sion bay'o net can teen' gar'ri son
corps pis'tol ep'au let coun'ter sign
in'fan try car'bine bag'gage court-mar'tial
cav'al ry mus'ket cais'son am mu ni'tion
ar til'ler y ri'fle biv'ou ac for ti fi ca'tion
en gi neers' can'non fur'lough in trench'ment

55

Wednes'day Ben'ja min Cu'ban Cin"cin na'ti
Feb'ru a ry Chi nese' Jap"a nese' Con nect'i cut
Dan'iel Chris'tian ize A'pril Min"ne so'ta
I tal'ia Eng'land Ken"ne bec' Min"ne ap'o lis
Brit'ish Swe'dish Ni"ca ra'gua Nar"ra gan'sett

56—TRANSPORTATION

car'goes tar'iff trans mit'ted leas'a ble
freight in stal'ment a mount' ex or'bi tant
route for'eign ti' tle du' tia ble
leak'age li'cense min'i mum charge'a ble
ship'ping trans ferred' busi'ness traf'fic
ton'nage vi'ce ver'sa re mit'tance des"ti na'tion

57—WORDS PERTAINING TO THE SICK

con″va les′cence	tem′per ance	al″co hol′ic	cur′a tive
phys′ic al ly	ail′ment	ben″e fi′cial	cleaned
lar′ynx	pes′ti lence	phy sique′	chlo′ro form
fa tigu′ ing	heart′rend″ing	suf′fer er	hem′or rhage
stim′u lus	vic′tim	con ta′gious	lau′da num
phar′ma cy	nurs′ing	par′a lyzed	neu ral′gi a
lin′i ment	liq′ue fy	mor′phine	pleu′ri sy
rheu′ma tism	quar″an tine′	liq′uor	a cous′tic

58

ex pense′	char′ging	des″ig na′tion	prob′a ble
in ad′e quate	plen′ti ful	am bi′tious	fa′vor a ble
as sur′ance	de scend′ant	av″a ri′cious	be com′ing
fi nan′cial ly	tax a′tion	po lit′ic al	nu′cle us
in tel′li gence	co in′ci dent	rep″re sent′	read′i ly
en rolled′	con ve′nience	so lic″i ta′tion	re′al ize
ex am″i na′tion	civ′i lized	pre′vi ous	im pugn′
oc cur′ring	em ploy′ment	dis cre′tion	siege

59—NEWSPAPER TERMS

dai′ly	ed′i tor	ed i to′ri al
morn′ing	jour′nal ist	lead′er
eve′ning	con trib′u tor	ar′ti cle
sem″i-week′ly	re port′er	i′tem
week′ly	cor re spond′ent	lo′cal
bimonth′ly	sub scrib′er	tel′e grams
month′ly	ad′ver ti″ser	no′tic es
ex chang′es	car′ri er	gos′sip
e di′tion	news′boy	fi nan′cial
pro pri′e tor	ex′tra	a muse′ments
pub′lish er	col′umn	mis cel la′ne ous

60—REVIEW

war′rior	cyn′ic al	fuch′si a	phlox
sa′chem	cler′ic al	a za′le a	cap′tain
skil′ful	proph′e cy	wretch′ed	trea′cle
pro′ceeds	proph′e sy	an″te ce′dent	pre cede′
con sign′ee	mer′ri ment	par′ti ci ple	pa′tience
Mis sis sip′pi	prom′is so″ry	de clar′a tive	de cease′

61

or'i fice	rheu'ma tism	jeal'ous y	fron"tier'
pal i sade'	aux il'ia ry	mal'ice	scal'lop
cav a lier'	lin'e a ment	vi'cious	su'ture
nau'ti lus	Cin cin na'ti	ec'sta sy	si'phon
se'cre cy	dys pep'si a	pen'sion	mus'cle
con'ten tion	pneu mo'ni a	mis spell'	e'qual ly

62

league	bat'ter y	im ag'ine	niche
gui tar'	sal'a ble	fric"as see'	sparse
cor'nice	tab'er na cle	lat'i tude	clique
fa çade'	syn'a gogue	mul'lein	na'dir
ac crue'	pre cen'tor	gor'geous	eye'let
pres'sure	cer'e mo ny	tor'toise	mi rage'

63

Mis sou'ri	Atch'i son	Great Brit'ain	Cay enne'
Ten"nes see'	Del'a ware	Chris'ten dom	Ra"cine'
Ha van'a	Mas"sa chu'setts	Por"tu guese'	Guin'ea
Mil wau'kee	Mis"sis sip'pi	Knox'ville	Sac"ra men'to
Ma ni'la	Eu"ro pe'an	Cal"i for'ni a	Vin cennes'

64—ARMY

re sist'ance	strat'e gy	en"gi neer'ing	un"der ta'king
rep'ri mand	dis charge'	co lo'ni al	vig'i lant
ir"re sist'i ble	haz'ard ous	col'umn	su pe'ri or
dis'ci pline	con ces'sion	fu'gi tive	con cil'i a to ry
bound'a ry	skil'ful	colo'nel	a ban'don
en'e my	vet'er an	u'til ize	fear'less ly
guard'ed	bat'tle	e quipped'	aux il'i ary
an tag'o nize	strug'gling	or'gan ized	tri'umph

65—HISTORY

pow'er ful	bul'le tin	as sist'ance	de fen'si ble
su prem'a cy	tour'na ment	baf'fled	rec'on cile
hos til'i ties	an ni'hi late	in"ef fi'cient	bar"ri cade'
his tor'ic al	dev'as tate	strat'a gem	can"non ade'
quar'rel	mer'ce na ry	as sail'ant	col'on y
brig"a dier'	scheme	a lert'	e mer'gen cies
sur ren'der	hon'or a ble	suc cess'ful	sur round'
ac'ci dent	serv'ice a ble	lac'er ate	al le'giance

66—BUSINESS

com mis'sion
el'i gi ble
cur'ren cy
can"cel a'tion
pur'chas a ble
mu'tu al
the'o ry
sur'plus
earn'ings
proc'ess
suc ceed'
ef face'a ble
rec'om pense
col lat'er al
com par'i son
de vel'op ment
as sess'a ble
leg'a cy
ex on'er ate
sup press'ing
ap prais'er
ir"re deem'able
in val'u a ble
in'ter view
con'fer ence
ex change'a ble
ac com'plish
wel'fare

67—BUSINESS

debts
sure'ties
dis qual'i fied
re paid'
ac crue'
in"ter fere'
ac com'mo date
fun"da men'tal
con spic'u ous
ex ag'ger ate
dis solve
mon op'o lize
cred'it
an nul'
un"der rate'
ex ceed'
ac'cu rate
an'nu al ly
for'feit ed
con trolled'
pro ce'dure
prompt
an'swer
com"pe ti'tion
per'son al
ig'nor ant
ap pear'ance
dem on'stra"ted

68

where as'
wheth'er
here"to fore'
so'journ
in'stance
per turbs'
al"to geth'er
doff
oc curred'
se cede'
u'sing
in dulge'
ex panse'
sol'dier ly
bar'ring
gra'ded
lim'it ed
sham'rock
them selves'
en a'bling
mid'dle
im ag'ine
dis pense'
as"cer tain'
thank'ful
peace'ful
per tain'ing
piece'meal"

69

crys'tal lize
choos'ing
through
meer'schaum
scin'til late
pa la'tial
re quir'ing
na"tion al'i ty
put'ting
sed'i ment
eas'i ly
versed
niche
be gin'ning
curt'sy
ad"van ta'geous
in i'ti a tive
ab hor'rence
fu til'i ty
en'vi ous
re gard'ing
sha'ky
em bed'ded
con fused'

70

mis lead'ing	gun'ner y	nup'tials	co″a lesce'
dough	bur'den	con ceive'	o paque'
the″o log'ic al	per haps'	feoff	fil'ial
af fords'	dis taste'ful	che root'	bour geois'
e nough'	re pair'ing	in″ad vert'ence	bi zarre'
fe'ver ish	pas'time″	fal la'cious	par'al lel

71

main'te nance	tend'en cy	priv'i lege	sup″ple men'ta ry
per vade'	mon op'o lies	ac cru'ing	com mo'di ous
in″cu ba'tion	con cede'	pe'ri od	nar'ra tive
dis tin'guish	in″ter cede'	con ferred'	im'ple ment
con trol'	ma te'ri al ly	se'cre cy	hy drau'lic
stead'i er	stud'y ing	leath'er	lau'rel

72—WORDS PERTAINING TO NAVIGATION

buoy'an cy	im per'iled	source	re gat'ta
fur'lough	aq'ue duct	pro pel'	cruis'er
stream	ex er'tion	trav'el ing	o″ce an'ic
ca nal'	ob'sta cles	voy'age	wa'ter course″
wa'ter	at'mos phere	schoon'er	har'bor
o'cean	ir″ri ga'tion	sea'far″ing	mar'i time
is'land er	jour'neys	nau'tic al	nav″i ga'tion

73

con″sti tu'tion	suf fi'cient	eq'ui ty	con sist'en cy
per mis'si ble	rar'e fy	ac cede'	as cend'
re spon″si bil'i ty	right'eous	ac quire'	pro cure'
re cur'rence	pre ten'tious	re ceive'	com mit'
ef fi'cien cy	in cip'i ent	en joined'	de pend'ent
es sen'tial	im'i tate	pre pared'	con cede'

74

car'i ca ture	pri'vate	im me'di ate ly
ru'di ment	vol'ume	sup″po si'tion
sim'i lar	reg″u lar'i ty	pos'i tive
pag'eant	de cide'	prom'i nent
in″tel lec'tu al	pro gress'ive	es pe'cial ly
ac knowl'edg ment	tem'po ra ry	ex treme'
pam'phlet	ad vi'sa ble	va'ri ous
cat'a log	ref'er ence	con sid'er

75

coup'le	pau'ci ty	nui'sance	sues
fro'zen	venge'ance	mar'riage	route
vict'uals	griev'ance	wretch'ed	choked
bou quet'	ni'tro gen	ap'pe tite	ra vine'
bru nette'	cap'il la ry	ha rangue'	rou tine'
drug'gist	es pe'cial ly	phy sique'	pat'ent

76—REVIEW

ca tal'pa	dis ap pear'	sep'a rate	sphinx
mi li'tia	dis'si pate	re ceived'	can'cer
ver'ti cal	glyc'er ine	sur'geon	ca tarrh'
cal'en dar	ben'e fit ed	sol'dier	cap size'
ex'er cise	nec'es sa ry	bag'gage	bea'con
con'science	re bel'lion	fur'lough	bod'ice

77

rec'i pe	in sur'er	fag'ot	dis cern'
def'i nite	ep'au let	for'feit	ex haust'
sac'ri lege	ma neu'ver	fer'rule	crip'ple
right'eous	ar'chi tect	for bade'	cau'tious
gra'cious	guar an tee'	sur'plice	symp'tom
en gi neer'	chrys'a lis	hic'cough	com'ment
chal'ice	con'fis cate	ton'sil	con'crete
chiv'al ry	con va les'cent	per spire'	poul'tice

78—REVIEW

cha rade'	bil'liards	al lege'	whey
po'rous	am'a teur	ax'i om	vig'or
plov'er	per se vere'	pur'ple	vic'ar
scep'ter	beau'te ous	ter'race	sol'der
lar'ynx	prej'u dice	spig'ot	bil'ious
tra'che a	ac com'plice	suc ceed'	su'mac

79

pit'y	con'duit	bre vier'	siege
a byss'	col'umn	wrin'kle	o'gre
sol'ace	brick'kiln	tres'tle	trip'le
er'ror	ban dan'na	ten'ant	sher'iff
busi'ness	pam'phlet	shud'der	pal'ace
re scind'	cat'er pil lar	res'er voir"	pi az'za
Af'ghan	con ge'ni al	wor'shiped	quin'ine
bi'cy cle	chi rop'o dist	chan de lier'	fla'grant

80

per suade′	per′fi dy	pur′ga to ry	purge
per form′	per se vere′	pur vey′or	pur′port
per′jure	per pet′u al	per′pe trate	pur′pose
per plex′	per son′i fy	per′ti nent	pur suit′

81—RAILROAD

mail	de′pot	lug′gage	tel′e graph
train	sig′nal	dis patch′	con duct′or
track	tick′et	sta′tion	post′al clerk
check	bun′dle	ex′press	pe ri od′ic als
freight	pack′age	brake′man	lo co mo′tive

82

at tain′a ble	el′i gi ble	con tempt′i ble	beau′te ous
pit′i a ble	suit′a ble	ter′ri ble	be gin′ning
a gree′a ble	al′ma nac	vis′i ble	be ha′vior
sec′re ta ry	boun′da ry	cred′i ble	ref′er a ble
spec′i men	as cer tain′	cer tif′i cate	cu ri os′i ty
ex pi ra′tion	ex treme′ly	dif′fer ent	treach′er y
grat′i tude	sum′moned	co lo′ni al	re it′er ate
ob serv′ance	re ceiv′a ble	ex cus′a ble	cer′e mony
av′er age	al read′y	al′pha bet	ac knowl′edge
ex pla na′tion	an′thra cite	ap pren′tice	rel′e vant

83—WORDS LIABLE TO BE CONFOUNDED

sculp′tor—a carver of stone	light′en ing—to make light
sculp′ture—carved work	light′ning—electric flash
proph′e sy—to foretell	pa′tients—sick people
proph′e cy—a prediction	pa′tience—endurance
pre cede′—to go before	dis ease′—illness
pro ceed′—to advance	de cease′—death

Write the following sentences, selecting the right word:

1. His (prophesy, prophecy) was fulfilled. 2. Let us run with (patience, patients) the race that is set before us.—*Bible*. 3. (Lightening, lightning) must always (proceed, precede) thunder. 4. Enjoy the kingdom after my (decease, disease). —*Shakespeare*. 5. Phidias was a famous (sculpture, sculptor) of ancient Greece. 6. The captain ordered the (lightning,

lightening) of the vessel. 7. The remedy is worse than the (disease, decease).—*Shakespeare.* 8. The physician cured many (patience, patients). 9. He forth on his journey did (precede, proceed). 10. The Greeks ornamented their temples with (sculptor, sculpture).

84—WORDS LIABLE TO BE CONFOUNDED

1 { ac cept'—to take
  { ex cept'—to leave out

2 { ad vice'—counsel
  { ad vise'—to give counsel

3 { at tend'ance—the persons who attend any service, etc.
  { at tend'ants—those who attend as servants, companions, etc.

4 { for'mer ly—time past
  { form'al ly—in a formal way

5 { sta'tion a ry—fixed
  { sta'tion er y—paper, pens, etc

6 { pop'u lous—full of people
  { pop'u lace—the people

Write the following sentences, using the right word from the list above:

China is a (6) country. Will you (1) my (2)? King Arthur had brave (3). The (6) swarmed in the streets. (4) buffaloes roamed over the Great Plains. A (5) engine drew a car to the top of the hill. The (3) was large at every lecture (1) the first. I shall no more (2) thee. A stationer sells(5). The meeting was (4) opened by the president.

85—WORDS LIABLE TO BE CONFOUNDED

ac cede'—to agree to
ex ceed'—to go beyond
af fect'—to act upon
ef fect'—to accomplish
ad di'tion—process of adding
e di'tion—publication
as say'—to test metals
es say'—to try
bal'lad—a song
bal'lot—a vote
close—to shut
clothes—articles of dress
cen'tu ry—hundred years
sen'try—a sentinel
cel'e ry—a vegetable
sal'a ry—wages
de scent'—a going down
dis sent'—to disagree
e lic'it—to draw out
il lic'it—unlawful

em'i nent—distinguished
im'mi nent—threatening
e lude'—to escape from
al lude'—to refer to
e rup'tion—a bursting forth
ir rup'tion—an invasion

em'i grate—to leave
im'mi grate—to move into
ex'er cise—to use
ex'or cise—to drive away
gla'cier—an ice field
gla'zier—a glass setter

86—WORDS LIABLE TO BE CONFOUNDED

in gen'ious—skilful
in gen'u ous—honest
jest'er—one who jests
ges'ture—action
lin'i ment—liquid ointment
lin'e a ment—a feature
lose—to suffer loss
loose—to untie
pas'tor—a minister
pas'ture—a field for cattle
pres'ence—nearness
pres'ents—gifts
pop'lar—kind of tree
pop'u lar—agreeable
prec'e dent—an example
pre ced'ence—superiority
pres'i dent—chief magistrate
par ti'tion—division

pe ti'tion—a request
rel'ic—a memorial
rel'ict—a widow
stat'ue—an image
stat'ure—height
stat'ute—a law
sur'plus—the remainder
sur'plice—clergyman's robe
se'ries—a succession
se'ri ous—solemn
spe'cies—a kind
spe'cious—plausible
track—a footstep
tract—a region [lands
ten'ure—manner of holding
ten'or—part in music
ve rac'i ty—truthfulness
vo rac'i ty—greediness

87

in″ex haust'i ble
for bear'ance
clem'en cy
re sus'ci tate
in fal'li ble
pre ca'ri ous
ne ces'si tate
un whole'some
con″sci en'tious
at tend'ant

im mu'ni ty
tre men'dous
awk'ward
in'tri cate
ex tor'tion
e nun'ci ate
in ten'tion al ly
in″ex cu'sa ble
em bar'rass

im pu'ta ble
ab'di cate
ob'sti na cy
in″ter rupt'
de ceiv'ing
dom″i neer'
ex cess'es
wrought
ret'i cent

88

ac cel'er ate	round'a bout"	search'ing
spon ta'ne ous	en croach'	sa gac'i ty
ve loc'i ty	in creas'ing	de scend'
for'ci ble	fore run'ner	sim"ul ta'ne ous
mov'a ble	shrewd'ness	un tir'ing ly
re vers'i ble	con tin'u ance	strat'e gy
fol'low er	mile'age	

89

taught	qui e'tus	sought	brought
men ag'e rie	caught	rev'er ent	hap'pened
val'leys	un couth'	stam'i na	si'ren
vict'uals	throt'tle	mar'riage a ble	me'ni al
let'tuce	no'tice a ble	in graft'ed	typ'i fy
re"tro cede'	del'i cate	lu'di crous	be siege'
ap'pe tite	serf'dom	dom'i nant	your self'

90—BUILDINGS

ware'house"	post' of"fice	home'stead	op'e ra-house
fac'tor ies	a sy'lum	saw'mill"	con serv'a to ry
a cad'e my	kin'der gar"ten	ten'e ment	man'sion
col'lege	sem'i na ry	brew'er ies	dwell'ing
a re'na	res'tau rant	li'bra ries	cot'tage

91

re frig'er a"tor	tel'e phone	mag"a zine'	dic'tion a ry
type'wri"ter	dy'na mo	pe"ri od'ic al	awn'ing
um brel'la	jew'el ry	can'o py	en cy"clo pe'di a
par'a chute	port fo'li os	pho'to graph	va lise'

92

moun"tain eer'	sum'moned	de signed'
im par'tial	fea'si ble	mon ot'o nous
in"de fen'si ble	ap"pre hen'sion	op'po site
dis"po si'tion	el'o quent	con cep'tion
ce ler'i ty	ad van'ta ges	hes'i tan cy
e nor'mous	com bi'ning	un"en light'ened
re cip'i ent	sat"is fac'to ry	com pel'ling
im'pu dence	sa lu'ta to ry	dis"ar range'
pen'ni less	su per'flu ous	a vow'al
in duct'ive		

93

pho nog′ra phy
chem′is try
ir″ri ga′tion
mer′can tile
con fec′tion ary
e lec′tric al
pot′ter ies
ma chin′er y
mil′li ner y
man″u fac′tur ing
ho″me o path′ic
ag″ri cul′tur al
ad′ver ti″sing
ar′chi tec″ture
au′to mo″biles
launch
trol′ley
ve′hi cles
bug′gy
steam′ en″gine
con vey′ance
bi′cy cle
am′bu lance
car′riage
mon′i tor
yacht
air-ship
mo′tor-car″

94—LAW

ju di′cial
a dopt′
in′sti tute
cen′sus
ver′i fy
dis cuss′
rec″om mend′
in″de pend′ent
es″ti ma′tion
cor rob″o ra′tion
prac′ti cal ly
sov′er eign ty
fis′cal
in′ju ry
scoun′drel
con vene′
rob′ber
ret′ro grade
hon′es ty
rogu′ish
ca reer′
im per′a tive
ma jor′i ty
emp′ty ing

95—WORDS PERTAINING TO WRITING

pen′man ship
leg′i ble
in tel′li gi ble
flour′ish es
in el′i gi ble
ver′ti cal
in del′i ble
fac′ile
spec″i fi ca′tion
com mu″ni ca′tion
wri′ting
cor″re spond′ence
sta′tion er″y
ac′cu ra cy
trace′a ble
rec′ord
cred′i bly
per fec′tion

96

nui′sance
in her′ent
com mit′ted
dis hon′or
em bar′rass ment
griev′ous
shunned
ma li′cious
pro hib′it ive
mea′ger
pre vent′ive
un rea′son a ble
in″ter vene′
de liv′er ance
ob′so lete
im′pu dence
dis sat′is fied
con ceit′ed
mis′chie vous
bla′ma ble
te na′cious
dis″a vow′al
hyp′o crite
in tox′i ca″ting
in″com pat′i ble
er ro′ne ous
mel′an chol y
dis″al low′
vy′ing
mys′ter y
im′bec ile
ex pel′

97—LAW

mort'gage	in i'ti ate	sub stan'tial
val'ue less	ex plic'it	sat"is fac'to ri ly
ad'ver tise"	un wield'y	ca"pa bil'i ties
share'hold"er	neu'tral ize	e quiv'a lent
fin"an cier'	bought	de sir'a ble
spec'u la"tor	in def'i nite	tech'nic al
cap'i ta list	pe cul'iar	pro fes'sion al
pro pri'et or	dis crep'ancy	im pla'ca ble

98

nec'es sa ries	sus'te nance	bel lig'er ent	fought
or'ches tra	fas'ci nate	per suade'	ap par'ent
pa vil'ion	vi cis'si tude	sou"brette'	e gre'gious
in au'gu rate	trip'li cate	con sen'sus	non'sense
glimpse	phi lip'pic	fu'ri ous ly	co er'cion
ar"ti fi'cial	in"ci den'tal	frac'tious	con"de scend'
re cep'ta cle	dropped	brought	hes'i tate
sam'ple	midg'et	in"de struc'ti ble	ex ist' ence

99

in ef'fa ble	off'set"	e quiv'o cal	prop'er ly
be lieve'	re scind'	pe cu'ni a ry	per ceive'
em'a nate	con cerned'	chief'ly	con'stant
be seech'	rou tine'	ben'e fit ed	man'i fold
lu'cra tive	al lege'	prim'i tive	un cer'tain
re trieve'	pa cif'ic	in an'i mate	nec'es sa ry
al low'ance	a chieve'	fore go'ing	me chan'ics

100

eight'i eth	eight	guer ril'la	il lic'it
sec'ond ly	e lev'enth	un touched'	cres'cent
six'teen"	nine'ty	par"ti al'i ty	a dieu'
thir'ty	coup'let	suc cinct'	eq'ui ta ble
fif'ti eth	un par'al leled	def'er ence	in"dis creet'

101

an″ni ver′sa ry
sub′urb
out′skirts″
dawn′ing
con tri′vance
mech′a nism
su″per se′ded
in ces′sant
au′di ence
sump′tu ous ly
en thu′si asm
pros′per ous
a man″u en′sis
in″flu en′tial
sen sa′tion al ism
grat″i fi ca′tion
de sir′a ble
o rig′i nate
lo cal′i ty
pre′cious
mag′ni tude
com mu′ni ty
eve′ning
ex u′ber ant

102

pur sue′
a lac′ri ty
ac ces′si ble
prec′i pice
thor′ough fare″
re cede′
stren′u ous
prox im′i ty
ir rel′e vant
det′ri ment
in dus′tri ous
ex ten′sion
pur′pose
de sir′ous
ex pect′an cy
jeop′ard y
guid′ance
bou′le vard
fa tigue′
re liev′ing

103—HISTORY

ep′och
ar″is toc′ra cy
rec″on noi′ter
com mand′er
pa tri′cian
os′tra cism
mar′shal
pa trol′ling
pen′sion
stand′ard
lieu ten′ant
yeo′man
he′roes
ser′geant
ter″ri to′ri al
in au′gu ral
a′lien ate
nul′li fy
ret′i nue
ju′bi lee
im pe′ri al ist
sta tis′tics
com′mis sa ry
me″di e′val

104

om niv′o rous
psy chol′o gy
pur su′ance
guid′ance
in el′i gi ble
im mov′a ble
im pan′el
gre ga′ri ous
mam′mal
al might′y
with al′
hon′or a ry
mal′ice
mon′key
coun′ter feit
trans′it
com′pos ite
sulk′y

105

con spir′a cy
tyr′an ny
in sep′a ra ble
du′te ous
de co′rous
tend′en cy
syn on′y mous
mel o′di ous
pre ten′tious
wit′ti cism
pro ceed′ing
e′qual ize
de mon′e tize
cat′e chize
vil′lain
wool′ly
def′i nite
a me′na ble
au′di ble
lab′o ra to″ry
vac′u um
scru′ti nize
mel o′de on
ces′sion

106

in dict'ment
ac"qui esce'
ver nac'u lar
tran'sient
con vey'ing
cri'sis
lil'ies

pri'mer
met'al
aux il'i a ry
Teu ton'ic
deg"ra da'tion
di"a bol'ic al
Phil'ip

Med"i ter ra'ne an
cop'y right"
hei'nous
Hel'en
dil'a to ry
dis crep'an cy
eu pho'ni ous

UNITED STATES GEOGRAPHY

INTRODUCTION

1. The object of this Paper is to prepare the student in the subject of Geography, as laid down by the United States Civil Service Commission at Washington, D. C., and to outline a clear method for the study of this subject.

2. In studying the following work, first memorize the definitions, then spread the map on a flat surface, study the general descriptions of the river systems, mountain systems, and the Great Lakes, as given in the text, and trace each statement on the map, noting the general direction of flow of rivers, the area drained by each system, the states included within the area, and the principal rivers belonging to each system. In tracing the mountain systems, note the general direction in which they extend, the principal ranges, peaks, and the states through which each system extends. In studying the Great Lakes, note carefully their position, comparative size and shape, their connection one with another, their outlet, and the states and country by which they are bounded.

3. In the tabulated synopses of the states and territories, we have given the ten largest cities, in the order of their population, and the principal physical features of each state and territory within the United States, such as the interior and boundary rivers, mountains, etc. In studying this synopses, trace, as before, each statement on the map. Locate each city, noting if it is situated on a river, lake, or ocean; trace each boundary or interior river its entire length, and read the description of each river as given in the text; also locate the mountains, lakes, etc. that are given. In studying

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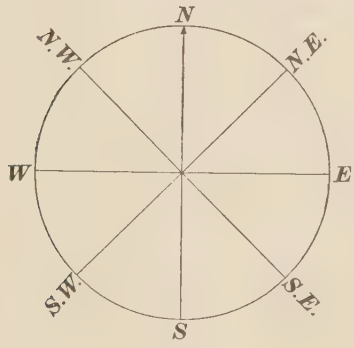
each state, always note its position, comparative size and shape, boundaries, capital, principal cities, and rivers.

4. For the convenience of the student, we group the states in their natural divisions, giving a short description of each group, and a tabulated synopsis of each state in the group; this is followed by Questions for Home Study. When the outline and principal features of the map have been familiarized, these questions should be taken up. If this work is thoroughly studied and each statement traced on the map until it forms a clear picture in the mind, the student is then prepared to answer the Examination Questions without any difficulty.

5. In locating places on the map, it is a help to have before you a drawing, similar to the one here given, in which the directions are marked. In finding direction, always start from the center and follow the line in the direction indicated. If the place is in the east from the center of the drawing, follow the line that leads to the east. If it lies half way between the north and east, the direction is northeast; or if half way between the east and south, it is southeast; and so on.



If you wish to find what state is in the extreme northeastern part of the United States, look at the drawing, find the center, and follow the line that points northeast; then find the center of the United States, and from that point follow the same direction as you did in the drawing and you will find Maine.

DEFINITIONS

6. **Geography** is a study of the earth's surface.

7. A **map** is a drawing or representation of the whole or a part of the earth's surface.

8. The different forms of land are: *continents*, *islands*, *peninsulas*, *capes*, and *isthmuses*.

A **continent** is one of the great bodies of land. There are six continents: North America, South America, Europe, Asia, Africa, and Australia.

An **island** is a smaller body of land entirely surrounded by water.

A **peninsula** is a body of land almost surrounded by water.

A **cape** is a point of land extending out into the water.

An **isthmus** is a narrow neck of land connecting two larger bodies of land.

9. The different forms of water are: *oceans*, *seas*, *bays*, *gulfs*, *straits*, *channels*, *lakes*, and *rivers*.

An **ocean** is the largest body of water.

A **sea** is a large body of water nearly shut in by land.

A **bay** or **gulf** is a body of water extending into the land.

A **strait** is a narrow body of water connecting two larger bodies of water.

A **channel** is a wide strait.

A **lake** is a body of water surrounded by land.

A **river** is a stream of fresh water that flows into the sea or other body of water.

10. The **source** of a river is the place where it rises. Its **mouth** is the place where it flows into some other body of water.

11. A **river system** is a large river having a number of tributary rivers.

12. A **desert** is a tract of land that is nearly or wholly barren.

13. An **archipelago** is a sea of islands.

14. A **mountain** is an elevation of land that rises considerably above the general surface.

15. A **mountain range** is a chain of mountains.

16. A **mountain system** consists of a number of mountain ranges extending in the same general direction.

DESCRIPTION OF THE UNITED STATES

17. The **United States** is situated in the south-central part of the continent of North America. It is bounded by Canada, Atlantic Ocean, Gulf of Mexico, Mexico, and the Pacific Ocean. The surface shows three natural divisions: the Eastern Highlands, the Western Highlands, and the Great Central Plain. The Eastern Highlands comprise the Appalachian Mountain system, the Western Highlands comprise the Rocky and Sierra Nevada systems, and the Great Central Plain is the region included between these highlands. It consists of a comparatively level surface called *prairies*. This territory is also called the Mississippi Basin, and is drained by the Mississippi River system.

The two mountain systems that constitute the Western Highlands nearly enclose a plateau, which is called the *Great Basin*. This Great Basin is drained by the Sacramento, Columbia, and the Colorado River systems.

The region east of the Eastern Highlands is drained by smaller river systems that flow into the Atlantic Ocean.

The United States consists of forty-six states, four territories, one district, and a number of detached possessions.

The territories are: Arizona, New Mexico, Alaska, and Hawaii.

The District of Columbia contains Washington, the capital of the United States.

The most important of the detached possessions are: the Philippines, Porto Rico, and Guam.

The largest state in the United States is Texas; the smallest is Rhode Island.

Kansas is nearly in the center of the United States.

Michigan consists of two peninsulas designated as the upper peninsula and the lower peninsula.

Key West, the most southerly city in the United States, is situated on the Florida Keys, which consist of a chain of islands south of Florida.

RIVER SYSTEMS OF THE UNITED STATES

18. The three principal river systems of the United States are: *Mississippi*, *Colorado*, and *Columbia*.

The **Mississippi River system**, which drains the largest area of any in the United States, consists of several secondary river systems that are tributary to the Mississippi River. The most important of them are: the Missouri, Ohio, Arkansas, and Red Rivers. The Mississippi, the principal river of the system, rises in the northern part of Minnesota, and flows south into the Gulf of Mexico. This river, with its tributaries, drains the Great Central Plain that lies between the Eastern and Western Highlands.

The **Missouri River system** drains the northern part of the United States that lies between the Mississippi River and the Rocky Mountains. The Missouri River, the principal river of the system, rises in the southwestern part of Montana, and flows generally east and southeast into the Mississippi River. Its principal tributaries are: Yellowstone, Dakota, Big Cheyenne, Platte, and Kansas Rivers.

The **Ohio River system** drains that part of the United States that lies south of the Great Lakes and between the Mississippi River and the Appalachian Mountain system. The Ohio, the principal river, is formed by the junction of the Monongahela and Alleghany Rivers in the western part of Pennsylvania, and flows southwest into the Mississippi River. Its principal tributaries are: Muskingum, Kanawha, Big Sandy, Wabash, Tennessee, and Cumberland Rivers.

The **Arkansas** and **Red River systems** drain a large region west of the Mississippi River and flow in a southeasterly direction. The Arkansas River rises in the central part of Colorado, in the Rocky Mountains, and flows southeast into the Mississippi River. The Canadian River is one of its tributaries. The Red River rises in the northern part of Texas and flows southeast into the Mississippi River.

The **Colorado River system** drains the southwestern part of the United States. The Colorado River is formed by the junction of the Green and Grand Rivers in the eastern

part of Utah, and flows southwest into the Gulf of California. Its principal tributaries are: the Gila and San Juan Rivers. The Colorado River flows through a high sandstone plateau through which it has cut a deep narrow channel with almost vertical sides; this channel is known as the *Grand Cañon.*

The **Columbia River system** drains the northwestern part of the United States. The Columbia River rises in Canada, and flows in a generally southwestern direction into the Pacific Ocean. The Snake and Willamette Rivers are two of its tributaries.

MOUNTAIN SYSTEMS OF THE UNITED STATES

19. The three great mountain systems are the *Sierra Nevada* and *Rocky Mountain systems* in the western part, and the *Appalachian Mountain system* in the eastern part.

The **Sierra Nevada system** extends near the coast through Washington, Oregon, and California. It includes the Sierra Nevada and Cascade Mountains.

A low coast range extends between the Sierra Nevada system and the Pacific Coast.

The **Rocky Mountain system** extends nearly across the United States from northwest to southeast at an average distance of 800 miles from the Pacific Ocean. Some of the mountain ranges included in the Rocky Mountain system are: Bitter Root, Wind River, and Big Horn Mountains.

Some of the peaks in the United States are: Long's, Pike's, and the Spanish Peaks, which are in Colorado.

The Bitter Root Mountains form the boundary between Idaho and Montana.

The Wind River Mountains are in the western part of Wyoming.

The Big Horn Mountains extend through Wyoming and the southern part of Montana.

The **Appalachian Mountain system** extends southwest from the Gulf of St. Lawrence nearly to the Gulf of Mexico. It consists of several parallel ranges, of which the Alleghany Mountains are the highest. Some of the ranges in the Appalachian Mountain system are: Alleghany, Blue,

Cumberland, Great Smoky, Adirondack, Catskill, Green, and the White Mountains.

Some of the peaks are: Mt. Washington in the White Mountains, Mt. Mansfield in the Green Mountains, and Mt. Marcy in the Adirondack Mountains.

The Alleghany Mountains extend from northeast to southwest through Pennsylvania, Maryland, West Virginia, and Virginia, and form the boundary between Tennessee and North Carolina.

The Blue Mountains extend through Pennsylvania, Maryland, Virginia, North Carolina, and Georgia.

The Cumberland Mountains extend through Kentucky and Tennessee.

The Great Smoky Mountains are a range of the Alleghany Mountains, and form part of the boundary between Tennessee and North Carolina.

The Adirondack Mountains form a group in the northeastern part of New York.

The Catskill Mountains form a group in the southeastern part of New York.

The Green Mountains extend from northeast to southwest, separating Canada from Maine and New Hampshire, and extending through Vermont.

The White Mountains form a group in the northern part of New Hampshire.

THE GREAT LAKES

20. The **Great Lakes** are in the northeastern part of North America, between the United States and Canada. They are five in number: Lake Superior, Lake Michigan, Lake Huron, Lake Erie, and Lake Ontario. Their outlet is the St. Lawrence River, which flows from Lake Ontario in a northeasterly direction into the Gulf of St. Lawrence.

Lake Superior is bounded by Canada, Michigan, Wisconsin, and Minnesota. It is connected with Lake Huron by St. Mary's River.

Lake Michigan is wholly within the United States and is bounded by Michigan, Indiana, Illinois, and Wisconsin. It is

connected with Lake Huron by the Straits of Mackinac. Green Bay, an arm of Lake Michigan, is in the northwestern part of the lake.

Lake Huron is bounded by Canada and Michigan. It is connected with Lake Erie by St. Clair River, Lake St. Clair, and Detroit River. Two important arms of Lake Huron are Georgian Bay, in the northeastern part, and Saginaw Bay in the southwestern part of the lake.

Lake Erie is bounded by Canada, New York, Pennsylvania, Ohio, and Michigan. It is connected with Lake Ontario by Niagara River.

Lake Ontario is bounded by Canada and New York. Its outlet is the St. Lawrence River.

DIVISIONS OF THE UNITED STATES

NEW ENGLAND STATES

21. The **New England States** are situated in the extreme northeastern part of the United States. They comprise *Maine*, *New Hampshire*, *Vermont*, *Massachusetts*, *Connecticut*, and *Rhode Island*. On account of their situation, abundant waterpower, and good harbors, they occupy an important position in regard to commerce and manufactures.

QUESTIONS FOR HOME STUDY

Where are the New England States?

Which of these states has the most coast?

Which has no coast?

What river flows into Penobscot Bay?

What is the capital of each of these states?

Which state has two capitals?

What mountains are in: (*a*) Vermont? (*b*) New Hampshire?

What states are separated by the Connecticut River?

Where are the White Mountains?

What is the largest city of: (*a*) Massachusetts? (*b*) Maine? (*c*) Vermont? Locate each city.

Where is: (*a*) Cape Cod? (*b*) Cape Ann?

MAINE

BOUNDARY: Canada, Atlantic Ocean, New Hampshire
CAPITAL: Augusta

Largest Cities		Interior Rivers	Boundary Rivers	Lakes	Bays
Portland,	Augusta,	Kennebec	St. John	Moosehead	Penobscot
Lewiston,	Bath,	Penobscot	St. Croix	Grand	Casco
Bangor,	Waterville,	Androscoggin	Salmon Falls		
Biddeford,	Rockland,		St. Francis		
Auburn,	Calais.				

NEW HAMPSHIRE

BOUNDARY: Canada, Maine, Atlantic Ocean, Massachusetts, Vermont
CAPITAL: Concord

Largest Cities		Interior Rivers	Boundary Rivers	Lakes	Mountains
Manchester,	Keene,	Merrimac	Connecticut	Winnipiseogee	White
Nashua,	Berlin,	Androscoggin	Salmon Falls		
Concord,	Rochester,				
Dover,	Laconia,				
Portsmouth,	Somersworth.				

VERMONT

BOUNDARY: Canada, New Hampshire, Massachusetts, New York
CAPITAL: Montpelier

Largest Cities		Interior Rivers	Boundary Rivers	Lakes	Mountains
Burlington,	Rockingham,	Otter Creek	Connecticut	Champlain	Green
Rutland,	St. Johnsbury,	Winooski			
Barre,	Bennington,				
Montpelier,	Colchester,				
St. Albans,	Brattleboro.				

MASSACHUSETTS

BOUNDARY: Vermont, New Hampshire, Atlantic Ocean, Connecticut, Rhode Island, New York
CAPITAL: Boston

Largest Cities		Interior Rivers	Islands	Bays	Capes
Boston,	Lynn,	Connecticut	Nantucket	Cape Cod	Cape Cod
Worcester,	Lawrence,		Martha's Vine-	Buzzards	Cape Ann
Fall River,	New Bedford,		yard	Boston	
Lowell,	Springfield,				
Cambridge,	Somerville.				

CONNECTICUT

BOUNDARY: Massachusetts, Rhode Island, Long Island Sound, New York
CAPITAL: Hartford

Largest Cities		Interior Rivers
New Haven,	Meriden,	Connecticut
Hartford,	New London,	Willimantic
Bridgeport,	Norwich,	Naugatuck
Waterbury,	Danbury,	
New Britain,	Stamford.	

RHODE ISLAND

BOUNDARY: Massachusetts, Atlantic Ocean, Connecticut
CAPITAL: Providence

Largest Cities		Interior Rivers	Islands	Bays	Sounds
Providence,	Central Falls,	Wood	Block	Narragansett	Block Island
Pawtucket,	East Providence,				
Woonsocket,	Cranston,				
Newport,	Westerly,				
Warwick,	Bristol.				

MIDDLE ATLANTIC STATES

22. This group of states borders on the Atlantic Ocean and lies between the New England States and the South Atlantic States. It includes *New York*, *Pennsylvania*, *New Jersey*, *Delaware*, *Maryland*, *Virginia*, *West Virginia*, and *District of Columbia*.

The ranges of the Appalachian Mountain system cross this section from northeast to southwest. The principal ranges are the Alleghany, Blue, and Cumberland Mountains.

The surface slopes in three general directions: the northern part toward the Great Lakes, the western part toward the Ohio River, and the eastern part toward the Atlantic Ocean.

QUESTIONS FOR HOME STUDY

What is the capital and largest city of each state?

What river forms the eastern boundary of Pennsylvania?

What large river flows through the eastern part of Pennsylvania? Where does it rise?

What two rivers unite to form the Ohio River? Where do they rise?

What lakes are on the northern boundary of New York?

Where are the: (*a*) Adirondack Mountains? (*b*) Catskill Mountains?

What is the largest river in New York? Where does it rise?

What large river is in Virginia? In what direction does it flow?

What and where is Washington?

What states bound: (*a*) Pennsylvania? (*b*) New York?

What two states are on the south bank of the Potomac River?

What two rivers form the western boundary of West Virginia?

Where is: (*a*) Chesapeake Bay? (*b*) Delaware Bay? (*c*) New York Bay?

Which is the smallest state in this section?

On what river is the capital of New York state situated? In what mountains does this river rise and into what bay does it flow?

NEW YORK

BOUNDARY: Canada, Vermont, Massachusetts, Connecticut, Atlantic Ocean, New Jersey, Pennsylvania, Lake Erie, Lake Ontario

CAPITAL: Albany

Largest Cities	Interior Rivers	Boundary Rivers	Lakes	Sounds	Mountains
New York, Troy, Buffalo, Utica, Rochester, Yonkers, Syracuse, Binghamton, Albany, Elmira.	Hudson Mohawk	St. Lawrence Niagara	Oneida George Cayuga Seneca Champlain	Long Island	Adirondack Catskill

PENNSYLVANIA

BOUNDARY: Lake Erie, New York, New Jersey, Delaware, Maryland, West Virginia, Ohio
CAPITAL: Harrisburg

Largest Cities		Interior Rivers	Boundary Rivers	Mountains
Philadelphia, Pittsburg, Scranton, Reading, Erie,	Wilkes-Barre, Harrisburg, Lancaster, Altoona, Johnstown.	Susquehanna Alleghany Monongahela Ohio	Delaware	Alleghany Blue

NEW JERSEY

BOUNDARY: New York, Atlantic Ocean, Delaware Bay, Pennsylvania, Delaware
CAPITAL: Trenton

Largest Cities		Interior Rivers	Boundary Rivers	Bays
Newark, Jersey City, Paterson, Camden, Trenton,	Hoboken, Elizabeth, Bayonne, Atlantic City, Passaic.	Raritan	Delaware	New York Delaware

DELAWARE

BOUNDARY: Pennsylvania, New Jersey, Delaware Bay, Atlantic Ocean, Maryland
CAPITAL: Dover

Largest Cities		Bays	
Wilmington,	Smyrna,	Delaware	
New Castle,	Laurel,		
Dover,	Seaford,		
Milford,	Georgetown,		
Lewes,	Middletown.		

MARYLAND

BOUNDARY: Pennsylvania, Delaware, Atlantic Ocean, Virginia, West Virginia
CAPITAL: Annapolis

Largest Cities		Interior Rivers	Boundary Rivers	Bays
Baltimore,	Cambridge,	Susquehanna	Potomac	Chesapeake
Cumberland,	Frostburg,			
Hagerstown,	Salisbury,			
Frederick,	Havre de Grace,			
Annapolis,	Westminster.			

VIRGINIA

BOUNDARY: Maryland, Atlantic Ocean, North Carolina, Tennessee, Kentucky, West Virginia
CAPITAL: Richmond

Largest Cities		Interior Rivers	Boundary Rivers	Mountains
Richmond,	Lynchburg,	Rappahannock	Potomac	Alleghany
Norfolk,	Portsmouth,	James		Blue Ridge
Petersburg,	Danville,			
Roanoke,	Alexandria,			
Newport News,	Manchester.			

WEST VIRGINIA

BOUNDARY: Pennsylvania, Maryland, Virginia, Kentucky, Ohio
CAPITAL: Charleston

Largest Cities		Interior Rivers	Boundary Rivers	Mountains
Wheeling,	Fairmont,	Great Kanawha	Ohio	Alleghany
Huntington,	Grafton,			
Parkersburg,	Moundsville,			
Charleston,	Bluefield,			
Martinsburg,	Benwood.			

DISTRICT OF COLUMBIA

BOUNDARY: Maryland, Virginia

Contains Washington, the capital of the United States.

SOUTH ATLANTIC AND GULF STATES

23. This group of states and territories is in the southern part of the United States. It consists of *Tennessee*, *North Carolina*, *South Carolina*, *Georgia*, *Florida*, *Alabama*, *Mississippi*, *Louisiana*, *Texas*, *Arkansas*, and *Oklahoma*.

The Appalachian Mountain system extends through Tennessee and North Carolina into Georgia. The surface slopes in two general directions, the eastern part, consisting of North Carolina, South Carolina, and a part of Georgia and Florida, slopes toward the Atlantic Ocean; all the remaining surface slopes toward the Gulf of Mexico.

QUESTIONS FOR HOME STUDY

What large river is on the southwestern boundary of this section? What gulf is on the south? What ocean is on the east?

What states bound Tennessee? What mountains are in the eastern part of this state?

Locate the following cities: Nashville, Chattanooga, Raleigh, Jackson, Montgomery, Charleston, Columbia, Atlanta, Tallahassee.

What large lake is in Florida?

What two rivers flow into Mobile Bay?

Where is the Savannah River? Into what does it flow?

By what large bodies of water is Florida bounded? By what states?

What city is at the mouth of the Cape Fear River?

What states bound Texas? What rivers?

Name three large rivers in Texas.

TENNESSEE

BOUNDARY: KENTUCKY, VIRGINIA, NORTH CAROLINA, GEORGIA, ALABAMA, MISSISSIPPI, ARKANSAS, MISSOURI
CAPITAL: NASHVILLE

Largest Cities		Interior Rivers	Boundary Rivers	Mountains
Memphis, Nashville, Knoxville, Chattanooga, Jackson,	Clarksville, Columbia, Bristol, Johnson City, Murfreesboro.	Tennessee Cumberland	Mississippi Tennessee	Cumberland Great Smoky

NORTH CAROLINA

BOUNDARY: Virginia, Atlantic Ocean, South Carolina, Georgia, Tennessee
CAPITAL: Raleigh

Largest Cities		Interior Rivers	Capes	Sounds	Mountains
Wilmington,	Greensboro,	Cape Fear	Hatteras	Albemarle	Blue Ridge
Charlotte,	Newbern,	Roanoke	Lookout	Pamlico	
Asheville,	Concord,		Fear		
Winston-Salem,	Durham,				
Raleigh,	Elizabeth City.				

SOUTH CAROLINA

BOUNDARY: North Carolina, Atlantic Ocean, Georgia
CAPITAL: Columbia

Largest Cities		Interior Rivers	Boundary Rivers	Mountains
Charleston,	Anderson,	Santee	Savannah	Blue Ridge
Columbia,	Rockhill,			
Greenville,	Union,			
Spartanburg,	Greenwood,			
Sumter,	Florence.			

GEORGIA

BOUNDARY: Tennessee, North Carolina, South Carolina, Atlantic Ocean, Florida, Alabama
CAPITAL: Atlanta

Largest Cities	Interior Rivers	Boundary Rivers	Swamp	Mountains
Atlanta, Athens, Savannah, Brunswick, Augusta, Americus, Macon, Rome, Columbus, Griffin.	Chattahoochee	Chattahoochee Savannah	Okefinokee	Blue Ridge

FLORIDA

BOUNDARY: Georgia, Atlantic Ocean, Gulf of Mexico, Alabama
CAPITAL: Tallahassee

Largest Cities	Interior Rivers	Boundary Rivers	Lakes	Capes
Jacksonville, Lake City, Pensacola, Gainesville, Key West, Ocala, Tampa, Palatka, St. Augustine, Fernandina.	Suwannee	St. Mary's Chattahoochee	Okeechobee	Sable

ALABAMA

BOUNDARY: TENNESSEE, GEORGIA, FLORIDA, GULF OF MEXICO, MISSISSIPPI
CAPITAL: MONTGOMERY

Largest Cities		Interior Rivers	Boundary Rivers	Bays
Mobile,	Huntsville,	Alabama	Chattahoochee	Mobile
Birmingham,	Florence,	Tombigbee		
Montgomery,	Bessemer,	Mobile		
Anniston,	Tuscaloosa,	Apalachee		
Selma,	Talladega.			

MISSISSIPPI

BOUNDARY: TENNESSEE, ALABAMA, GULF OF MEXICO, LOUISIANA, ARKANSAS
CAPITAL: JACKSON

Largest Cities		Interior Rivers	Boundary Rivers
Vicksburg,	Columbus,	Pearl	Mississippi
Meridian,	Biloxi,	Yazoo	Pearl
Natchez,	Yazoo City,		Tennessee
Jackson,	McComb,		
Greenville,	Hattiesburg.		

LOUISIANA

BOUNDARY: Arkansas, Mississippi, Gulf of Mexico, Texas
CAPITAL: Baton Rouge

Largest Cities		Interior Rivers	Boundary Rivers	Lakes	Bays
New Orleans, Shreveport, Baton Rouge, New Iberia, Lake Charles,	Alexandria, Monroe, Crowley, Donaldsonville, Plaquemine.	Mississippi Red	Mississippi Sabine Pearl	Pontchartrain	Atchafalaya Barataria

TEXAS

BOUNDARY: Oklahoma, Arkansas, Louisiana, Gulf of Mexico, Mexico, New Mexico
CAPITAL: Austin

Largest Cities		Interior Rivers	Boundary Rivers	Bays
San Antonio Houston, Dallas, Galveston, Ft. Worth,	Austin, Waco, El Paso, Laredo, Denison.	Brazos Colorado Pecos Nueces	Rio Grande Red Sabine	Galveston Corpus Christi Matagorda

ARKANSAS

BOUNDARY: Missouri, Tennessee, Mississippi, Louisiana, Texas, Oklahoma

CAPITAL: Little Rock

Largest Cities	Interior Rivers	Boundary Rivers
Little Rock, Texarkana, Fort Smith, Jonesboro, Pine Bluff, Fayetteville, Hot Springs, Eureka Springs, Helena, Mena.	Arkansas	Mississippi Red

OKLAHOMA

BOUNDARY: Colorado, Kansas, Missouri, Arkansas, Texas, New Mexico

CAPITAL: Guthrie

Largest Cities	Interior Rivers	Boundary Rivers
Oklahoma City, Guthrie, Shawnee, Enid, Tulsa, Lawton, Ardmore, Elreno, South McAlester, Hobart.	Arkansas Canadian	Red

Name the capitals of each of the following states and tell on what waters they are located: Texas, Louisiana, Arkansas, Tennessee.

What large lake is in Louisiana?

What two bays are east of Texas?

CENTRAL STATES

24. The states of this group extend from the Eastern almost to the Western Highlands. On this account they are called the **Central States.** They include *Michigan*, *Wisconsin*, *Ohio*, *Indiana*, *Illinois*, *Kentucky*, *Minnesota*, *Missouri*, *Iowa*, *Kansas*, *Nebraska*, *South Dakota*, and *North Dakota.*

MICHIGAN

BOUNDARY: Lake Superior, Lake Huron, Canada, Lake St. Clair, Lake Erie, Ohio, Indiana, Lake Michigan, Wisconsin

CAPITAL: Lansing

Largest Cities		Interior Rivers	Bays
Detroit,	Jackson,	Grand	Saginaw
Grand Rapids,	Battle Creek,		
Saginaw,	Muskegon,		
Bay City,	Lansing,		
Kalamazoo,	Port Huron.		

WISCONSIN

BOUNDARY: Lake Superior, Michigan, Illinois, Iowa, Minnesota

CAPITAL: Madison

Largest Cities		Interior Rivers	Boundary Rivers	Bays
Milwaukee,	Madison,	Wisconsin	Mississippi	Green
Superior,	Sheboygan,	Chippewa		
Racine,	Green Bay,			
Oshkosh,	Eau Claire,			
La Crosse,	Fond du Lac.			

OHIO

BOUNDARY: Michigan, Lake Erie, Pennsylvania, West Virginia, Kentucky, Indiana

CAPITAL: Columbus

Largest Cities		Interior Rivers	Boundary Rivers
Cleveland,	Youngstown,	Muskingum	Ohio
Cincinnati,	Akron,	Scioto	
Toledo,	Springfield,		
Columbus,	Canton,		
Dayton,	Hamilton.		

INDIANA

BOUNDARY: Lake Michigan, Michigan, Ohio, Kentucky, Illinois

CAPITAL: Indianapolis

Largest Cities		Interior Rivers	Boundary Rivers
Indianapolis,	Muncie,	Wabash	Ohio
Evansville,	New Albany,		Wabash
Ft. Wayne,	Anderson,		
Terre Haute,	Richmond,		
South Bend,	Lafayette.		

ILLINOIS

BOUNDARY: Wisconsin, Lake Michigan, Indiana, Kentucky, Missouri, Iowa

CAPITAL: Springfield

Largest Cities		Interior Rivers	Boundary Rivers
Chicago,	East St. Louis,	Illinois	Mississippi
Peoria,	Joilet,	Kaskaskia	Ohio
Quincy,	Aurora,		Wabash
Springfield,	Bloomington,		
Rockford,	Elgin.		

KENTUCKY

BOUNDARY: Illinois, Indiana, Ohio, West Virginia, Virginia, Tennessee, Missouri
CAPITAL: Frankfort

Largest Cities		Interior Rivers	Boundary Rivers	Mountains
Louisville,	Owensboro,	Green	Mississippi	Cumberland
Covington,	Henderson,		Ohio	
Newport,	Frankfort,		Big Sandy	
Lexington,	Bowling Green,			
Paducah,	Hopkinsville.			

MINNESOTA

BOUNDARY: Canada, Lake Superior, Wisconsin, Iowa, South Dakota, North Dakota
CAPITAL: St. Paul

Largest Cities		Interior Rivers	Boundary Rivers	Lakes
Minneapolis,	Mankato,	Mississippi	Mississippi	Red
St. Paul,	St. Cloud,	Minnesota	Red River of	Itasca
Duluth,	Faribault,		the North	Mille Lacs
Winona,	Red Wing,			
Stillwater,	Brainerd.			

MISSOURI

BOUNDARY: Iowa, Illinois, Kentucky, Tennessee, Arkansas, Oklahoma, Kansas, Nebraska

CAPITAL: Jefferson City

Largest Cities	Interior Rivers	Boundary Rivers	Mountains
St. Louis, Sedalia, Kansas City, Hannibal, St. Joseph, Jefferson City, Joplin, Carthage, Springfield, Webb City.	Missouri	Mississippi Missouri	Ozark

IOWA

BOUNDARY: Minnesota, Wisconsin, Illinois, Missouri, Nebraska, South Dakota

CAPITAL: Des Moines

Largest Cities		Interior Rivers	Boundary Rivers
Des Moines,	Burlington,	Des Moines	Mississippi
Dubuque,	Council Bluffs,		Missouri
Sioux City,	Clinton,		Des Moines
Davenport,	Ottumwa,		
Cedar Rapids,	Waterloo.		

KANSAS

BOUNDARY: Nebraska, Missouri, Oklahoma, Colorado

CAPITAL: Topeka

Largest Cities		Interior Rivers	Boundary Rivers
Kansas City,	Lawrence,	Arkansas	Missouri
Topeka,	Fort Scott,	Kansas	
Wichita,	Galena,		
Leavenworth,	Pittsburg,		
Atchison,	Hutchinson.		

NEBRASKA

BOUNDARY: South Dakota, Iowa, Missouri, Kansas, Colorado, Wyoming

CAPITAL: Lincoln

Largest Cities		Interior Rivers	Boundary Rivers
Omaha,	Nebraska City,	Platte	Missouri
Lincoln,	Fremont,		
South Omaha,	Hastings,		
Beatrice,	Kearney,		
Grand Island,	York.		

SOUTH DAKOTA

BOUNDARY: North Dakota, Minnesota, Iowa, Nebraska, Wyoming, Montana
CAPITAL: Pierre

Largest Cities		Interior Rivers	Boundary Rivers	Mountains
Sioux Falls,	Deadwood.	Missouri	Missouri	Black Hills
Lead,	Yankton,	Dakota		
Aberdeen,	Huron,	Big Cheyenne		
Mitchell,	Brookings,			
Watertown	Madison.			

NORTH DAKOTA

BOUNDARY: Canada, Minnesota, South Dakota, Montana
CAPITAL: Bismarck

Largest Cities		Interior Rivers	Boundary Rivers	Lakes
Fargo,	Grafton,	Missouri	Red River of the North	Devil's Lake
Grand Forks,	Wahpeton,	Dakota		
Bismarck,	Dickinson,			
Jamestown,	Devil's Lake,			
Valley City,	Mandan.			

QUESTIONS FOR HOME STUDY

What great lakes form part of the northern boundary of this section? Which lake is the largest?

What large lake is entirely within this section?

Name and locate the capital of each state.

Name three lakes in Minnesota. What large river rises in Minnesota?

What three capital cities are located on the Missouri River?

Which state forms two peninsulas and is nearly surrounded by water? What bay indents the eastern coast of this state?

Bound Indiana. What river is between Indiana and Kentucky?

What important commercial city is in Illinois? On what lake is it situated?

Name two rivers in each of the following states: Illinois, Wisconsin, Minnesota, North Dakota, South Dakota.

What river forms the eastern boundary of Nebraska?

What states are separated from Kentucky by the Ohio River? Describe the Ohio River.

What river lies between Indiana and Illinois?

What mountains are in the southwestern part of South Dakota?

Locate Minneapolis, Dubuque, Sioux City, St. Louis, Lacrosse, Racine, Chicago, Rock Island, Evansville, Lexington, Louisville, Cleveland, Cincinnati, Deadwood, Fargo, Atchison.

ROCKY MOUNTAIN AND PACIFIC STATES AND TERRITORIES

25. This group covers the entire Western Highlands of the United States. The Rocky Mountain and the Sierra Nevada systems extend through this section and nearly enclose a plateau, called the Great Basin. The states and territories included in this section are: *Washington*, *Oregon*, *California*, *Idaho*, *Wyoming*, *Montana*, *Utah*, *Arizona*, *Nevada*, *Colorado*, and *New Mexico*.

WASHINGTON

BOUNDARY: Canada, Idaho, Oregon, Pacific Ocean
CAPITAL: Olympia

Largest Cities		Interior Rivers	Boundary Rivers	Mountains	Sounds
Seattle,	Everett,	Columbia	Columbia	Cascade Range	Puget
Tacoma,	Ballard,				
Spokane,	Olympia,				
Bellingham,	Aberdeen,				
WallaWalla,	Port Townsend.				

OREGON

BOUNDARY: Washington, Idaho, Nevada, California, Pacific Ocean
CAPITAL: Salem

Largest Cities		Interior Rivers	Boundary Rivers	Mountains	Lakes
Portland,	The Dalles,	Willamette	Columbia	Cascade Range	Klamath
Astoria,	Oregon City,		Snake	Coast Range	
Baker City,	Eugene,				
Pendleton,	Albany,				
Salem,	La Grande.				

CALIFORNIA

BOUNDARY: Oregon, Nevada, Arizona, Mexico, Pacific Ocean
CAPITAL: Sacramento

Largest Cities		Interior Rivers	Boundary Rivers	Mountains	Valleys
San Francisco,	San Diego,	Sacramento	Colorado	Sierra Nevada	Dead
Los Angeles,	Stockton,	San Joaquin		Coast Range	Yosemite
Oakland,	Alameda,				
Sacramento,	Berkeley,				
San Jose,	Fresno.				

IDAHO

BOUNDARY: Canada, Montana, Wyoming, Utah, Nevada, Oregon, Washington
CAPITAL: Boise City

Largest Cities		Interior Rivers	Boundary Rivers	Mountains
Boise City,	Wardner,	Salmon	Snake	Bitter Root
Pocatello,	Montpelier,	Snake		
Lewiston,	Weiser,			
Moscow,	Idaho Falls,			
Wallace,	Hailey.			

WYOMING

BOUNDARY: Montana, South Dakota, Nebraska, Colorado, Utah, Idaho
CAPITAL: Cheyenne

Largest Cities		Interior Rivers	Mountains	Parks
Cheyenne, Laramie, Rock Springs, Rawlins, Evanston,	Sheridan, Green River, Diamondville, Casper, Kemmerer.	North Platte Big Horn	Rocky Black Hills	Yellowstone, or National, Park

MONTANA

BOUNDARY: Canada, North Dakota, South Dakota, Wyoming, Idaho
CAPITAL: Helena

Largest Cities		Interior Rivers	Mountains
Butte,	Bozeman,	Missouri	Rocky
Great Falls,	Billings,	Yellowstone	
Helena,	Livingston,		
Anaconda,	Walkerville,		
Missoula,	Kalispell.		

UTAH

BOUNDARY: Idaho, Wyoming, Colorado, Arizona, Nevada
CAPITAL: Salt Lake City

Largest Cities		Interior Rivers	Lakes
Salt Lake City,	Springville,	Colorado	Great Salt Lake
Ogden,	Eureka,	Green	
Provo City,	Brigham,	Grand	
Logan,	Spanish Fork,		
Park City,	American Fork.		

ARIZONA

BOUNDARY: Utah, New Mexico, Mexico, California, Nevada
CAPITAL: Phœnix

Largest Cities		Interior Rivers	Boundary Rivers
Tucson,	Jerome,	Colorado	Colorado
Bisbee,	Nogales,	Gila	
Phœnix,	Yuma,		
Prescott,	Florence,		
San Carlos,	Globe.		

NEVADA

BOUNDARY: Oregon, Idaho, Utah, Arizona, California
CAPITAL: Carson City

Largest Cities		Interior Rivers	Boundary Rivers	Lakes
Reno,	Winnemucca,	Humboldt	Colorado	Walkers
Virginia City,	Delamar,			Humboldt
Carson City,	Eureka,			
Lovelock,	Austin,			
Wadsworth,	Elko.			

COLORADO

BOUNDARY: Wyoming, Nebraska, Kansas, Oklahoma, New Mexico, Utah
CAPITAL: Denver

Largest Cities		Interior Rivers	Mountains	Mountain Peaks
Denver,	Boulder,	Arkansas	Rocky	Long's
Pueblo,	Trinidad,	Grand		Pike's
Colorado Springs,	Victor,			Spanish
Leadville,	Canon City,			
Cripple Creek,	Florence.			

NEW MEXICO

BOUNDARY: Colorado, Oklahoma, Texas, Mexico, Arizona

CAPITAL: Santa Fe

Largest Cities	Interior Rivers	Mountains
Albuquerque, East Las Vegas, Santa Fe, Silver City, Las Vegas, Roswell, Raton, Socorro, Gallup, Mora.	Rio Grande Rio Pecos	Rocky

QUESTIONS FOR HOME STUDY

What ocean forms the western boundary of this section? What two mountain systems extend through it? What are the principal rivers? What country is north of it?

Name and locate the capital of each state and territory in this section.

Bound Washington. What sound is in the northwestern part of Washington? Name and locate the largest city in Washington?

What river partially separates Oregon from Washington?

What mountains form the boundary between Montana and Idaho?

Name two rivers in each of the following states: California, Arizona, New Mexico, Montana, Wyoming, and Utah.

Name three mountain peaks in Colorado, two ranges in Wyoming, and two ranges in California.

Where is Yellowstone, or the National, Park?

What large lake is in Utah? Name three cities near this lake.

Where is the Grand Cañon of the Colorado River?

Locate Yosemite Valley and Dead Valley.

Where is San Francisco?

What three cities are situated on or near the southwestern coast of California?

Locate: Wallawalla, Ft. Assinniboine, Provo City, Lewiston, Laramie, Canyon City, Delamar, Virginia City, Oakland, San Jose, San Carlos, Albuquerque.

IMPORTANT RIVERS IN THE UNITED STATES

26. The **Alabama River** is formed by the junction of two smaller rivers in the central part of Alabama. It flows southwest through Alabama into Tombigbee River.

The **Alleghany River** rises in the northern part of Pennsylvania and flows in a generally southwestern direction through New York and Pennsylvania into the Ohio River.

The **Altamaha River** is formed by the junction of two streams in the south-central part of Georgia. It flows southeast through Georgia into the Atlantic Ocean.

The **Androscoggin River** is formed by the junction of two rivers in northeastern New Hampshire and flows in a generally southeastern direction through New Hampshire and Maine into the Kennebec River.

The **Apalachicola River** is formed by the junction of the Chattahoochee and Flint Rivers in northern Florida. It flows south across Florida into the Gulf of Mexico.

The **Arkansas River** rises in Colorado in the Rocky Mountains. It flows in a southeasterly direction through Colorado, Kansas, Oklahoma, and Arkansas into the Mississippi River.

The **Big Cheyenne River** is formed by the junction of the North and South forks in the southwestern part of South Dakota and flows northeast through South Dakota into the Missouri River.

The **Big Horn River** rises in central Wyoming and flows north through Wyoming and Montana into the Yellowstone River.

The **Big Sandy River** is formed by the junction of two streams on the boundary between Kentucky and West Virginia. It flows north between Kentucky and West Virginia into the Ohio River.

The **Brazos River** rises in the northwestern part of Texas and flows southeast through Texas into the Gulf of Mexico.

The **Canadian River** rises in the northern part of New Mexico and flows east through New Mexico, Texas, and Oklahoma into the Arkansas River.

The **Cape Fear River** is formed by the junction of two streams in the central part of North Carolina. It flows southeast through North Carolina into the Atlantic Ocean.

The **Chattahoochee River** rises in the mountains in the northern part of Georgia. It flows southwest through Georgia, then south, separating Georgia from Alabama and Florida, into the Apalachicola River.

The **Chippewa River** rises in northern Wisconsin and flows southwest through Wisconsin into the Mississippi River.

The **Colorado River** rises in northwestern Texas and flows southeast through Texas into Matagorda Bay.

The **Colorado River** is formed by the junction of the Green and Grand Rivers in the eastern part of Utah and flows in a southwesterly direction into the Gulf of California, crossing Utah, Arizona, and Mexico and separating California from Arizona, and partially separating Nevada and Mexico from Arizona.

The **Columbia River** rises in southern Canada in the Rocky Mountains. It flows in a southerly and westerly direction into the Pacific Ocean. It flows through Canada, crosses Washington, and forms part of the boundary between Washington and Oregon.

The **Connecticut River** rises in northern New Hampshire and flows south into Long Island Sound, crossing New Hampshire, Massachusetts, and Connecticut, and forming the boundary between Vermont and New Hampshire.

The **Cumberland River** rises in the Cumberland Mountains in the eastern part of Kentucky and flows west through Kentucky and Tennessee into the Ohio River.

The **Dakota,** or **James, River** rises in the central part

of North Dakota and flows south through North and South Dakota into the Missouri River.

The **Delaware River** rises in the southeastern part of New York and flows south, through New York, separating Pennsylvania from New York and New Jersey, and New Jersey from Delaware, then flowing into Delaware Bay.

The **Des Moines River** rises in the southwestern part of Minnesota and flows southeast into the Mississippi River, crossing Minnesota and Iowa, and forming the boundary between Iowa and Missouri.

The **Detroit River** connects Lake St. Clair and Lake Erie and flows between Michigan and Canada.

The **Gila River** rises in the southwestern part of New Mexico and flows west through New Mexico and Arizona into the Colorado River.

The **Grand River** rises in the south-central part of Michigan and flows northwest through Michigan into Lake Michigan.

The **Grand River** rises in the central part of Colorado and flows southwest through Colorado and Utah into the Colorado River.

The **Great Kanawha River** is formed by two streams in the central part of West Virginia and flows northwest through West Virginia into the Ohio River.

The **Rio Grande** rises in southwestern Colorado, flows in a southeasterly direction, into the Gulf of Mexico, crossing Colorado and New Mexico, and forming the boundary between Texas and Mexico.

The **Green River** rises in central Kentucky and flows in a northwesterly direction through Kentucky into the Ohio River.

The **Green River** rises in the western part of Wyoming and flows south through Wyoming, Colorado, and Utah into the Colorado River.

The **Housatonic River** rises in the northwestern part of Massachusetts and flows south through Massachusetts and Connecticut into Long Island Sound.

The **Hudson River** rises in the Adirondack Mountains in

the northeastern part of New York and flows south through New York and between New Jersey and New York into New York Bay.

The **Humboldt River** rises in the northeastern part of Nevada and flows southwest through Nevada into Humboldt Lake.

The **Illinois River** is formed by the junction of three rivers in the north-central part of Illinois and flows southwest through Illinois into the Mississippi River.

The **James River** is formed by the junction of two streams in the western part of Virginia and flows east through Virginia into Chesapeake Bay.

The **Kansas River** is formed by the junction of two rivers in the eastern part of Kansas, and flows east through Kansas into the Missouri River.

The **Kaskaskia River** rises in the eastern part of Illinois and flows southwest across Illinois into the Mississippi River.

The **Kennebec River** rises in Moosehead Lake in the northwestern part of Maine and flows south through Maine into Sheepscott Bay.

The **Merrimac River** is formed by the junction of two streams in central New Hampshire and flows south into Massachusetts, then northeast through Massachusetts into the Atlantic Ocean.

The **Minnesota River** rises in the western part of Minnesota and flows east through Minnesota into the Mississippi River.

The **Missouri River** is formed by the junction of three small streams in the northwestern part of Montana in the Rocky Mountains and flows generally east and southeast into the Mississippi River. It flows through Montana, North Dakota, South Dakota, and Missouri, and separates Nebraska from South Dakota, Iowa and Missouri, and Kansas from Missouri.

The **Mississippi River** rises in the Mississippi Springs in the vicinity of Lake Itasca in northern Minnesota, and flows in a southerly direction into the Gulf of Mexico. The states of Wisconsin, Illinois, Kentucky, Tennessee,

Mississippi, and parts of Minnesota and Louisiana are on the east bank; Iowa, Missouri, Arkansas, and parts of Minnesota and Louisiana are on the west bank.

The **Mohawk River** rises in central New York and flows east through New York into the Hudson River.

The **Monongahela River** rises in the northern part of West Virginia and flows north through West Virginia and Pennsylvania into the Ohio River.

The **Muskingum River** is formed by the junction of two streams in the eastern part of Ohio and flows south through Ohio into the Ohio River.

The **Naugatuck River** rises in the northwestern part of Connecticut and flows south through Connecticut into the Housatonic River.

The **Niagara River** flows from Lake Erie into Lake Ontario between New York and Canada.

The **North Platte River** rises in the northern part of Colorado, and flows north through Colorado and Wyoming, then east through Wyoming and Nebraska into the Platte River.

The **Nueces River** rises in the southwestern part of Texas and flows east through Texas into Corpus Christi Bay.

The **Ohio River** is formed by the junction of the Alleghany and Monongahela Rivers in the western part of Pennsylvania and flows in a southwesterly direction into the Mississippi River. The states of Ohio, Indiana, Illinois, and part of Pennsylvania are on the north bank; West Virginia, Kentucky, and part of Pennsylvania are on the south bank.

Otter Creek rises in the southwestern part of Vermont and flows north, through Vermont, into Lake Champlain.

The **Pearl River** rises in the central part of Mississippi and flows south through Mississippi, and between Mississippi and Louisiana, into the Gulf of Mexico.

The **Pecos River** rises in the northern part of New Mexico and flows in a southeasterly direction through New Mexico and Texas into the Rio Grande.

The **Penobscot River** rises near the northwestern

boundary of Maine and flows in a generally eastern then southeastern direction into Penobscot Bay.

The **Platte River** is formed by the junction of the North Platte and South Platte Rivers in central Nebraska and flows in an easterly direction through Nebraska into the Missouri River.

The **Potomac River** rises in the northeastern part of West Virginia and flows southeast into Chesapeake Bay, separating Maryland from West Virginia and Virginia.

The **Rappahannock River** rises in the Blue Mountains in Virginia and flows southeast through Virginia into Chesapeake Bay.

The **Raritan River** rises in the northern part of New Jersey and flows southeast through New Jersey into the Atlantic Ocean.

The **Red River** rises in northern Texas and flows east into the Mississippi River, crossing Texas, Arkansas, and Louisiana and separating Oklahoma from Texas, and partially separating Oklahoma and Arkansas from Texas.

The **Red River of the North** rises in Elbow Lake, Minnesota, flows north between Minnesota and North Dakota, and through Canada into Lake Winnipeg.

The **Roanoke River** is formed by the junction of two streams in the southern part of Virginia and flows east through Virginia and North Carolina into Albemarle Sound.

The **Sabine River** rises in northeastern Texas and flows southeast through Texas and between Texas and Louisiana into the Gulf of Mexico.

The **Sacramento River** rises in the northern part of California and flows south through California into San Francisco Bay.

The **Salmon River** rises in the central part of Idaho and flows northwest through Idaho into the Snake River.

The **Salmon Falls River** rises between New Hampshire and Maine and flows south between Maine and New Hampshire into the Piscataqua River.

The **Santee River** is formed by the junction of two

streams in the central part of South Carolina and flows southeast through South Carolina into the Atlantic Ocean.

The **San Joaquin River** rises in the eastern part of California and flows northwest through California into the Pacific Ocean.

The **Savannah River** is formed by the junction of two streams on the boundary between South Carolina and Georgia and flows southeast between South Carolina and Georgia into the Atlantic Ocean.

The **Snake River** rises in northern Wyoming, in the Rocky Mountains, and flows northwest through the states of Wyoming and Idaho, north between Idaho and Oregon, and west through Washington into the Columbia River.

The **St. Clair River** flows from Lake Huron into Lake St. Clair, and forms the boundary between Michigan and Canada.

The **St. Croix River** rises on the boundary between Maine and Canada and flows southeast into Passamaquoddy Bay, forming part of the boundary between Maine and Canada.

The **St. Francis River** rises on the boundary between Maine and Canada and flows southeast into the St. John River, forming part of the boundary between Maine and Canada.

The **St. John River** rises in the highlands between Maine and Canada and flows northeast through Maine, then southeast, forming part of the boundary between Maine and Canada, then southeast through Canada into the Bay of Fundy.

The **St. Lawrence River** rises in Lake Ontario and flows northeast between New York and Canada and through Canada into the Gulf of St. Lawrence.

The **St. Mary's River** rises in Okefinokee Swamp, between Georgia and Florida, and flows east into the Atlantic Ocean. It forms part of the boundary between Georgia and Florida.

The **St. Mary's River** connects Lake Superior and Lake Huron. It flows between Michigan and Canada.

The **Susquehanna River** rises in Lake Otsego in the central part of New York and flows south through New York, Pennsylvania, and Maryland into Chesapeake Bay.

The **Suwannee River** rises in the southern part of Georgia and flows southwest through Georgia and Florida into the Gulf of Mexico.

The **Tennessee River** rises in the southwestern part of North Carolina, in the Blue Mountains, and flows northwest through North Carolina, Tennessee, Alabama, and Kentucky into the Ohio River.

The **Thames River** is formed by the junction of three streams in the southeastern part of Connecticut and flows south through Connecticut into the Atlantic Ocean.

The **Tombigbee River** rises in the northeastern part of Mississippi and flows south through Mississippi and Alabama into the Mobile River.

The **Wabash River** rises in western Ohio and flows southwest through Ohio and Indiana, and between Illinois and Indiana, into the Ohio River.

The **Willamette River** rises in the Cascade Mountains in the western part of Oregon and flows north through Oregon into the Columbia River.

The **Willimantic River** rises in the northeastern part of Connecticut and flows south through Connecticut into the Quinebaug River.

The **Wisconsin River** rises on the northern boundary of Wisconsin and flows southwest through Wisconsin into the Mississippi River.

The **Yazoo River** is formed by the junction of two rivers in the western part of Mississippi and flows southwest through Mississippi into the Mississippi River.

The **Yellowstone River** rises in Yellowstone, or the National, Park, in Wyoming, and flows northeast through Yellowstone Park, Montana, and North Dakota into the Missouri River.

CANALS

27. Below is a table showing the principal canals of the United States used for commercial purposes:

Canals	Length Miles	Location
Albemarle and Chesapeake	44	Norfolk, Va., to Currituck Sound, N. C.
Augusta	9	Savannah River, Ga., to Augusta, Ga.
Black River	35	Rome, N. Y., to Lyons Falls, N. Y.
Cayuga and Seneca	25	Montezuma, N. Y., to Cayuga and Seneca Lakes, N. Y.
Champlain	81	Whitehall, N. Y., to West Troy, N. Y.
Chesapeake and Delaware	14	Chesapeake City, Md., to Delaware City, Del.
Chesapeake and Ohio	184	Cumberland, Md., to Washington, D. C.
Companys	22	Mississippi River, La., to Bayou Black, La.
Delaware and Raritan	66	New Brunswick, N. J., to Trenton, N. J.
Delaware Division	60	Easton, Pa., to Bristol, Pa.
Des Moines Rapids	7½	At Des Moines Rapids, Mississippi River.
Dismal Swamp	22	Connects Chesapeake Bay with Albemarle Sound.
Erie	387	Albany, N. Y., to Buffalo, N. Y.
Fairfield	4½	Alligator River to Lake Mattimuskeet, N. C.
Galveston and Brazos	38	Galveston, Tex., to Brazos River, Tex.
Hocking	42	Carroll, Ohio, to Nelsonville, Ohio.
Illinois and Michigan	102	Chicago, Ill., to La Salle, Ill.
Illinois and Mississippi	4½	Around lower rapids of Rock River, Ill. Connects with Mississippi River.
Lehigh Coal and Navigation Co.	108	Coalport, Pa., to Easton, Pa.
Louisville and Portland	2½	At Falls of Ohio River, Louisville, Ky.
Miami and Erie	274	Cincinnati, Ohio, to Toledo, Ohio.
Morris	103	Easton, Pa., to Jersey City, N. J.
Muscle Shoals and Elk R. Shoals	16	Big Muscle Shoals, Tenn., to Elk River Shoals, Tenn.
Newberne and Beaufort	3	Clubfoot Creek to Harlow Creek, N. C.
Ogeechee	16	Savannah River, Ga., to Ogeechee River, Ga.
Ohio	317	Cleveland, Ohio, to Portsmouth, Ohio.
Oswego	38	Oswego, N. Y., to Syracuse, N. Y.
Pennsylvania	193	Columbia, Northumberland, Wilkes-Barre, Huntingdon, Pa.
Portage Lake and Lake Superior	25	From Keweenaw Bay to Lake Superior.
Port Arthur	7	Port Arthur, Tex., to Gulf of Mexico.
Santa Fe	10	Waldo, Fla., to Melrose, Fla.
Sault Ste. Marie (ship canal)	3	Connects Lakes Superior and Huron at St. Mary's River.
Schuylkill Navigation Company	108	Mill Creek, Pa., to Philadelphia, Pa.
Sturgeon Bay and Lake Michigan	1½	Between Green Bay and Lake Michigan.
St. Mary's Falls	1½	Connects Lakes Superior and Huron at Sault Ste. Marie, Mich.
Susquehanna and Tidewater	45	Columbia, Pa., to Havre de Grace, Md.
Walhonding	25	Rochester, Ohio, to Roscoe, Ohio.
Welland (ship canal)	26¾	Connects Lake Ontario and Lake Erie.

INSULAR POSSESSIONS OF THE UNITED STATES

PHILIPPINES

28. The **Philippine group** lies off the southern coast of Asia and numbers about two thousand islands, covering an area of 140,000 square miles, equal to the combined area of New York, New Jersey, Maine, New Hampshire, Vermont, Massachusetts, Rhode Island, and Connecticut.

The larger islands of this group are given below in the order of their size. The largest city on each island is given opposite the name of each island.

PHILIPPINE ISLANDS

Islands	Largest City	Islands	Largest City
Luzon	Manila	Leyte	Masin
Mindanao	Davao	Negros	Tolon
Samar	Basey	Cebu	Cebu
Panay	Iloilo	Sulu	Sulu
Mindoro	Calapan		

PORTO RICO

29. The island of **Porto Rico** is one of the West Indies group and lies about 50 miles east of the island of Hayti. This island is about one-half the size of the state of New Jersey. San Juan on the northeastern coast is the largest city, and Ponce on the southern coast is the next largest city.

GUAM

30. The island of **Guam** is the largest of the Ladrone group. It is situated east of the island of Luzon in the Pacific Ocean. Agana is the largest city on the island.

TUTUILA

31. The island of **Tutuila** is one of the Samoan group situated in the Southern Pacific Ocean northeast of New Zealand. The largest city is Pago-Pago.

HAWAIIAN ISLANDS

32. The **Hawaiian Islands** are situated in the North Pacific Ocean southwest of the United States.

The Hawaiian group is composed of the following islands, arranged according to area:

HAWAIIAN ISLANDS

Islands	Largest City	Islands	Largest City
Hawaii	Hilo	Molokai	Wailau
Maui	Lahania	Lanai	Keomuku
Oahu	Honolulu	Niihau	No cities
Kauai	Lihue	Kahoolawe	No cities

IMPORTANT CITIES OF THE UNITED STATES ACCORDING TO POPULATION

MAINE

Portland	Rockland	Eastport
Lewiston	Calais	Brunswick
Bangor	Westbrook	Brewer
Biddeford	South Portland	Caribou
Auburn	Saco	Houlton
Augusta	Sanford	Belfast
Bath	Old Town	Eden
Waterville	Gardiner	Ellsworth

NEW HAMPSHIRE

Manchester	Rochester	Littleton
Nashua	Laconia	Milford
Concord	Somersworth	Derry
Dover	Claremont	Franklin Falls
Portsmouth	Franklin	Haverhill
Keene	Lebanon	Lancaster
Berlin	Exeter	Pembroke

VERMONT

Burlington	Colchester	Brandon
Rutland	Brattleboro	Morristown
Barre	Bellows Falls	Fairhaven
Montpelier	Hartford	Essex
St. Albans	Winooski	Newbury
Rockingham	Poultney	Castleton
St. Johnsbury	Lyndon	Bristol
Bennington	West Rutland	Enosburg

MASSACHUSETTS

Boston	Brockton	Quincy
Worcester	Haverhill	Waltham
Fall River	Salem	Pittsfield
Lowell	Chelsea	Brookline
Cambridge	Malden	Chicopee
Lynn	Newton	Northampton
Lawrence	Fitchburg	Medford
New Bedford	Taunton	Newburyport
Springfield	Gloucester	Woburn
Somerville	Everett	Beverly
Holyoke	North Adams	Clinton

CONNECTICUT

New Haven	Manchester	Putnam
Hartford	Naugatuck	South Norwalk
Bridgeport	Middletown	Thompson
Waterbury	Willimantic	East Hartford
New Britain	Torrington	Bristol
Meriden	Derby	Norwalk
New London	Rockville	Groton
Norwich	South Manchester	Huntington
Danbury	Winsted	Westhaven
Stamford	Wallingford	Plainfield
Ansonia	Enfield	New Milford

RHODE ISLAND

Providence	Central Falls	Coventry
Pawtucket	East Providence	Warren
Woonsocket	Cranston	Manville
Newport	Westerly	Lonsdale
Warwick	Bristol	Valley Falls

NEW YORK

New York	Kingston	Middletown
Buffalo	Poughkeepsie	Watervliet
Rochester	Cohoes	Ithaca
Syracuse	Jamestown	Ogdensburg
Albany	Oswego	Glens Falls
Troy	Watertown	Lansingburg
Utica	Mount Vernon	Saratoga Springs
Yonkers	Amsterdam	Hornellsville
Binghamton	Niagara Falls	Belmont
Elmira	Gloversville	Dunkirk
Schenectady	Lockport	Corning
Auburn	Rome	Geneva
Newburg	New Rochelle	Peekskill

PENNSYLVANIA

Philadelphia	York	Plymouth
Pittsburg	Williamsport	Carbondale
Scranton	New Castle	Mahanoy City
Reading	Easton	Oil City
Erie	Norristown	South Bethlehem
Wilkes-Barre	Shenandoah	Mount Carmel
Harrisburg	Shamokin	Dunmore
Lancaster	Lebanon	Pittston
Altoona	Pottsville	Homestead
Johnstown	Braddock	Columbia
Allentown	Bradford	Nanticoke
McKeesport	Hazleton	Steelton
Chester	Pottstown	Wilkinsburg

NEW JERSEY

Newark	West Hoboken	Millville
Jersey City	East Orange	Phillipsburg
Paterson	New Brunswick	Bloomfield
Camden	Perth Amboy	Hackensack
Trenton	Plainfield	Long Branch
Hoboken	Union	Rahway
Elizabeth	Montclair	Burlington
Bayonne	Bridgeton	W. Orange
Atlantic City	Morristown	Gloucester
Passaic	Kearney	South Amboy
Orange	Harrison	Englewood

DELAWARE

Wilmington	Lewes	Georgetown
Newcastle	Smyrna	Middletown
Dover	Laurel	Harrington
Milford	Seaford	Newark

MARYLAND

Baltimore	Frostburg	Chestertown
Cumberland	Salisbury	Elkton
Hagerstown	Havre de Grace	Brunswick
Frederick	Westminster	Lonaconing
Annapolis	Crisfield	Pocomoke
Cambridge	Easton	Laurel

VIRGINIA

Richmond	Portsmouth	Winchester
Norfolk	Danville	Fredericksburg
Petersburg	Alexandria	Berkley
Roanoke	Manchester	Bristol
Newport News	Staunton	Suffolk
Lynchburg	Charlottesville	Harrisonburg

WEST VIRGINIA

Wheeling	Martinsburg	Bluefield
Huntington	Fairmont	Benwood
Parkersburg	Grafton	Clarksburg
Charleston	Moundsville	Hinton

TENNESSEE

Memphis	Clarksville	Cleveland
Nashville	Columbia	Dyersburg
Knoxville	Bristol	Harriman
Chattanooga	Johnson City	Union City
Jackson	Murfreesboro	Morristown

NORTH CAROLINA

Wilmington	Greensboro	Salisbury
Charlotte	Newbern	Goldsboro
Asheville	Concord	Washington
Winston-Salem	Durham	Fayetteville
Raleigh	Elizabeth City	Gastonia

SOUTH CAROLINA

Charleston	Anderson	Newberry
Columbia	Rockhill	Orangeburg
Greenville	Union	Georgetown
Spartanburg	Greenwood	Beaufort
Sumter	Florence	Chester

GEORGIA

Atlanta	Athens	Waycross
Savannah	Brunswick	Valdosta
Augusta	Americus	Thomasville
Macon	Rome	Albany
Columbus	Griffin	Marietta

FLORIDA

Jacksonville	St. Augustine	Palatka
Pensacola	Lake City	Fernandina
Key West	Gainesville	Apalachicola
Tampa	Ocala	Tallahassee

ALABAMA

Mobile	Huntsville	Eufaula
Birmingham	Florence	New Decatur
Montgomery	Bessemer	Gadsden
Anniston	Tuscaloosa	Opelika
Selma	Talladega	Phœnix

MISSISSIPPI

Vicksburg	Greenville	McComb
Meridian	Columbus	Hattiesburg
Natchez	Biloxi	Water Valley
Jackson	Yazoo City	Corinth

LOUISIANA

New Orleans	Monroe	Thibodaux
Shreveport	Crowley	Houma
Baton Rouge	Donaldsonville	Opelousas
New Iberia	Plaquemine	Franklin
Lake Charles	Gretna	Patterson
Alexandria	Lafayette	Natchitoches

TEXAS

San Antonio	Sherman	Greenville
Houston	Beaumont	Terrell
Dallas	Paris	Brownsville
Galveston	Corsicana	Brenham
Ft. Worth	Palestine	Hillsboro
Austin	Tyler	Texarkana
Waco	Gainesville	Bonham
El Paso	Marshall	Ennis
Laredo	Cleburne	Weatherford
Denison	Temple	Corpus Christi

ARKANSAS

Little Rock	Texarkana	Paragould
Fort Smith	Jonesboro	Newport
Pine Bluff	Fayetteville	Camden
Hot Springs	Eureka Springs	Arkadelphia
Helena	Mena	Van Buren

OKLAHOMA

Oklahoma City	Ardmore	Stillwater
Guthrie	Elreno	Muskogee
Shawnee	South McAlester	Norman
Enid	Hobart	Anadarko
Tulsa	Durant	Wagoner
Lawton	Kingfisher	Perry

MICHIGAN

Detroit	Manistee	Delray
Grand Rapids	Alpena	Coldwater
Saginaw	Traverse City	Monroe
Bay City	Pontiac	Wyandotte
Kalamazoo	Adrian	St. Joseph
Jackson	Owasso	Grand Haven
Battle Creek	Holland	Calumet
Muskegon	Ypsilanti	Ishpeming
Lansing	Ludington	Sault Sainte Marie
Port Huron	Mt. Clemens	Escanaba
Flint	Cadillac	Menominee
Ann Arbor	Cheboygan	Marquette
West Bay City	Benton Harbor	Iron Wood

WISCONSIN

Milwaukee	Eau Claire	Beloit
Superior	Fond du Lac	Manitowoc
Racine	Appleton	Merrill
Oshkosh	Kenosha	Stevens Point
La Crosse	Marinette	Chippewa Falls
Madison	Ashland	Watertown
Sheboygan	Wausau	Waukesha
Green Bay	Janesville	Antigo

OHIO

Cleveland	Lorain	Cambridge
Cincinnati	Steubenville	Wellston
Toledo	Marietta	Delaware
Columbus	Chillicothe	Martins Ferry
Dayton	Ashtabula	Fostoria
Youngstown	Piqua	Salem
Akron	Massillon	Defiance
Springfield	Ironton	Niles
Canton	Marion	Galion
Hamilton	Tiffin	Conneaut
Zanesville	Bellaire	Norwalk
Lima	Middletown	Circleville
Sandusky	Lancaster	Kenton
Newark	Alliance	Urbana
Portsmouth	Elyria	Mount Vernon
Mansfield	Xenia	Bellefontaine
Findlay	Warren	Bucyrus
East Liverpool	Fremont	Norwood

INDIANA

Indianapolis	Logansport	Peru
Evansville	Elkhart	Columbus
Ft. Wayne	Michigan City	Madison
Terre Haute	Elwood	Goshen
South Bend	Hammond	Brazil

INDIANA—*Continued*

Muncie	Jeffersonville	Alexandria
New Albany	Kokomo	Laporte
Anderson	Vincennes	Frankfort
Richmond	Huntington	Shelbyville
Lafayette	Wabash	Connersville
Marion	Washington	Crawfordsville

ILLINOIS

Chicago	Rock Island	Cairo
Peoria	Evanston	Ottawa
Quincy	Galesburg	La Salle
Springfield	Belleville	Matton
Rockford	Moline	Waukegan
East St. Louis	Danville	Champaign
Joliet	Jacksonville	Lincoln
Aurora	Alton	Pekin
Bloomington	Streator	Kewanee
Elgin	Kankakee	Dixon
Decatur	Freeport	Oak Park

KENTUCKY

Louisville	Frankfort	Winchester
Covington	Bowling Green	Richmond
Newport	Hopkinsville	Harris
Lexington	Ashland	Danville
Paducah	Maysville	Middlesboro
Owensboro	Bellevue	Mayfield
Henderson	Dayton	Georgetown

MINNESOTA

Minneapolis	Faribault	Austin
St. Paul	Red Wing	New Ulm
Duluth	Brainerd	Crookston
Winona	Rochester	Albert Lea
Stillwater	Fergus Falls	St. Peter
Mankato	Little Falls	Hastings
St. Cloud	Owatonna	Anoka

MISSOURI

St. Louis	Jefferson City	Chillicothe
Kansas City	Carthage	Aurora
St. Joseph	Webb City	Kirksville
Joplin	Moberly	Columbia
Springfield	St. Charles	De Soto
Sedalia	Nevada	Brookfield
Hannibal	Independence	Trenton

IOWA

Des Moines	Ottumwa	Fort Madison
Dubuque	Waterloo	Iowa City
Sioux City	Muscatine	Creston
Davenport	Keokuk	Mason City
Cedar Rapids	Fort Dodge	Centerville
Burlington	Marshalltown	Oelwein
Council Bluffs	Oskaloosa	Cedar Falls
Clinton	Boone	Atlantic

KANSAS

Kansas City	Parsons	Wellington
Topeka	Ottawa	Chanute
Wichita	Newton	Osawatomie
Leavenworth	Arkansas City	Abilene
Atchison	Salina	Cherryvale
Lawrence	Argentine	Eldorado
Fort Scott	Iola	Olathe
Galena	Winfield	Manhattan
Pittsburg	Coffeyville	Concordia
Hutchinson	Independence	Horton
Emporia	Junction City	Rosedale

NEBRASKA

Omaha	Hastings	Fairbury
Lincoln	Kearney	Falls City
South Omaha	York	Holdredge

NEBRASKA—*Continued*

Beatrice	Plattsmouth	Blair
Grand Island	Norfolk	Auburn
Nebraska City	North Platte	Wymore
Fremont	Columbus	Alliance

SOUTH DAKOTA

Sioux Falls	Yankton	Vermilion
Lead	Huron	Hot Springs
Aberdeen	Brookings	Webster
Mitchell	Madison	Rapid City
Watertown	Pierre	Milbank
Deadwood	Canton	Redfield

NORTH DAKOTA

Fargo	Valley City	Devil's Lake
Grand Forks	Grafton	Mandan
Bismarck	Wahpeton	Minot
Jamestown	Dickinson	Larimore

WASHINGTON

Seattle	Walla Walla	Aberdeen
Tacoma	Everett	Port Townsend
Spokane	Ballard	North Yakima
Bellingham	Olympia	Vancouver

OREGON

Portland	Salem	Albany
Astoria	The Dalles	La Grande
Baker City	Oregon City	Ashland
Pendleton	Eugene	Grants Pass

CALIFORNIA

San Francisco	Berkeley	San Bernardino
Los Angeles	Fresno	Santa Cruz
Oakland	Pasadena	Pomona

CALIFORNIA—*Continued*

Sacramento	Riverside	Santa Ana
San Jose	Vallejo	Bakersfield
San Diego	Eureka	Redlands
Stockton	Santa Rosa	Grass Valley
Alameda	Santa Barbara	Napa

IDAHO

Boise City	Wallace	Idaho Falls
Pocatello	Wardner	Hailey
Lewiston	Montpelier	Grangeville
Moscow	Weiser	Rexburg

WYOMING

Cheyenne	Evanston	Casper
Laramie	Sheridan	Kemmerer
Rock Springs	Green River	Newcastle
Rawlins	Diamondville	Cambria

MONTANA

Butte	Missoula	Walkerville
Great Falls	Bozeman	Kalispell
Helena	Billings	Red Lodge
Anaconda	Livingston	Belt

UTAH

Salt Lake City	Park City	Spanish Fork
Ogden	Springville	American Fork
Provo City	Eureka	Lehi City
Logan	Brigham	Payson

ARIZONA

Tucson	San Carlos	Florence
Bisbee	Jerome	Globe
Phœnix	Nogales	Williams
Prescott	Yuma	Winslow

NEVADA

Reno	Wadsworth	Austin
Virginia City	Winnemucca	Elko
Carson City	Delamar	Dayton
Lovelock	Eureka	Tuscarora

COLORADO

Denver	Trinidad	Durango
Pueblo	Victor	Aspen
Colorado Springs	Canon City	Central City
Leadville	Florence	Ft. Collins
Cripple Creek	Salida	Greeley
Boulder	Grand Junction	Colorado City

NEW MEXICO

Albuquerque	Gallup	Socorro
Santa Fe	East Las Vegas	Mora
Las Vegas	Silver City	Taós
Raton	Roswell	Las Cruces

ALASKA

Nome	Sitka	Douglas
Skagway	Ft. Wrangle	Point Hope
Juneau	St. Michael	Karluk

ONE HUNDRED LARGEST CITIES IN THE UNITED STATES

New York, N. Y.
Chicago, Ill.
Philadelphia, Pa.
St. Louis, Mo.
Boston, Mass.
Baltimore, Md.
Pittsburg, Pa.
Cleveland, Ohio
Buffalo, N. Y.
San Francisco, Cal.
Cincinnati, Ohio
New Orleans, La.
Detroit, Mich.
Milwaukee, Wis.
Washington, D. C.
Newark, N. J.
Jersey City, N. J.
Louisville, Ky.
Minneapolis, Minn.
Providence, R. I.
Indianapolis, Ind.
Kansas City, Mo.
St. Paul, Minn.
Rochester, N. Y.
Denver, Colo.
Toledo, Ohio
Columbus, Ohio
Worcester, Mass.
Syracuse, N. Y.
New Haven, Conn.
Paterson, N. J.
Fall River, Mass.
St. Joseph, Mo.
Omaha, Neb.
Los Angeles, Cal.
Memphis, Tenn.
Scranton, Pa.
Lowell, Mass.
Albany, N. Y.
Cambridge, Mass.
Portland, Ore.
Atlanta, Ga.
Grand Rapids, Mich.
Dayton, Ohio
Richmond, Va.
Nashville, Tenn.
Seattle, Wash.
Hartford, Conn.
Reading, Pa.
Wilmington, Del.
Camden, N. J.
Trenton, N. J.
Troy, N. Y.
Bridgeport, Conn.
Lynn, Mass.
Oakland, Cal.
Lawrence, Mass.
New Bedford, Mass.
Des Moines, Iowa
Springfield, Mass.
Somerville, Mass.
Hoboken, N. J.
Evansville, Ind.
Manchester, N. H.
Utica, N. Y.
Peoria, Ill.
Charleston, S. C.
Savannah, Ga.
Salt Lake City, Utah
San Antonio, Tex.
Duluth, Minn.
Erie, Pa.
Elizabeth, N. J.
Wilkes-Barre, Pa.
Kansas City, Kans.
Harrisburg, Pa.
Portland, Me.
Yonkers, N. Y.
Norfolk, Va.
Waterbury, Conn.
Holyoke, Mass.
Fort Wayne, Ind.
Youngstown, Ohio
Houston, Tex.
Covington, Ky.
Akron, Ohio
Dallas, Tex.
Saginaw, Mich.
Lancaster, Pa.
Lincoln, Neb.
Brockton, Mass.
Binghamton, N. Y.
Augusta, Ga.
Honolulu, Hawaii
Pawtucket, R. I.
Altoona, Pa.
Wheeling, W. Va.
Mobile, Ala.
Birmingham, Ala.
Little Rock, Ark.

GAUGING AND ELEMENTARY PHYSICS

GAUGING

1. Introduction.—The United States laws require that all spirits distilled within its boundaries shall be measured by a representative of the government, known as an **internal-revenue gauger.** After an applicant for this position is appointed, but before he is assigned to duty, he must "take an oath faithfully to perform his duties and shall give bond, with one or more sureties satisfactory to the Commissioner of Internal Revenue, for the faithful discharge of the duties assigned to him by law or regulations; and the penal sum of said bond shall not be less than five thousand dollars; neither a distiller, rectifier, nor wholesale liquor dealer will be accepted as surety on the bond of any gauger or storekeeper."

The first efforts of an applicant for the position of gauger, after attending a civil-service examination, should be to acquire a thorough knowledge of all the requirements of the laws and regulations relating to the office. The important bearing of the duties of such a position on the revenues and commercial interests of the country imperatively demands that carefulness and accuracy of execution should be observed at all times. Such knowledge can be readily acquired by making a study of the Gaugers' Manual, which can be obtained without cost by applying to the Commissioner of Internal Revenue, Washington, D. C.

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2. Definitions.—Gauging, as applied to the United States internal-revenue service, is the method of measuring the capacities or contents of casks, tubs, or cisterns.

A **cask** resembles two frustums of a cone with their larger bases placed together, Fig. 1.

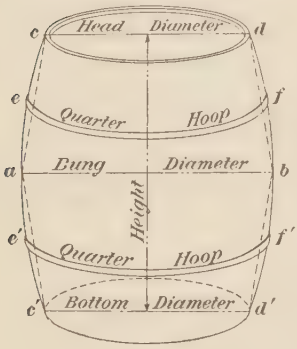


Fig. 1

The **bung diameter** of a cask is the diameter measured half way between the two ends; it is usually the greatest diameter, as shown at *a b*, Fig. 1.

The **head diameter** of a cask is the diameter measured at the head or bottom of a cask between the chimes, as shown at *c d* and *c′ d′*, Fig. 1.

The **mean diameter** of a cask is the mean between the bung diameter and the head diameter. In the United States internal-revenue service, it depends on the variety of cask.

3. Varieties of Casks.—Casks are divided into three classes, according to the curvature of the staves at what is

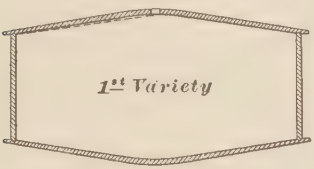


Fig. 2

Fig. 3

Fig. 4

termed the *quarter hoop*, as shown at *e f* and *e′ f′*, Fig. 1, midway between the bung and the chime.

The casks having the least curvature are termed the **first variety,** Fig. 2; those having medium curvature, the **second variety,** Fig. 3; and those having the greatest curvature, the **third variety,** Fig. 4. When no variety of cask is mentioned, the second variety is assumed to be the one meant, as it is the one most commonly used.

4. The mean diameter of the first-variety casks is found by multiplying the difference between the head diameter and the bung diameter by the decimal .55 and adding the product to the head diameter; the sum will be the mean diameter.

The mean diameter of the second variety is found by multiplying the difference between the head and the bung diameter by the decimal .63 and adding the product to the head diameter; while the mean diameter of the third variety is found by multiplying the difference between the head and the bung diameter by the decimal .70 and adding the product to the head diameter.

EXAMPLE.—The head diameter of a second-variety cask is 19 inches and the bung diameter is 22 inches; what is the mean diameter?

SOLUTION.—Mean diameter is

$$.63 \times (22 - 19) + 19 = 20.89 \text{ in.} \quad \text{Ans.}$$

5. Capacity.—The **capacity** of a cask, tub, or cistern is the number of gallons that such a vessel will contain when filled. The unit of measurement considered in expressing the capacity of a vessel is the United States wine gallon, which contains 231 cubic inches. The unit of measurement in Great Britain is the Imperial gallon, which contains 277 cubic inches. The beer gallon contains 282 cubic inches.

GAUGING INSTRUMENTS

6. The instruments used in gauging are:

1. *Standard Gauging Rod.*—The instrument known as the Prime and McKean's combination *gauging rod*, Fig. 5, is prescribed by the Commissioners of Internal Revenue for determining the capacity of casks when the same is not determined by the weighing beam.

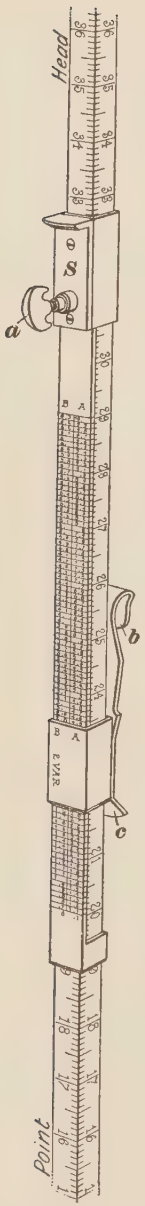


FIG. 5

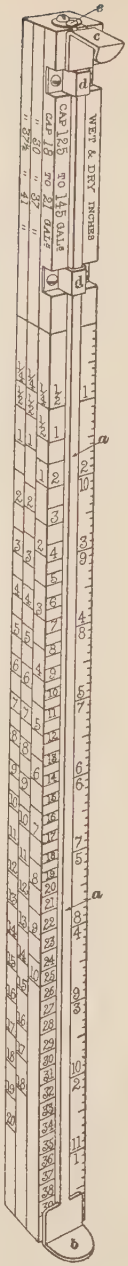


FIG. 6

2. *Wantage Rod.*—For determining the *wantage* or *ullage* in casks, Alexander's improved *wantage rod*, Fig. 6, is prescribed.

Wantage or **ullage** is the quantity that a vessel lacks of being full.

3. *Weighing Beam.*—The Fairbanks weighing beam, Fig. 7, has been adopted for use by the internal-revenue

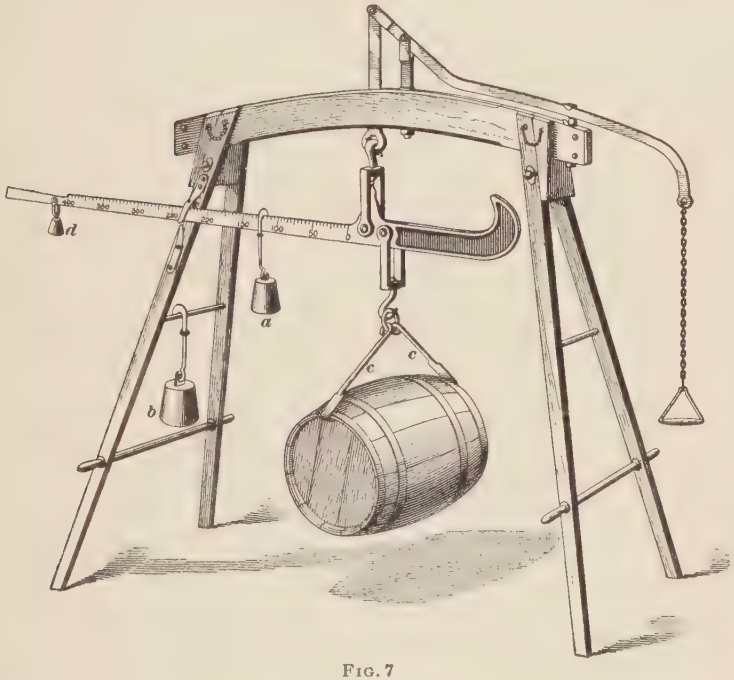


FIG. 7

service. Accompanying each weighing beam are two poises (one *a* of 8 pounds and another *b* of 16 pounds), a set of barrel hooks *c*, and a counterpoise *d* to the barrel hooks. This combination is termed a *weighing beam complete.*

4. *Hydrometers.*—The standard *hydrometers* for weighing or determining the proof of distilled spirits are supplied for the use of internal-revenue gaugers by the government.

DESCRIPTION AND USE OF INSTRUMENTS

7. Gauging Rod.—The **gauging rod,** Fig. 8, used in the internal-revenue service, is a nickel-plated square steel bar about 5 feet long, pointed on one end. The four sides of the bar are graduated into inches and tenths of an inch for convenience in taking measurements. The rod has five attachments, each of which is used to find some dimension or quantity of a cask. The slide *S* is used to find the mean diameter of a cask. This attachment has two graduated surfaces and two open sides, so that the quantities on the graduated surfaces can be read in connection with the graduations on the bar. A setscrew *a* is provided to lock the slide in any desired position. Ordinarily, when using the mean-diameter slide, it is placed on the rod with the setscrew *a* toward the head of the rod and the other end of the slide toward the point of the rod. An overslide *O* with a clip attached is used to find the bung diameter of a cask. The overslide is made sufficiently large to fit over the slide *S* and can therefore be used in connection with it. A head-slide *H* is used to determine the diameter of the head of a cask and an attachment consisting of the fixed caliper arm *N* and the movable caliper arm *M* is used to determine the height of a cask.

The rod is prepared for determining the mean diameter of a cask by first placing the mean-diameter scale *S*, Fig. 8, on the rod with the screw attachment *a* toward the head of the rod and with the manufacturer's stamp on the scale in a line with that on the rod (this latter condition must be observed in adjusting all the parts of the combination rod). The head-slide *H* is then slipped over the point of the rod with the stationary projection *P* pointing upwards. Having thus prepared the rod, seize it with one hand above the mean-diameter scale, place the other hand on the head-slide, and rest the point of the rod on the inner side of the lower chime of the cask close against the head, then bring the projection of the head-slide against the inner side of the upper chime, holding the rod at an angle of 45°, as shown in

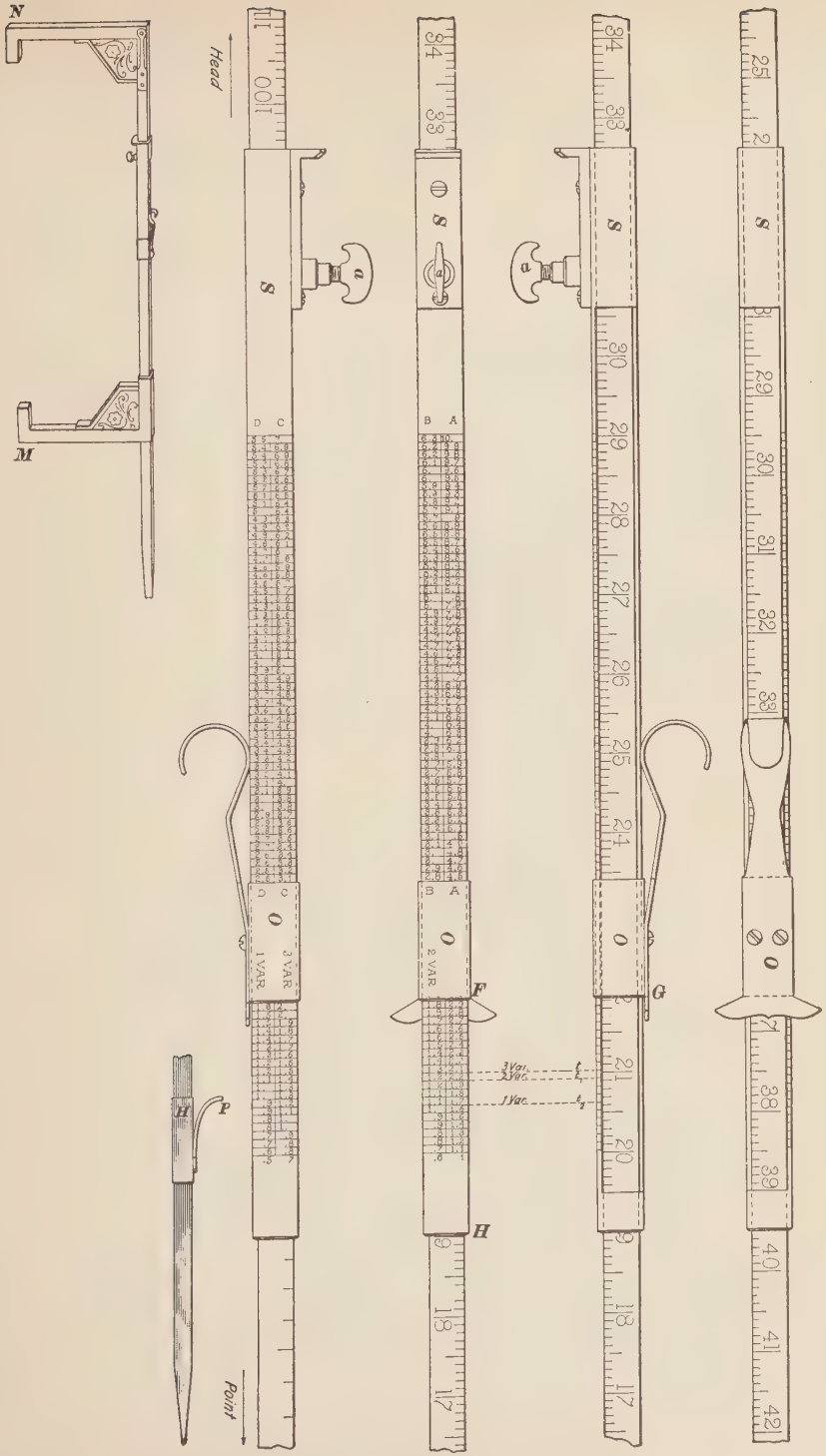


FIG. 8

Fig. 9.* Hold the head-slide firmly in place and raise the mean-diameter scale to the position shown on the face of the rod immediately above the head-slide, so as to leave sufficient space for the head diameter to be conveniently read. Hold the head-slide firmly in place and gently move the mean-diameter scale downwards until it rests on the head-slide; when the mean-diameter slide is in this position, turn the thumbscrew at the top to secure it in place. The bottom of the mean-diameter scale will then indicate the same reading on the rod as the top of the head-slide. Next remove the head-

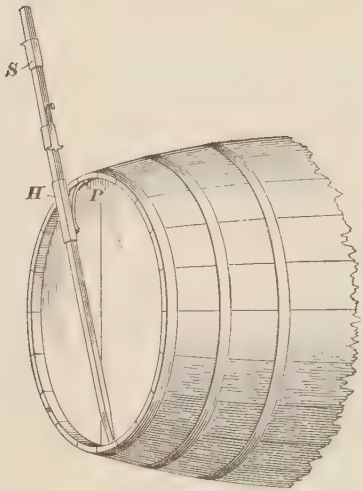


Fig. 9

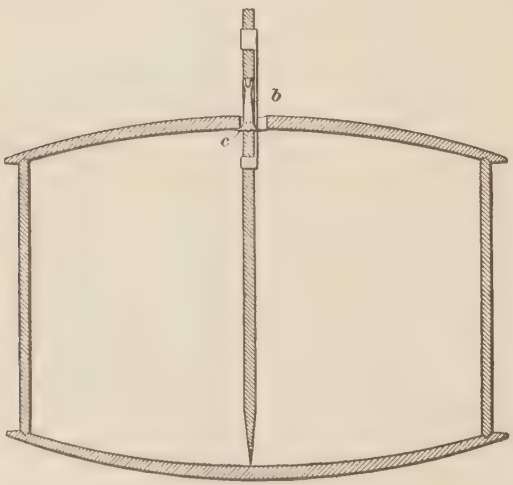


Fig. 10

slide from the rod and insert the rod in the cask at the bung

*The rod should always be applied to the stamped head of the cask in case but one head is measured.

hole, the point resting on the bottom stave opposite the bung hole, as shown in Fig. 10. The overslide *b* is then drawn up until the lip *c* strikes the under side of the bung stave. The rod is then removed from the cask, the overslide being retained in the same position and raised so that the figures on the scale at the bottom of the overslide can be easily read. Care must be observed that the figures read are in the column corresponding to the variety of the cask being gauged. Then find the same figures in column *A* of the scale and follow the line *above* the figures, turning the rod to the left to the corresponding line on the face of the rod, which latter line will indicate the mean diameter.

Illustration.—Suppose that the rod shows the head diameter to be 19 inches, as shown at *H*, Fig. 8, and the bung diameter to be 22 inches, as shown at *G*. The difference between the two diameters (3 inches) will appear in the *A* column at the foot *F* of the overslide, and 1.9 in the *B* or second-variety column, 2 in the *C* or third-variety column, and 1.6 in the *D* or first-variety column. By following the line above, 1.9, 2, and 1.6 in the *A* column on the mean-diameter scale to the right-hand side of the rod, the corresponding line will indicate a mean diameter of 20.9 inches, 21 inches, and 20.6 inches, respectively, as shown at *t*, t_1, and t_2.

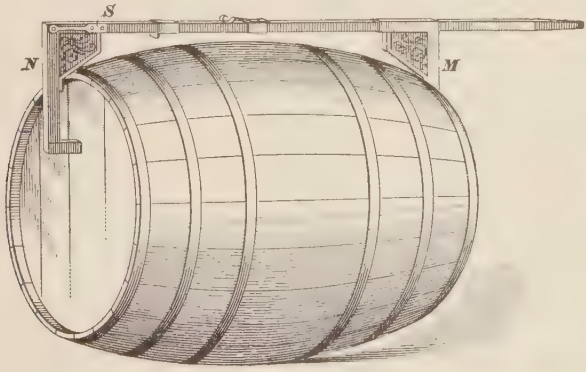


Fig. 11

8. The rod is prepared for finding the length of casks by sliding the fixed caliper arm *N*, Fig. 11, having the spring *S* on the head of the rod, taking care to have the spring bolt

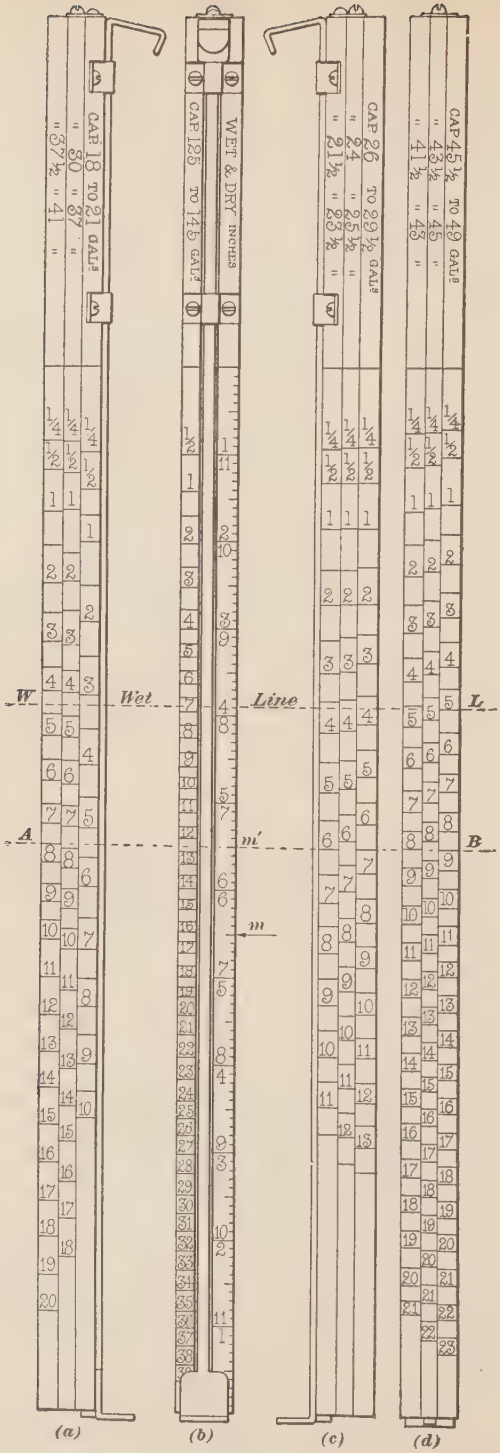

CAP 18 TO 21 GALS
" 87½ " 41 "
" 80 " 87 "
CAP 135 TO 145 GALS
WET & DRY INCHES
CAP 26 TO 29½ GALS
" 24 " 25½ "
" 21½ " 23½ "
CAP 45½ TO 49 GALS
" 43½ " 45 "
" 41½ " 43 "
W
Wet
Line
L
A
m'
m
B
(a)
(b)
(c)
(d)

FIG. 12

enter the hole in the rod so that it will remain stationary. Next slide the movable caliper arm *M* over the point of the rod, so that this arm will be in line with the other. Apply the rod as follows: Standing at the head of the cask, support the rod in the middle with the left hand, holding the top of the sliding caliper arm with the right hand. Place the rod on the cask in a line with the bung stave, allowing it to rest on the braces of the caliper arms, the fixed caliper arm being lightly held against the head of the cask; then move, with the right hand, the sliding caliper arm against the other head of the cask. The length of the cask will be indicated by the figures on the top of the rod at the inside of the movable arm. The rod is graduated so as to allow for heads $\frac{3}{4}$ inch thick.

9. Wantage Rod.—The wantage rod is graduated on four sides, as shown in Fig. 12, so as to measure the wantage, or ullage, in casks more than half filled varying in capacity from 18 to 145 gallons. When casks are not full, only the actual wantage, as determined by the wantage rod, is allowed. In a gauger's official report, fractional parts of a gallon less than .5 or $\frac{1}{2}$ are not reported, if more than .5 or $\frac{1}{2}$ they are reported as 1.

In using this rod, the following instructions should be observed: Take the wantage rod, Fig. 6, in the right hand with the metal rod *a* resting entirely in the groove; insert the rod in the cask through the bung hole with the lip *b* against the under side of the bung stave; with the thumb and forefinger of the left hand, clasp the handle *c* at the head of the metal part in the manner shown in Fig. 13 (*a*). Hold the rod in a perpendicular position with the lip *b*, Fig. 6, firmly pressed against the under side of the bung stave, turn with the right hand the eccentric button *e* on top of the rod, and move the wooden part of the rod downwards as far as the metal guides *d*, Fig. 6, will permit, as shown in Fig. 13 (*b*). Withdraw the rod to its former position at once without jarring the contents of the cask, remove it from the cask and read the wet line made by the liquid on the scale, which shows the

capacity of the cask containing the liquid. The graduation nearest the wet line should always be read. When the wet line is midway between graduations, the line that indicates the greater wantage should be read.

FIG. 13

10. In Fig. 12, the line WL represents the wet line made by a liquid placed in various-sized casks, the top of the liquid in each cask being at the same distance from the bung stave. The wantage in each of these casks would be represented by the graduations nearest the line WL. Beginning at the left, the wantage in a cask whose capacity is between $37\frac{1}{2}$ and 41 gallons would be $4\frac{1}{2}$ gallons; in a cask between 30 and 37 gallons, it would be 4 gallons; in a cask between 18 and 21 gallons, it would be 3 gallons; in a cask between 125 and 145 gallons, it would be 7 gallons; and so on across.

11. To find the wantage in a cask that is less than half full, pull out the metal part of the rod as far as the guides will permit, secure it in this position by turning the eccentric button into the notch in the back of the metal rod. Insert the entire rod into the cask until the wooden part rests on the bottom stave opposite the bung hole, as shown in Fig. 14;

withdraw it at once and the wet line on the wet-and-dry inch column, Fig. 12, counting from the bottom upwards, will show the number of inches and tenths of inches of spirits in the cask. Take the same number of inches and tenths of inches in the same column, counting from the top downwards; follow with the eye this line around the rod to the corresponding line on the scale that contains the capacity of the cask being measured.

FIG. 14

ILLUSTRATION.—Suppose that the point *m*, Fig. 12, represents the wet line in the wet-and-dry column on the rod after it has been withdrawn from a cask. This point appears to be 5.5 inches above the bottom of the rod, or, in other words, there are 5.5 inches of liquid in the cask. If the same measure, 5.5 inches, is taken from the top downwards in the same column, it will come to a point *m'*. If this point is followed around the rod it will appear as the line *AB*. In casks varying in size, as shown on the rod containing 5.5 inches of liquid, this liquid will measure, beginning at the left, $7\frac{1}{2}$, $7\frac{1}{2}$, $5\frac{1}{2}$, $12\frac{1}{2}$, 6, 6, $6\frac{1}{2}$, 8, $8\frac{1}{2}$, $8\frac{1}{2}$ gallons, respectively.

RULES FOR GAUGING CASKS, CISTERNS, OR TUBS

12. Rule for Finding the Capacity of a Cask.—*To find the capacity of a cask, multiply the square of the mean diameter, in inches, by the decimal .0034 and multiply this product by the length of the cask, in inches; the result will be the capacity of the cask, in wine gallons.*

EXAMPLE.—Find the capacity of a cask of the second variety, the diameter of which is 18.9 inches at the head and 21.9 inches at the bung, and whose length is 28.9 inches.

SOLUTION.—Head diameter is 18.9 in., bung diameter is 21.9 in., difference is $21.9 - 18.9 = 3$ in.; $3 \times .63 = 1.89$ in., the amount to be added to the head diameter. $18.9 + 1.89 = 20.79$ in., the mean diameter,

$$20.79 \times 20.79 \times .0034 \times 28.9 = 42.5 \text{ gal. Ans.}$$

13. Rule for Gauging Cisterns or Tubs.—*To find the capacity of a cistern or tub, take the dimensions, in inches and tenths of an inch. Add together the square of the head diameter, Fig. 15, the square of the bottom diameter, and four times the square of the midway diameter (ascertained by adding the top and bottom diameters together and dividing by 2), and divide the sum by 6, which gives the square of the true mean diameter; multiply this by the height of the cistern, and the product will be the capacity, in cylindrical inches. As there are 294 cylindrical inches in a gallon, divide this last product by 294, and the quotient will be the number of gallons contained in the cistern.*

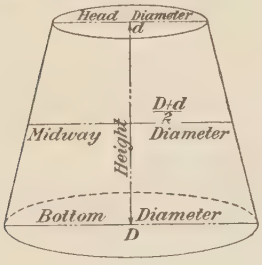


FIG. 15

A **cylindrical inch** is equal to the capacity of a cylinder 1 inch in diameter and 1 inch high; it is less than a cubic inch, 294 cylindrical inches being equal to 231 cubic inches.

EXAMPLE.—What is the capacity, in gallons, of a tub whose bottom diameter is 10 feet, top diameter 9 feet, and height 8 feet 4 inches?

SOLUTION.—

Top diameter, 108 in.
Bottom diameter, 120 in.
Divided by 2) 228 in.
Midway diameter, 114 in.
Height, 100 in.

$108 \times 108 = 11664$
$120 \times 120 = 14400$
$114 \times 114 \times 4 = 51984$
$78048 \div 6 = 13{,}008$
$13{,}008 \times 100 \div 294 = 4{,}424.5$ gal. Ans.

14. Should the cistern be full, warped so as not to be a perfect circle, or otherwise in such condition that the diameter of the bottom cannot be taken, the following rule, though not mathematically correct, is for all practical purposes sufficiently so, the difference being shown only in large cisterns, where the difference between the top and bottom diameters is considerable.

Rule.—*Divide the outside circumference of the cistern half way between the bottom and top, in inches, by 3.1416 (or multiply by 7 and divide by 22), and the result will be the mean outside diameter, in inches. Deduct from this twice the thickness of the staves, and the result will be the mean inside diameter. Multiply this sum by itself and by the height, in inches, and the product by .0034; the final product will be the capacity of the cistern, in gallons.*

EXAMPLE.—What is the capacity, in gallons, of a cistern having the dimensions of the tub given in the example in Art. **13** and whose outside circumference is 367.5 inches?

SOLUTION.— $367.5 \div 3.1416 = 117 - 3$ in. $= 114$ in.;
$114 \times 114 \times 100 = 1{,}299{,}600 \times .0034 = 4{,}418$ gal. Ans.

ELEMENTARY PHYSICS

INTRODUCTORY

DEFINITIONS

15. Natural and Physical Science.—Science, which may be defined as a classified knowledge of nature, is divided into *natural* and *physical science.* **Natural science** concerns itself with the external form and internal structure of bodies. **Physical science** considers only the matter of which these bodies are composed.

Geology, mineralogy, botany, and zoology, which investigate the form and structure of the earth, of minerals, of plants, and of animals, respectively, are natural sciences. Physics and chemistry, which consider the properties of matter itself, whether it is light or heavy, hard or soft, combustible or incombustible, are physical sciences.

16. Matter is anything that possesses weight; that is, is acted on by gravitation. In studying matter, physical science considers: (1) The division of which matter is capable;

(2) the attractions by which these particles are held together; (3) the motions that these particles may have.

Science recognizes three divisions of matter—*masses*, *molecules*, and *atoms*. A **mass**, or body, of matter is any portion of matter appreciable by the senses. A **molecule** is the smallest particle of matter that a body can be divided into without losing its identity. An **atom** is an indivisible portion of matter. Atoms unite to form molecules; a collection of molecules forms a mass, or a body.

The sun and the grain of sand are masses of matter; the smallest particles of sugar or of salt that still show the properties of these substances, respectively, are molecules of sugar or of salt. The still more minute particles of carbon or hydrogen, and of oxygen, that make up the molecule of sugar, or those of chlorine and of sodium that compose the molecule of salt, are atoms.

17. Bodies, which are collections of molecules, exist in three forms or conditions: *solid*, *liquid*, and *gaseous*.

A **solid body** is one whose molecules change their relative positions with great difficulty; as iron, wood, stone, etc.

A **liquid body** is one whose molecules tend to change their relative positions easily. Liquids readily adapt themselves to the vessel that contains them, and their upper surface always tends to become level. Water, mercury, etc. belong to this class.

A **gaseous body**, or a **gas**, is one whose molecules tend to separate from one another; as air, oxygen, etc.

Gaseous bodies are sometimes called *aeriform* (air-like) *bodies* and are divided into two classes: *permanent gases* and *vapors*. A **permanent gas** is one that remains a gas at ordinary temperatures and pressures. A **vapor** is a body that at ordinary temperature is a liquid or a solid, but when heat is applied becomes a gas. By means of heat, nearly all bodies may be finally vaporized.

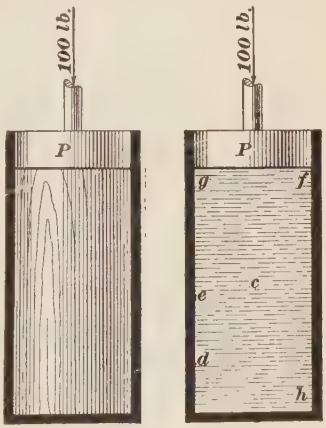
MECHANICS OF FLUIDS

HYDROSTATICS

18. Hydrostatics treats of liquids at rest under the action of forces.

19. Liquids are very nearly *incompressible.* A pressure of 15 pounds per square inch compresses water less than $\frac{1}{20000}$ of its volume.

20. Hydrostatic Pressure.— Fig. 16 represents two cylindrical vessels of exactly the same size. The vessel (*a*) is fitted with a wooden block of the same size as, and free to move in, the cylinder; the vessel (*b*) is filled with water, whose depth is the same as the length of the wooden block in (*a*). Both vessels are fitted with air-tight pistons *P*, each of whose areas are 10 square inches.



(*a*) FIG. 16 (*b*)

Suppose, for convenience, that the weights of the pistons, block, and water are neglected, and that a force of 100 pounds is applied to both pistons. The pressure per square inch will be $100 \div 10 = 10$ pounds. In the vessel (*a*), this pressure will be transmitted to the bottom of the vessel, and will be 10 pounds per square inch; it is easy to see that there will be no pressure on the sides. In the vessel (*b*), an entirely different result is obtained. The pressure on the bottom will be the same as in the other case, that is, 10 pounds per square inch, but, owing to the fact that the molecules of the water are perfectly free to move, this pressure of 10 pounds per square inch is *transmitted in every direction with the same intensity;* that is to say, the pressure at

any point *c*, *d*, *e*, *f*, *g*, *h*, etc., due to the force of 100 pounds, is exactly the same, and equals 10 pounds per square inch. In the foregoing explanation, it was assumed that the water possessed no weight. As a matter of fact, however, the weight of water must be taken into consideration in practice, and the pressure due to the weight of a column of water must be added to the applied pressure. Under such conditions, the pressure will be found greatest at the bottom of the cylinder and will decrease in intensity toward the top of the cylinder.

21. The fact that water transmits pressure in every direction with equal intensity may be easily proved, experimentally, by means of an apparatus like that shown in Fig. 17. Let the area of the piston *a* be 20 square inches; of *b*, 7 square inches; of *c*, 1 square inch; of *d*, 6 square inches; of *e*, 8 square inches; and of *f*, 4 square inches.

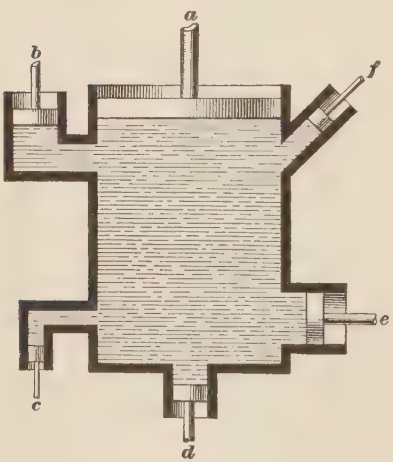


FIG. 17

If the pressure due to the weight of the water is neglected, and a force of 5 pounds is applied at *c* (whose area is 1 square inch), a pressure of 5 pounds per square inch will be transmitted in all directions, and in order that there shall be no movement, a force of $6 \times 5 = 30$ pounds must be applied at *d*, 40 pounds at *e*, 20 pounds at *f*, 100 pounds at *a*, and 35 pounds at *b*.

If a force of 99 pounds were applied to *a*, instead of 100 pounds, the piston *a* would rise, and the other pistons *b*, *c*, *d*, *e*, and *f* would move inwards; but, if the force applied to *a* were 100 pounds, they would all be in equilibrium. If 101 pounds were applied at *a*, the pressure per square inch

would be $101 \div 20 = 5.05$ pounds, which would be transmitted in all directions; and, since the pressure due to the load on c is only 5 pounds per square inch, it is now evident that the piston a will move downwards, and the pistons b, c, d, e, and f will be forced outwards.

22. Pascal's Law.—The whole of the preceding explanation of water pressure may be summed up as follows:

The pressure per unit of area exerted anywhere on a liquid is transmitted undiminished in all directions and acts with the same force on all surfaces, in a direction at right angles to those surfaces.

This law, first discovered by Pascal, and accordingly named after him, is the most important one in hydrostatics. Its meaning should be thoroughly understood.

EXAMPLE.—If the area of the piston e, Fig. 17, were 8.25 square inches and a force of 150 pounds were applied to it, what forces would have to be applied to the other pistons to keep the water in equilibrium, assuming that their areas were the same as given before?

SOLUTION.— $150 \div 8.25 = 18.182$ lb. per sq. in., nearly

$20 \times 18.182 = 363.64$ lb., force to balance a
$7 \times 18.182 = 127.274$ lb., force to balance b
$1 \times 18.182 = 18.182$ lb., force to balance c
$6 \times 18.182 = 109.092$ lb., force to balance d
$4 \times 18.182 = 72.728$ lb., force to balance f

} Ans.

23. *The pressure due to the weight of a liquid may be downwards, upwards, or sidewise.*

24. Downward Pressure.—In Fig. 18, the pressure on the bottom of the vessel (a) is, of course, equal to the weight of the water it contains. If the area of the bottom of the vessel (b) and the depth of the liquid contained in it are the same as in the vessel (a), the pressure on the bottom of (b) will be the same as on the bottom of (a). Suppose that the bottoms of the vessels (a)

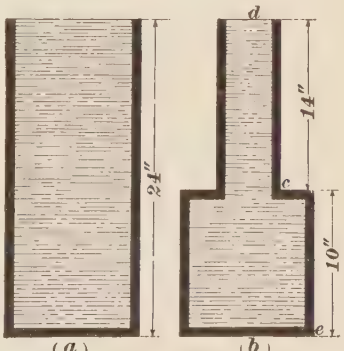


FIG. 18

and (b) are each 6 inches square, and that the part $c\,d$ in the vessel (b) is 2 inches square, and that they are filled with water. Then, since 1 cubic foot of water weighs 62.5 pounds, the weight of 1 cubic inch of water is $62.5 \div 1{,}728 = .03617$ pound. The number of cubic inches in (a) is $6 \times 6 \times 24 = 864$ cubic inches; and the weight of the water is $864 \times .03617 = 31.25$ pounds. Hence, the total pressure on the bottom of the vessel a is 31.25 pounds, or

$$\frac{31.25}{6 \times 6} = .868 \text{ pound per square inch}$$

The pressure in (b) due to the weight contained in the part $b\,c$ is

$$6 \times 6 \times 10 \times .03617 = 13.02 \text{ pounds}$$

The weight of the part contained in $c\,d$ is

$$2 \times 2 \times 14 \times .03617 = 2.0255 \text{ pounds}$$

and the weight per square inch of area in $c\,d$ is

$$2.0255 \div 4 = .5064 \text{ pound}$$

25. According to Pascal's law, this weight (pressure) is transmitted equally in all directions; therefore, an extra weight of .5064 pound is imposed on every square inch of the bottom of (b); the area of this is $6 \times 6 = 36$ square inches, and the pressure on it due to the water contained in $c\,d$ is, therefore, $36 \times .5064 = 18.23$ pounds; thus, we have a total pressure on the bottom of vessel (b) of $13.02 + 18.23 = 31.25$ pounds, the same as in vessel (a). As a result of this law, there is also an upward pressure of .5064 pound acting on every square inch of the top of the enlarged part (b).

If an additional pressure of 10 pounds per square inch were applied to the upper surface of both vessels, the total pressure on each bottom would be

$$31.25 + (6 \times 6 \times 10) = 31.25 + 360 = 391.25 \text{ pounds}$$

If this pressure were to be obtained by means of a weight placed on each piston [as shown in Fig. 16 (a) and (b)], a weight of $6 \times 6 \times 10 = 360$ pounds would have to be put on the piston in the vessel in Fig. 18 (a) and one of $2 \times 2 \times 10 = 40$ pounds on the piston in the vessel (b).

26. General Law for Downward Pressure on Bottom of Any Vessel.

Law.—*The pressure on the bottom of a vessel containing a fluid is independent of the shape of the vessel, and is equal to the weight of a column of the fluid, the area of whose base is equal to that of the bottom of the vessel and whose altitude is the distance between the bottom and the upper surface of the fluid, increased by the pressure per unit of area on the upper surface of the fluid multiplied by the area of the bottom of the vessel, in case there is any pressure on the surface.*

27. Suppose that the vessel (*b*) in Fig. 18 is inverted, as shown in Fig. 19, the pressure on the bottom will still be .868 pound per square inch, but it will require a weight of 3,490 pounds on a piston at the upper surface to make the pressure on the bottom 391.25 pounds, instead of a weight of 40 pounds, as in the other case.

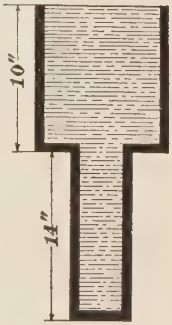


FIG. 19

EXAMPLE.—A vessel filled with salt water, weighing .037254 pound per cubic inch, has a circular bottom 13 inches in diameter. The top of the vessel is fitted with a piston 3 inches in diameter, on which is laid a weight of 75 pounds; what is the total pressure on the bottom, if the depth of the water is 18 inches?

SOLUTION.—Applying the rule, we have

$$13 \times 13 \times .7854 \times 18 \times .037254 = 89.01 \text{ lb.}$$

the pressure due to the weight of the water.

$$\frac{75}{3 \times 3 \times .7854} = 10.61 \text{ lb. per sq. in.}$$

due to the weight on the piston.

$$13 \times 13 \times .7854 \times 10.61 = 1{,}408.29 \text{ lb.}$$

$$1{,}408.29 + 89.01 = 1{,}497.3 \text{ lb.} = \text{total pressure.} \quad \text{Ans.}$$

28. Upward Pressure.—In Fig. 20 is represented a vessel of exactly the same size as that shown in Fig. 19. There is no upward pressure on the surface *c* due to the weight of the water in the large part *c d*, but there is an upward pressure on *c* due to the weight of the water in the small part *b c*. The pressure per square inch due to the

weight of the water in $b\,c$ was found, in Art. **24,** to be .5064 pound; the area of the upper surface c of the large part $c\,d$ is

$$(6 \times 6) - (2 \times 2) = 36 - 4$$
$$= 32 \text{ square inches}$$

and the total upward pressure due to the weight of the water is $.5064 \times 32 = 16.2$ pounds.

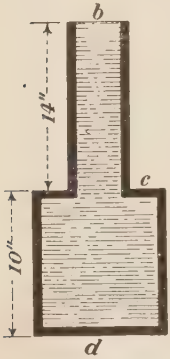


FIG. 20

If an additional pressure of 10 pounds per square inch were applied to a piston fitting in the top of the vessel, the total upward pressure on the surface c would be

$$16.2 + (32 \times 10) = 336.2 \text{ pounds}$$

29. General Law for Upward Pressure.

Law.—*The upward pressure on any submerged horizontal surface equals the weight of a column of the liquid whose base has an area equal to the area of the submerged surface and whose altitude is the distance between the submerged surface and the upper surface of the liquid, increased by the pressure per unit of area on the upper surface of the fluid multiplied by the area of the submerged surface, in case of any pressure on the upper surface.*

EXAMPLE.—A horizontal surface 6 inches by 4 inches is submerged in a vessel of water 26 inches below the upper surface; if the pressure on the water is 16 pounds per square inch, what is the total upward pressure on the horizontal surface?

SOLUTION.—Applying the rule, we get for the upward pressure due to the weight of the water, and

$$6 \times 4 \times 26 \times .03617 = 22.57 \text{ lb.}$$

and for the upward pressure due to the outside pressure of 16 lb. per sq. in.,

$$6 \times 4 \times 16 = 384 \text{ lb.}$$

Therefore, $384 + 22.57 = 406.57$ lb., the total upward pressure.

Ans.

30. Lateral (Sidewise) Pressure.—Suppose that the top of the vessel shown in Fig. 21 is 10 inches square and that each of the projections a and b is 1 inch square. The pressure per square inch on the bottom of the vessel due to the weight of the liquid is $1 \times 1 \times 18 \times$ the weight of a

cubic inch of the liquid. The pressure at a depth equal to the distance of the upper surface of b is $1 \times 1 \times 17 \times$ the weight of a cubic inch of the liquid.

Since both of these pressures are transmitted in every direction, they are also transmitted sidewise, and the *pressure per unit of area on the projection b is a mean between the two* and equals $1 \times 1 \times 17\frac{1}{2}$ $\times$ the weight of a cubic inch of the liquid.

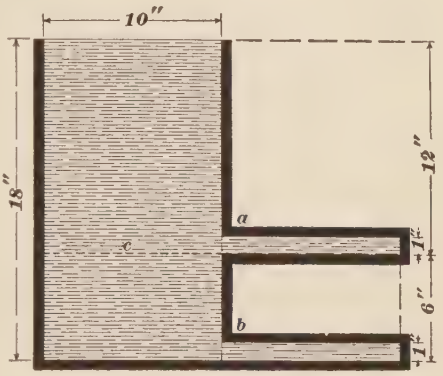


FIG. 21

To find the lateral pressure on the projection a, imagine that the dotted line c is the bottom of the vessel; then the conditions will be the same as in the preceding case, except that the depth is not so great.

The lateral pressure on a is thus seen to be $1 \times 1 \times 11\frac{1}{2}$ $\times$ the weight of a cubic inch of the liquid.

EXAMPLE.—A well 3 feet in diameter and 20 feet deep is filled with water: (*a*) What is the pressure on a strip of the wall 1 inch wide, extending around the well and having its center 1 foot from the bottom? (*b*) What is the pressure on the bottom? (*c*) What is the upward pressure per square inch 2 feet 6 inches from the bottom?

SOLUTION.—(*a*) The area of the strip is

$$1 \times 36 \times 3.1416 = 113.1 \text{ sq. in.}$$

The total pressure on the strip is

$$113.1 \times 19 \times 12 \times .03617 = 932.71 \text{ lb.} \quad \text{Ans.}$$

(*b*) The pressure per square inch is $932.71 \div 113.1 = 8.247$ lb., nearly. The pressure on the bottom is

$$36 \times 36 \times .7854 \times 20 \times 12 \times .03617 = 8{,}836 \text{ lb.} \quad \text{Ans.}$$

(*c*) $20 - 2.5 = 17.5$. The upward pressure per square inch 2 ft. 6 in. from the bottom is

$$1 \times 17.5 \times 12 \times .03617 = 7.596 \text{ lb.} \quad \text{Ans.}$$

31. A tall vessel a having a stop-cock b near its base and arranged to float on the water, as shown in Fig. 22,

illustrates the effects of lateral pressure. When this vessel is filled with water, the lateral pressures at any two points of the surface of the vessel opposite each other are equal. Being equal and acting in opposite directions, they balance each other, and no motion can result; but if the stop-cock is opened, there will be no resistance to that pressure acting on the surface equal to the area of the opening, and it will cause the water to flow out, while its equal and opposite force will cause the vessel to move through the water in a direction opposite to that of the spouting water.

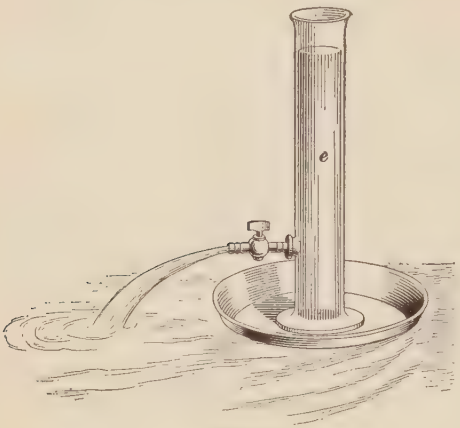


FIG. 22

32. Since the pressure on the bottom of a vessel due to the weight of the liquid is dependent only on the height

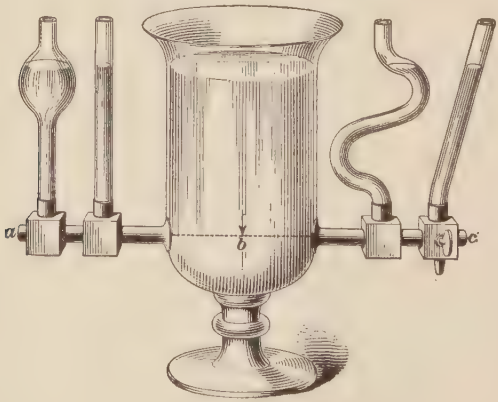


FIG. 23

of the liquid, and not on the shape of the vessel, it follows that if a vessel has a number of radiating tubes, as shown in

Fig. 23, the water in each tube will be on the same level, no matter what may be the shape of the tubes. For, if the water were higher in one tube than in the others, the downward pressure on the bottom due to the height of the water in this tube would be greater than that due to the height of the water in the other tubes. Consequently, the upward pressure would also be greater, the equilibrium would be destroyed, and the water would flow from this tube into the vessel, and rise in the other tubes until it was at the same level in all, when it would be in equilibrium. This principle is expressed in the familiar saying, *water seeks its level.*

The foregoing principle explains why city water reservoirs are located on high elevations, and why water on leaving the hose nozzle spouts so high.

If there where no resistance by friction and air, the water would spout to a height equal to the level of the water in the reservoirs. If a long pipe whose length was equal to the vertical distance between the nozzle and the level of the water in the reservoir were attached to the nozzle, the water would just reach the end of the pipe. If the pipe were lowered slightly, the water would trickle out. Fountains, canal locks, and artesian wells are examples of the application of this principle.

EXAMPLE.—The water level in a city reservoir is 150 feet above the level of the street; what is the pressure of the water per square inch on the hydrant?

SOLUTION.— $1 \times 150 \times 12 \times .03617 = 65.106$ lb. per sq. in. Ans.

NOTE.—In measuring the height of the water to find the pressure that it produces, the *vertical height*, or distance between the level of the water and the point considered, is always taken; this vertical height is called the *head*. The weight of a column of water 1 inch square and 1 foot high is $62.5 \div 144 = .434$ pound, nearly. Hence, if the depth (head) is given, the pressure per square inch may be found by multiplying the depth in feet by .434. The constant .434 is the one ordinarily used in practical calculations.

33. In Fig. 24, let the area of the piston *a* be 1 square inch; of *b*, 40 square inches. According to Pascal's law, 1 pound placed on *a* will balance 40 pounds placed on *b*.

Suppose that *a* moves downwards 10 inches, then 10 cubic inches of water will be forced into the tube *b*. This will be distributed over the entire area of the tube *b*, in the form of

a cylinder whose cubical contents must be 10 cubic inches,

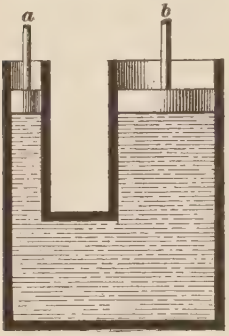


Fig. 24

whose base has an area of 40 square inches, and whose altitude must be $10 \div 40 = \frac{1}{4}$ inch; that is, a movement of 10 inches of the piston *a* will cause a movement of $\frac{1}{4}$ inch in the piston *b*. This is another illustration of the well-known principle of machines: *The force multiplied by the distance through which it acts equals the weight multiplied by the distance through which it moves*, since, if 1 pound on the piston *a* represents the force *P*, the equivalent weight *W* on *b* may be obtained from the equation $W \times \frac{1}{4} = P \times 10$, whence $W = 40\ P = 40$ pounds.

BUOYANT EFFECTS OF WATER

34. In Fig. 25 is shown a 6-inch cube entirely submerged in water. The lateral pressures are equal and in opposite directions. The upward pressure acting on the lower surface of the cube is $6 \times 6 \times 21 \times .03617$; the downward pressure acting on the top of the cube is $6 \times 6 \times 15 \times .03617$; and the difference is $6 \times 6 \times 6 \times .03617$, which equals the volume of the cube in cubic inches times the weight of 1 cubic inch of water. That is, the upward pressure exceeds the downward pressure by the weight of a volume of water equal to the volume of the body.

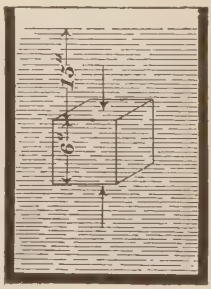


Fig. 25

This excess of upward pressure over the downward pressure acts against gravity; that is, the water presses the body upwards with a greater force than it presses it downwards; consequently, *if a body is immersed in a fluid, it will lose in weight an amount equal to the weight of the fluid it displaces.* This is called the **principle of Archimedes,** because it was first stated by him.

This principle may be experimentally demonstrated with the beam scales, as shown in Fig. 26. From one scale pan suspend a hollow cylinder of metal *t*, and below that a solid cylinder *a* of the same size as the hollow part of the upper cylinder. Put weights in the other scale pan until they exactly balance the two cylinders. If *a* is immersed in water, the scale pan containing the weights will descend, showing that *a* has lost some of its weight. Now, fill *t* with water, and the volume of water that can be poured into *t* will equal that displaced by *a*. The scale pan that contains the weights will gradually rise until *t* is filled, when the scales again balance.

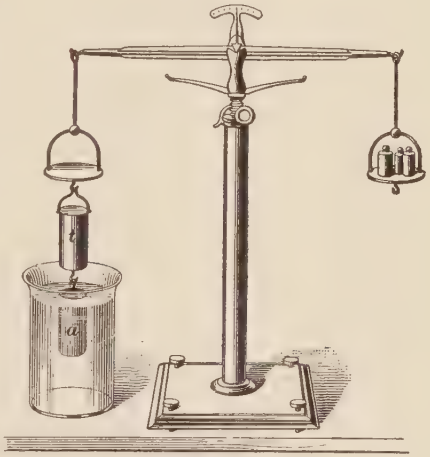


Fig. 26

If a body is lighter than the liquid in which it is immersed, the upward pressure will cause it to rise and project partly out of the liquid, until the weight of the body and the weight of the liquid displaced are equal. If the immersed body is heavier than the liquid, the downward pressure plus the weight of the body will be greater than the upward pressure, and the body will fall downwards until it touches bottom or meets an obstruction. If the weights of equal volumes of the liquid and the body are equal, the body will remain stationary and will be in equilibrium in any position or depth beneath the surface of the liquid.

35. An interesting experiment in confirmation of the foregoing facts may be performed as follows: Drop an egg into a glass jar filled with fresh water. The mean density of the egg being a little greater than that of the water, it will

fall to the bottom of the jar. Now, dissolve salt in the water, stirring it so as to mix the fresh and salt water. The salt water will presently become denser than the egg and the egg will rise. Now, if fresh water is poured in until the egg and water have the same density, the egg will remain stationary in any position that it may be placed below the surface of the water.

36. The principle of Archimedes gives a very easy and accurate method of finding the volume of an irregularly shaped body. Thus, subtract its weight in water from its weight in air and divide by .03617; the quotient will be the volume in cubic inches; or divide by 62.5 and the quotient will be the volume in cubic feet.

If the specific gravity of the body is known, its cubical contents can be found by dividing its weight by its specific gravity, and then dividing again by either .03617 or 62.5.

EXAMPLE.—A certain body has a specific gravity of 4.38 and weighs 76 pounds; how many cubic inches are there in the body?

SOLUTION.— $\frac{76}{4.38 \times .03617} = 479.72$ cu. in. Ans.

GRAVITATION

37. Every body in the universe exerts on every other body a certain attractive force that tends to draw the two bodies together; this attractive force is called **gravitation.** If a body is held in the hand, a downward pull is felt, and if released, the body will fall to the ground. This pull is commonly called **weight,** but it really is the attraction between the earth and the body.

38. Specific Gravity.—The ratio between the weight of a body and the weight of a like volume of water is called its **specific gravity.** The formula for the specific gravity of a liquid is therefore as follows:

$$\text{specific gravity} = \frac{\text{actual weight of a liquid}}{\text{weight of an equal volume of water}}$$

The actual weight of a liquid can be found when its volume and its specific gravity are known since the actual

weight is equal to the weight of an equal volume of water multiplied by the specific gravity of the liquid. The weight of water is usually taken as 8.355 pounds to the gallon.

EXAMPLE.—What is the weight of a liquid whose specific gravity is .79416 that fills a cistern whose capacity is 1,200 gallons?

SOLUTION.—If the cistern holds 1,200 gal. of liquid, it will hold 1,200 gal. of water.

$1{,}200 \times 8.355 = 10{,}026$ lb., weight of water;

$10{,}026 \times .79416 = 7{,}962.25$ lb., weight of liquid. Ans.

39. Determining Specific Gravity of Liquids.—To find the specific gravity of any particular liquid, compared with that of water, it is only necessary to weigh equal bulks and divide the weight of the liquid by the weight of the water. To weigh equal bulks of liquid, the simplest and most accurate way is to weigh them in succession in the same vessel, taking care that it is equally full on both occasions.

FIG. 27

The vessel commonly used when small quantities are to be weighed is shown in Fig. 27. It is easily made by blowing a bulb on a glass tube. On that portion of the tube that is narrowed by being drawn out over a flame, a scratch is made with a diamond or a file. The bulb is filled up to the scratch with the liquid, and is then weighed, emptied, cleaned, dried, filled with water, and again weighed. In these experiments, care should always be taken that both liquids have the same temperature. This is easily accomplished by immersing the bulb filled with liquid for some time in water, part of which is later used in the second weighing.

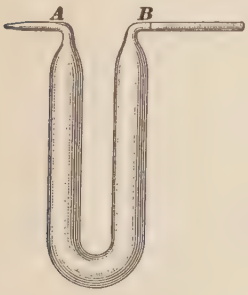


FIG. 28

A form of apparatus, devised by Doctor Sprengel, for determining the specific gravity of liquids, consists of an elongated **U** tube, Fig. 28, the ends of which terminate in the two capillary tubes A, B, bent at right angles in opposite

directions. The shorter tube *A* is narrower at the end than the longer one. The horizontal part of the wider tube is marked near the bend with a fine line. The **U** tube is filled by suction, the little bulb apparatus, Fig. 29, having been previously attached to the narrow capillary tube by a piece of rubber tubing. It is then detached from the bulb, placed in water almost to the bends of the capillary tubes, left there until it has assumed the temperature of the water, and after careful adjustment of the volume of the liquid up to the mark in the wider capillary tube, it is taken out, dried, and weighed.

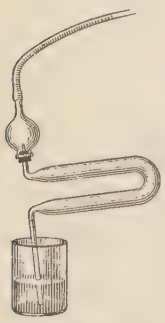


FIG. 29

40. Hydrometers.—Instruments called hydrometers are in general use for determining quickly and accurately the specific gravities of liquids and some forms of solids. There are two kinds; viz., hydrometers of constant weight, as Baumé's; hydrometers of constant volume, as Nicholson's.

A hydrometer of constant weight is shown in Fig. 30. It consists of a glass tube, near the bottom of which are two bulbs. The lower and smaller bulb is loaded with mercury or shot, so as to cause the instrument to remain in a vertical position when placed in the liquid. The upper bulb is filled with air, and its volume is such that the whole instrument is lighter than an equal volume of water.

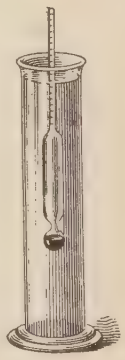


FIG. 30

The point to which the hydrometer sinks when placed in water is usually marked, the tube being graduated above and below in such a manner that the specific gravity of the liquid can be read directly. It is customary to have two instruments: one with the zero point near the top of the stem, for use in liquids heavier than water, and the other with the zero point near the bulb, for use in liquids lighter than water. These instruments are more commonly used for determining the degree of concentration or dilution of certain liquids, as

acids, alcohol, milk, solutions of sugar, etc., rather than their actual specific gravities. They are known as *acidimeters*, *alcoholmeters*, *lactometers*, *saccharimeters*, etc., according to the use to which they are put. These instruments are graded in different ways according to the purpose for which they are made. They either show directly the specific gravity of the liquid, that of water being called 1, or they are graduated in *degrees Baumé*, *Twaddel*, etc.

The hydrometer used by the United States internal-revenue service for determining the percentage of proof of alcoholic liquids is so graduated that the scale indicates zero when immersed in pure water at 60° F.; 200 in absolute alcohol at 60° F.; and 100 in proof spirits at 60° F. The introduction of any substance into an alcoholic liquid either increases or diminishes the specific gravity. Consequently, the hydrometer will sink lower in the mixture if its specific gravity is less than that of the alcoholic liquid before the introduction of any substance. If the specific gravity of the mixture is increased by the introduction of any substance, the hydrometer will not sink so far as it did in the alcoholic liquid.

41. Using a Hydrometer.—In order to use a hydrometer, proceed as follows: Fill the hydrometer jar to the mark with the liquid under examination, allow the filled jar to stand for about 2 minutes, then return the liquid into the original container; fill again the hydrometer jar; let it stand for a minute or two. Now take the temperature of the liquid and note. Carefully drop the hydrometer in the liquid, let it come to rest, and read off the graduation mark. This is sometimes rather difficult, as air bubbles gather around the stem of the hydrometer and the liquid rises up the stem owing to capillary attraction. It has become the rule to read the first line below the surface and to call this the true specific gravity.

42. Capillary Attraction.—Capillary attraction is due to the molecular forces, adhesion and cohesion, which produce disturbing effects on the surface of a liquid. When a solid

body is immersed in a liquid, the liquid is elevated above or depressed below the general level.

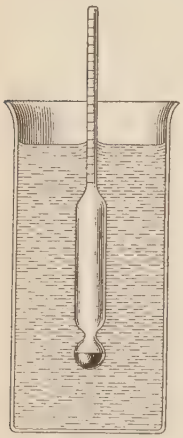


FIG. 31

Adhesion is the molecular attraction exerted between the surfaces of bodies in contact by which they stick together.

Cohesion is the force that unites adjacent molecules of the same nature so that the body resists being pulled to pieces.

The liquid in the vessel, Fig. 31, is elevated in a concave curve by the side of the hydrometer because the adhesion of the molecules of the liquid for those of the solid (glass hydrometer) is greater than the cohesion between the molecules of the liquid. The internal-revenue service requires that the first line below the general surface be read on the hydrometer.

PNEUMATICS

PRESSURE OF GASES

43. **Pneumatics** is that branch of mechanics that treats of the properties and pressures of gases.

44. The most striking feature of all gases is their great expansibility. *If we inject a quantity of gas, however small, into a vessel, it will expand and fill that vessel.* If a bladder or a football is partly filled with air and placed under a glass jar (called a receiver), from which the air has been exhausted, the bladder or football will immediately expand, as shown in Fig. 32. The

FIG. 32

force that a gas always exerts when confined in a limited space is called *tension*. The word **tension** in this case means pressure, and is only used in this sense in reference to gases.

45. As water is the most common type of fluids, so air is the most common type of gases. It was supposed by the ancients that air had no weight, and it was not until about the year 1650 that the contrary was proved. The ratio of the weight of a volume of air at 60° F., under atmospheric pressure, to that of an equal volume of water under the same conditions, is about 1 : 816; that is, under these conditions air is only about $\frac{1}{816}$ as heavy as water. It has been shown that if a body is immersed in water and weighs less than the volume of water displaced, the body will rise and project partly out of the water. The same is true, to a certain extent, of air. If a vessel made of light material is filled with a gas lighter than air, so that the total weight of the vessel and gas is less than the air they displace, the vessel will rise. It is on this principle that baloons are made.

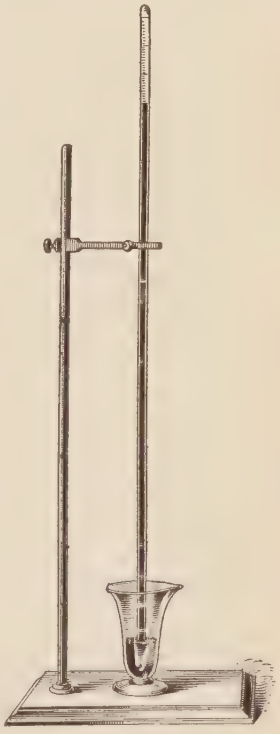


Fig. 33

46. Pressure of the Atmosphere.—Since air has weight, it is evident that the enormous quantity of air that constitutes the atmosphere must exert a considerable pressure on the earth. This is easily proved by taking a long glass tube closed at one end and filling it with mercury. If the finger is placed over the open end so as to keep the mercury from running out, the tube inverted and placed in a cup of mercury, as shown in Fig. 33, the mercury will fall, then rise, and after a few oscillations

will come to rest at a height of 29.92, or roughly 30 inches, at sea level, above the top of the mercury in the cup.

Now, if the atmosphere has weight, it must press on every square unit of the surface of the mercury in the cup with equal intensity, except on that part of the surface covered by the tube. According to Pascal's law, which is given in Art. **22**, this pressure is transmitted equally in all directions. There being nothing in the tube, except the mercury, to counterbalance the upward pressure of the air, the mercury falls in the tube until it exerts a downward pressure on the upper surface of the mercury in the cup sufficiently great to counterbalance the upward pressure produced by the atmosphere. In order that there shall be equilibrium, the pressure of the air per unit of area on the upward surface of the mercury in the cup must be equal to the pressure exerted per unit of area by the mercury inside the tube at the level of the surface of the mercury in the cup.

Suppose that the area of the inside of the tube is 1 square inch, then, since mercury is 13.6 times as heavy as water, the weight of the mercurial column is

$$.03617 \times 13.6 \times 30 = 14.7574 \text{ pounds}$$

The actual height of the mercury is a little less than 30 inches, and the actual weight of a cubic inch of distilled water is a little less than .03617 pound. When these considerations are taken into account, the average weight of the mercurial column at the level of the sea, when the temperature is 60° F., is 14.69 pounds, or practically 14.7 pounds. Since this weight, when exerted on 1 square inch of the liquid in the glass, just produces equilibrium, it is plain that the pressure of the outside air is 14.7 pounds on every square inch of surface.

47. Vacuum.—The space between the upper end of the tube and the upper surface of the mercury is called a **vacuum**, meaning that it is an entirely empty space and does not contain any substance—solid, liquid, or gaseous. If there were a gas of some kind there, no matter how small the quantity might be, it would expand and fill the space,

and its tension would cause the column of mercury to fall and become shorter, according to the amount of gas or air present. The condition then existing in the space would be called a **partial vacuum.**

PUMPS

48. Pumps are machines for lifting or conveying fluids.

49. How Water Flows Into a Pump.—Pumps are frequently so located that the fluid must flow into the pump cylinder by atmospheric pressure on the surface of the fluid external to the suction pipe; that is, by the action of the pump, a vacuum of more or less perfection is produced in the pump chamber. If the end of the suction pipe, which is the pipe connecting the pump chamber with the water, is submerged, the excess of pressure on the surface of the water outside of the suction pipe will cause the water to rise in the suction pipe until the pressure due to the weight of the column equals the pressure of the atmosphere.

50. Theoretical Lift of a Pump.—The pressure of the atmosphere is constantly changing. For practical purposes, the pressure at sea level is taken as 30 inches of mercury, or 14.7 pounds pressure per square inch. Since a pressure of 1 pound per square inch is equal to that exerted by a column of water 2.309 feet high, the theoretical height that water can be raised by a perfect vacuum at sea level is $14.7 \times 2.309 = 33.94$ feet. Since the atmospheric pressure becomes less as the altitude increases, it follows that the greater the altitude, the less the theoretical and practical lift by atmospheric pressure will be. To find the theoretical height, in feet, to which water can be lifted at any altitude, multiply the barometric reading in inches by 1.133.

51. For water holding foreign substances in suspension, or for other liquids, the theoretical lift can be found by dividing the theoretical height to which water can be lifted at the existing atmospheric pressure, as shown by the barometer, by the specific gravity of the liquid.

52. Actual Lift of a Pump.—Since a perfect vacuum cannot be obtained on account of mechanical imperfections, air contained in the water, and the vapor of the water itself, the actual height to which it can be lifted is only about .82 of the theoretical height, which ratio is good only for pure water.

53. The Air Pump.—The **air pump** is an instrument for removing air from a given space. A section illustrating the principal parts of a pump is shown in Fig. 34 and a com-

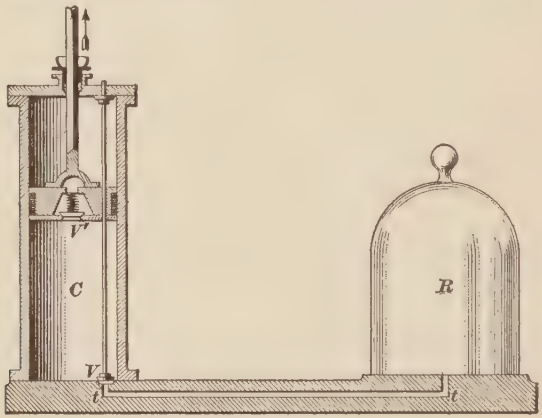


FIG. 34

plete apparatus is shown in Fig. 35. The closed vessel R is called the **receiver,** and the space that it encloses is that from which it is desired to remove the air. The receiver usually is made of glass, and the edges are ground so as to be perfectly air-tight; when made in the form shown, it is called a *bell-jar receiver.* The receiver rests on a horizontal plate, in the center of which is an opening communicating with the pump cylinder C by means of the passage $t\,t$. The pump piston accurately fits the cylinder, and has a valve V' opening upwards. Where the passage $t\,t$ joins the cylinder, another valve V is placed, which also opens upwards. When the piston is raised, the valve V' closes, and, since no air can get into the cylinder from above, the piston leaves a vacuum behind it. The pressure on V being now removed, the

tension of the air in the receiver R causes V to rise; the air in the receiver and passage $t\,t$ then expands so as to occupy the additional space provided by the upward movement of the piston. The piston is now pushed down, the valve V closes, the valve V' opens, and the air in C escapes. The lower valve V is sometimes supported, as shown in Fig. 34, by a metal rod passing through the piston and fitting it somewhat tightly. When the piston is raised or lowered, this rod moves with it. A button near the upper end of the rod confines its motion within very narrow limits, the piston sliding on the rod during the greater part of the journey. In the complete form of the apparatus shown in Fig. 35, communication between receiver and pump is made by means of the tube t.

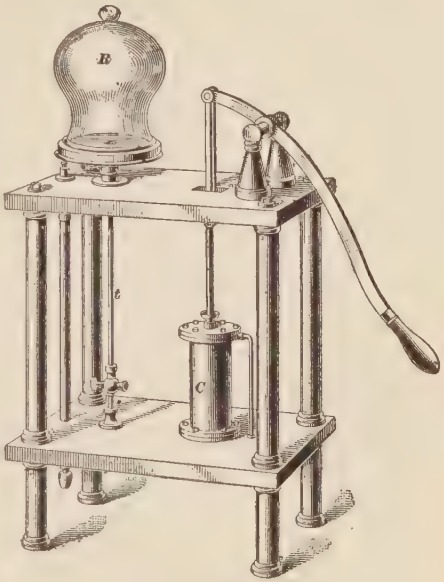


FIG. 35

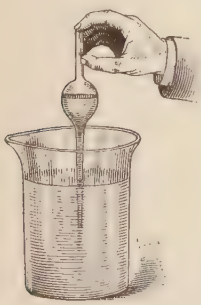


FIG. 36

54. Pipette.—A **pipette,** sometimes called a *thief*, is a glass tube, usually with a bulb or enlargement to it, drawn to a point at one end. This instrument is, as a rule, used to fill the hydrometer jar. It is filled by sucking the liquid up in it and covering the upper end with the finger, as shown in Fig. 36. Then, by a slight release of the pressure of the finger the liquid is allowed to flow slowly out.

THE SIPHON

55. Theory of the Siphon.—The action of the **siphon** illustrates the effect of atmospheric pressure. The siphon is simply a bent tube having unequal legs, open at both ends, and is used to convey a liquid from a higher point to a lower one over an intermediate point that is higher than either. In Fig. 37, *a* and *b* are two vessels, *b* being lower than *a*, and *a c b* is the bent tube, or siphon. Suppose this tube to be filled with water and placed in the vessels as shown, with the short leg *a c* in the vessel *a*. The water will flow from the vessel *a* into the vessel *b* as long as the level of the water in *b* is below the level of the water in *a* and the level of the water in *a* is above the lower end of the tube *a c*. The atmospheric pressure on the surfaces of *a* and *b* tends to force the water up the tubes *a c* and *b c*. When the siphon is filled with water, each of these pressures is counteracted in part by the pressure of the water in that leg of the siphon that is immersed in the water on which the pressure is exerted. The atmospheric pressure opposed to the weight of the longer column of water will, therefore, be resisted more strongly than that opposed to the weight of the shorter column; consequently, the pressure exerted on the shorter column will be greater than that on the longer column, and this excess of pressure will produce motion.

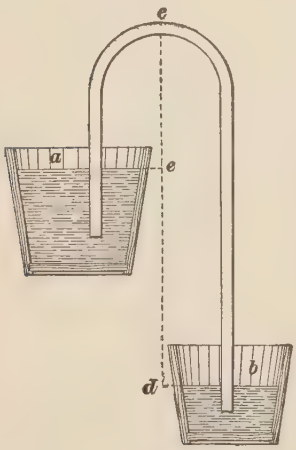
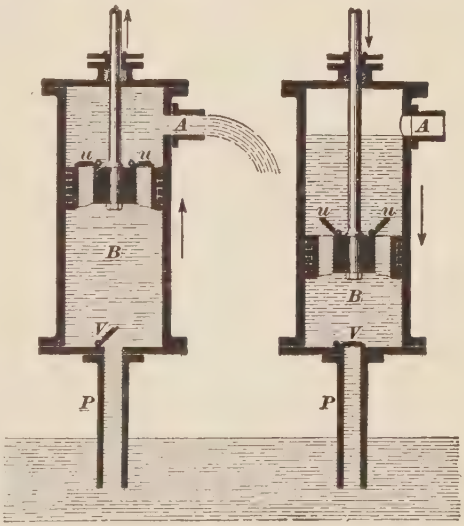


FIG. 37

56. Classification of Pumps.—Pumps may be divided into three general divisions, according to the service they perform; viz., *suction pumps*, *lifting pumps*, and *force pumps*. They may also be divided into two general classes—*single-acting* and *double-acting pumps*—according as they take water

on one side or on both sides of the water piston. According to the arrangement of the pump cylinders, they are classified as *simple*, *duplex*, or *triplex pumps*. As pumps displace the liquids in various ways, they may also be divided according to the method of displacement, into *reciprocating*, *centrifugal*, and *rotary pumps*. Reciprocating pumps only will be considered here.

57. The Suction Pump.—A section of an ordinary suction pump is shown in diagrammatic form in Fig. 38. Suppose that in (*a*) the piston, or **bucket** as it is commonly termed, is at the bottom of the cylinder and just on the point of moving upwards in the direction of the arrow. As the piston rises, it leaves a partial vacuum behind it, and the atmospheric pressure on the surface of the water in the well causes it to rise in the pipe *P*. The water rushes up the pipe and lifts the suction valve *V*, filling the space in the cylinder *B* caused by the displacement of the piston. When the end of the piston stroke has been reached, the water entirely fills the space between the bottom of the piston and the bottom of the cylinder, also the pipe *P*. The instant the piston begins its down stroke, the water in the chamber *B* begins to flow back into the well, and its downward flow forces the valve *V* to its seat, thus preventing any further escape of the water. As the piston descends, the water must give way to it, and since the suction valve *V* is closed, the bucket valves *u*, *u*



(*a*) FIG. 38 (*b*)

must open, and thus allow the water to pass through the piston, as shown in (*b*). When the piston has reached the end of its downward stroke and commenced its upward movement again, the water flowing through the piston quickly closes the valves *u*, *u*. All the water resting on the top of the piston is then lifted by the piston on its upward stroke and discharged through the spout *A*; the valve *V* again opens and the water fills the space below the piston, as before.

58. It is evident that the distance between the piston when at the top of its stroke and the surface of the water in the well must not exceed 34 feet, the highest column of water that the pressure of the atmosphere will sustain, since, otherwise, the water in the pipe will not rise and fill the cylinder as the piston ascends. In practice, this distance shall not exceed 28 feet, because there is a little air left between the bottom of the piston and the bottom of the cylinder, a little air leaks through the valves, which are not perfectly air-tight, and a pressure is needed to raise the valve against its own weight, which, of course, acts downwards.

There are many varieties of the suction pump, differing principally in the construction of the valves and piston, but the principle is the same in all.

59. The Lifting Pump.—In some cases, it is desired to raise water higher than it can be forced by the pressure of the atmosphere into the chamber of a simple suction pump, such as is shown in Fig. 38. To accomplish this, the pump chamber with its bucket and valves *u*, *u* are set at a distance above the supply not exceeding that to which the air will successfully force the water. A closed pipe *P'*, Fig. 39, called the *delivery*, or *discharge*, *pipe*, is then led from the upper part of the chamber to the point where it is desired to deliver the water. Such a pump is often called a **lifting pump.**

In order to prevent the leakage of water around the piston rod, a stuffingbox *S* is provided. The lower end of the discharge pipe *P'* is sometimes fitted with a valve *c* to prevent the water flowing back into the pump chamber; this valve is

not essential to the operation of the pump, however, since the valve V prevents the water in the chamber and discharge pipe from flowing back during the downward motion of the piston.

While it is customary to consider lifting pumps and suction pumps as two types of pumps, there is in reality no difference in their operation, as a careful study of Figs. 38 and 39 will show. The only difference is that the water is discharged by a suction pump at the level of the pump, while a lifting pump discharges the water above the level of the pump.

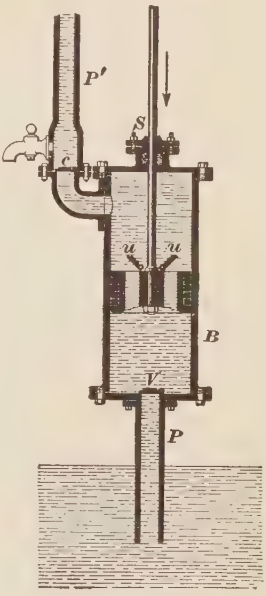
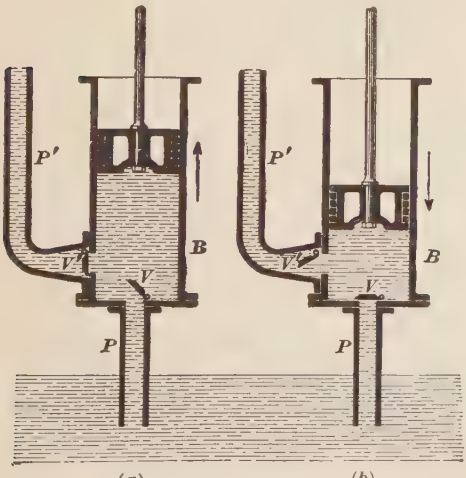


FIG. 39

60. Force Pumps.—The force pump differs from the lifting pump in one important particular, that is, in the fact that its piston is solid. A section of a force pump is shown in Fig. 40. As the piston ascends, as shown in (a), the pressure of the atmosphere forces the water up the suction pipe P; the water opens the suction valve V and flows into the pump cylinder. When the piston moves down, as shown in (b), the suction valve is closed and the delivery valve V'' opened. The water in the pump cylinder is now forced up the delivery pipe P'. When the



(a) FIG. 40 (b)

piston again begins to move upwards, the delivery valve is closed by the water above it and the suction valve opened by the pressure of the atmosphere on the water below it.

61. Plunger Pumps.—When force pumps are used to convey water to great heights or to force water against heavy pressures, the great pressure of the water makes it extremely difficult to keep the water from leaking past the piston, and the constant repairing and renewal of the piston packing becomes a nuisance and involves serious expense. To obviate this drawback, plunger pumps have been designed, one of which is shown diagrammatically in Fig. 41. The action does not differ in any way from that of the piston force pump. During the up stroke of the plunger, the suction valve is open and the delivery valve is closed; during the down stroke, the suction valve is closed and the delivery valve is open.

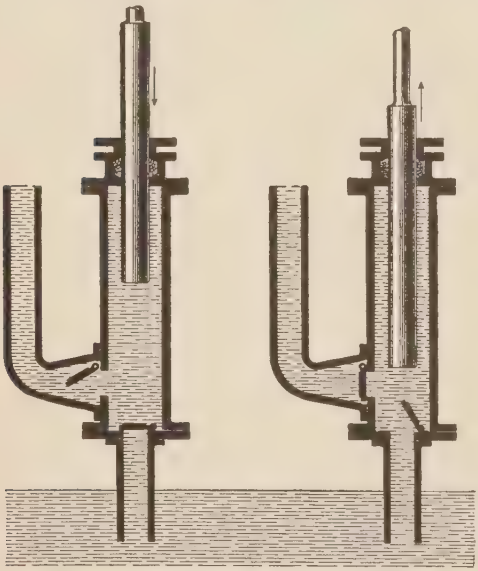


FIG. 41

62. The force pumps shown so far are **single-acting,** that is, the water is forced into the delivery pipe only during the down stroke or forward stroke of the piston or plunger. Force pumps, either of the piston or plunger pattern, may be constructed so as to force water into the delivery pipe both during the forward and return stroke. They are then called **double-acting.**

63. A **double-acting force pump** of the piston pattern is shown in Fig. 42. Such a pump has two sets of suction valves and delivery valves, one set for each side of the piston. With the piston moving in the direction of the arrow, the pressure of the atmosphere forces the water up the suction pipe P into the left-hand end of the pump cylinder, the left-hand suction valve opens and the left-hand delivery valve is closed. The piston in moving to the right, displaces the water in the right-hand end of the pump cylinder; as a consequence, the right-hand suction valve is closed and the right-hand delivery valve opens. The water now flows up the delivery pipe P'.

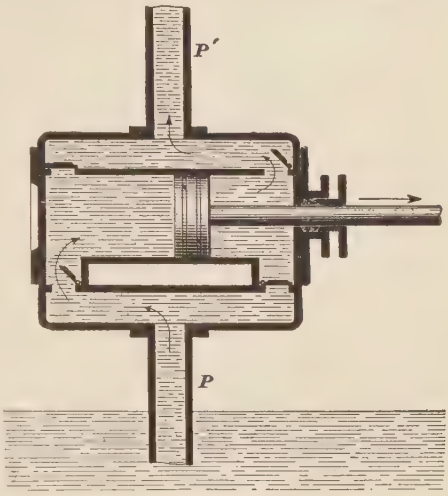


FIG. 42

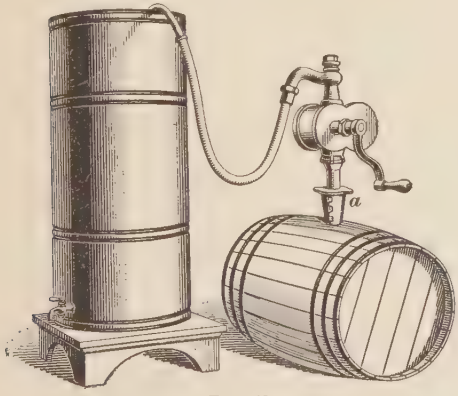


FIG. 43

Imagine that the piston is at the end of its stroke and commences to move to the left. Its first movement promptly closes the left-hand suction valve and opens the left-hand delivery valve. It also closes the right-hand delivery valve and opens the right-hand suction valve. It is thus seen that with the arrangement given, which shows the principle of operation of all double-acting pumps, the piston will

discharge water both during the forward and the return stroke. While the pump shown is a horizontal pump, it may be vertical as well.

64. In Fig. 43 is shown a hand force pump used by oil and liquor dealers. This pump is adapted to all the uses of a lift or force pump. The barrel attachment *A* is tapering and can be used in different-sized casks. With connecting pipe or hose, the fluids can be emptied from the cask and transferred from the cellar to any part of a building.

HEAT

NATURE OF HEAT

65. Although scientists differ as to the exact nature of **heat,** it is assumed that heat is a form of energy and also a kind of motion. It is not the purpose to enter into the various theories regarding heat, but as much of the generally accepted theory will be given as will be necessary to make clear the principles that are to follow.

The minute molecules of which all bodies are composed play a very important part in the theory of heat. Each molecule attracts the molecules surrounding it in a manner similar to the attraction between the earth and bodies near its surface, only with a much greater force in proportion to their sizes. Without going into any theory regarding the precise nature of heat, it will be taken for granted that each molecule has a rapid vibratory motion to and fro, and that all molecules are kept from one another by the attractive force between them; this attractive force is known as *cohesion;* without it, everything throughout the universe would instantly crumble into the finest dust.

The exact shape of molecules has not been determined, but it is generally supposed that they are spherical and that they are a considerable distance apart, compared with their size. Applying heat to a body, increases the vibrations of the molecules in proportion to the amount of heat applied. In consequence of this increase, the distance through which a

molecule moves is increased, and the force of cohesion is proportionally lessened. If enough heat is added to a solid, the force of cohesion is so far overcome that the body becomes liquid, i. e., it melts; and if still more intense heat is applied, the once solid body becomes a vapor. So long as it is kept at this temperature, the force of cohesion has no effect, because the number of vibrations has been so far increased that the distance between any two molecules has become too great for the force of cohesion to act. If the vapor is cooled, the number of vibrations and also the distance between any two molecules will decrease, the force of cohesion will begin to act, and the body will become a liquid. If cooled further and a sufficient quantity of heat is removed (in other words, if the number of vibrations is so far decreased that the molecules are comparatively near together), the body will become a solid and will remain in this state until the temperature is again increased to the melting point.

If a body is heated and brought near the hand, the sensation of warmth is felt; if heat is removed from this same body and it is again brought near the hand, the sensation of cold is felt. The heat that thus manifests itself is called *sensible heat*, because any change from any state to a hotter or colder one is indicated at once by the sense of feeling. The more sensible heat a body possesses, the hotter it is; the more sensible heat taken away from it, the colder it is.

TEMPERATURE

66. The most common heat measure that we have is that gained by means of the sensation of warmth it produces. According to the character of this sensation, a body is said to be cold, warm, or hot. These terms all refer to the power that one body has of communicating heat to other bodies. The measure of this power is termed *temperature*, which may be more exactly embodied in the following definition: *The* **temperature** *of a body is a measure of the intensity of its heat, and is further defined as the thermal state of a body considered with reference to its power of communicating heat to other bodies.*

67. The Thermometer.—The thermometer is an instrument for measuring temperature. For scientific purposes, sensation is not a sufficiently accurate method of measuring temperature; accordingly, temperature is usually measured by the effect produced by heat on certain bodies. Among the effects produced, one of the most convenient to measure is the expansion that most bodies experience with a rise of temperature. This expansion is distinctly perceptible in solids; it occurs to a greater extent in liquids, and most of all with gases. For the general purposes of temperature measurement, mercury and alcohol are the most convenient substances, the former because it boils only at a very high temperature, and the latter because it freezes only at the most intense cold produced by ordinary means. Mercury boils at a temperature of 662° F., and solidifies at a temperature of −40° F. Alcohol boils at a temperature of 172° F. and freezes at a temperature of −203° F. One of these liquids, enclosed in a suitable vessel, constitutes the temperature-measuring instrument termed a **thermometer**—the mercurial thermometer being used to measure temperatures between −97° F. and 212° F., and the alcohol thermometer to measure temperatures below −97° F.

In constructing a thermometer, a bulb is blown at one end of a glass tube having a very narrow bore; the bulb and a portion of this tube are next filled with carefully purified mercury, which is boiled so that all air and moisture is driven out of the tube; the open end is then hermetically sealed by fusing the glass itself. The bulb and a portion of the tube are thus filled with mercury while the remainder of the tube is a vacuum, save for the presence of a minute quantity of mercury vapor. On heating the bulb of this instrument, the mercury expands and rises in the stem. Heat has a tendency to so distribute itself throughout any body or series of bodies as to make them of the same temperature; consequently, if the thermometer is placed in contact with the body whose temperature it is desired to measure, a redistribution of heat occurs until the two are at the same temperature. That is to say, if the body is the colder it receives

heat from the thermometer, and if it is hotter it yields heat to the thermometer, until the temperatures of the two are the same. The two being in efficient contact, this stage is indicated by the mercury becoming stationary in the thermometer tube. Now, the mercury is constant for any one temperature; therefore, to register temperature, it is only necessary to have, further, a scale or series of graduations attached to the stem of the instrument, by which the temperature may always be read.

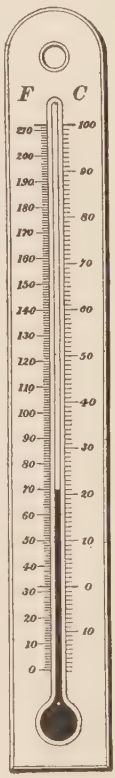


Fig. 44

68. Thermometer Scales.—In Fig. 44 is shown a mercurial thermometer with two sets of graduations on it. The one on the left *F* is the *Fahrenheit thermometer*, so named after its inventor, and is the one commonly used in this country and in England; the one on the right *C* is the *centigrade thermometer*, and is used by scientists throughout the world on account of the graduations being better adapted for calculations.

In graduating thermometers, the two fixed points of temperature almost universally employed are the temperatures of melting ice, and of the steam from boiling water at atmospheric pressure. Certain simple precautions being taken, these temperatures are always constant. It is, in addition, necessary to graduate the thermometers so as to register temperatures intermediate between these two points, and also below and above them. The most convenient system of graduation is that of Celsius, known as the centigrade scale. In this scale, the distance between the melting point of ice (or the freezing point of water, as it is commonly called) and the temperature of steam at atmospheric pressure (this latter temperature is more commonly described as being that of the boiling point of water) is divided into 100 equal graduations, or *degrees*. The freezing point is called zero, or 0°, and the boiling point 100°. Degrees of

the same value are carried above and below the boiling and freezing points, respectively; the temperature below 0° is considered negative, and is counted downwards from zero; thus, 10° below freezing point is −10°, and so on. Degrees above the boiling point are simply counted upwards from zero; thus, 10° above the boiling point is 110°, and so on.

In graduating Fahrenheit thermometers, the distance between the freezing and boiling points is divided into 180 equal parts, and degrees of the same size are carried above and below the boiling and freezing points. Fahrenheit assumed that the greatest cold attainable was 32° below the freezing point, and accordingly took that point as his zero and reckoned from it upwards. The freezing point thus became 32° F., and the boiling point 32 + 180 = 212°. Degrees below the Fahrenheit zero are reckoned downwards from zero and are considered as negative.

In Germany and Russia, temperature is reckoned on the *Réaumur scale*, in which the freezing point is 0° and the boiling point is 80°.

69. Changing Values From One Thermometer Scale to Another.—It is frequently necessary to be able to compare the readings on Fahrenheit and centigrade scales, and to translate temperatures from one to the other. For example, what would 80° C. be on the Fahrenheit scale?

Since the number of degrees between the freezing point and the boiling point on the centigrade scale is 100, and on the Fahrenheit 180, it is evident that, if F equals number of degrees Fahrenheit and C equals number of degrees centigrade,

$$F : C = 180 : 100, \text{ or } F = \frac{180}{100} C = \frac{9}{5} C;$$

also,

$$C = \frac{100}{180} F = \frac{5}{9} F.$$

Therefore, to change centigrade temperatures into their corresponding Fahrenheit values:

Rule I.—*Multiply the temperature, centigrade, by $\frac{9}{5}$ and add 32°; the result will be the temperature, Fahrenheit.*

To change Fahrenheit temperatures into their corresponding centigrade values:

Rule II.—*Subtract 32° from the temperature, Fahrenheit, multiply by* $\frac{5}{9}$, *and the result will be the temperature, centigrade.*

Expressing these two rules by means of formulas, let

t_c = temperature, centigrade;
t_f = temperature, Fahrenheit.

$$t_f = \tfrac{9}{5} \times t_c + 32° \qquad (1)$$

$$t_c = \tfrac{5}{9} \times (t_f - 32°) \qquad (2)$$

EXAMPLE 1.—Change into Fahrenheit temperatures: (*a*) 100° C.; (*b*) 4° C.; (*c*) − 40° C.

SOLUTION.—(*a*) $t_f = \frac{9}{5} \times t_c + 32 = \frac{9}{5} \times 100 + 32 = 212°$ F. **Ans.**
(*b*) $t_f = \frac{9}{5} \times 4 + 32 = 39.2°$ F. **Ans.**
(*c*) $t_f = \frac{9}{5} \times -40 + 32 = -40°$ F. **Ans.**

EXAMPLE 2.—Change into their corresponding centigrade temperatures: (*a*) 60° F.; (*b*) 32° F.; (*c*) − 20° F.

SOLUTION.—(*a*) $t_c = (t_f - 32) \times \frac{5}{9} = (60 - 32) \times \frac{5}{9} = 15\frac{5}{9}°$ C. **Ans.**
(*b*) $t_c = (32 - 32) \times \frac{5}{9} = 0°$ C. **Ans.**
(*c*) $t_c = (-20 - 32) \times \frac{5}{9} = -28\frac{8}{9}°$ C. **Ans.**

70. Expansion of Bodies by Heat.—The volume of any body—solid, liquid, or gaseous—is always changed if the temperature is changed; nearly all bodies expand when heated and contract when cooled. Although the expansion of solids and liquids is very nearly uniform throughout all the ranges of temperature, water is a marked exception to the general rule. If water is cooled from its boiling point, it continually contracts until it reaches 39.2° F., when it begins to expand, until it freezes at 32° F. On the other hand, if water at 32° F. is heated, it contracts until it reaches 39.2° F., when it commences to expand. Therefore, the density of water is greatest where this change occurs.

In taking the specific gravity of liquids, the temperature of such liquids has to be taken into consideration; in the internal-revenue service, the standard of comparison is water at a temperature of 39.2° F. (4° C.). In determining specific gravities of liquids, however, they are usually kept at a temperature of 60° F. (15.5° C.). As it is impossible in

practical work, such as gauging, etc., to obtain always the exact temperature of 60° F., special tables are computed, which show the necessary corrections that have to be made if the temperature is below or above the standard.

There is, of course, a limit of temperature beyond which the specific gravity of liquids cannot be taken correctly. Alcohol, for instance, boils at about 78° C. (172° F.), while water boils at 100° C. (212° F.), and if a mixture of alcohol and water at a temperature near the boiling point of alcohol were taken, erroneous results would be obtained owing to the unequal expansion of the liquids, due to the different temperatures of their boiling point and the evaporation that takes place.

71. Evaporation.—Evaporation may be defined as the transformation of liquid molecules into gaseous ones, and is due to the absorption of heat that raises the temperature of the liquid molecules to that point of latent heat at which they become gasified. There are two kinds of evaporation: namely, *surface evaporation* and *boiling*. The former is entirely due to the influence of the conditions of the atmosphere. A liquid exposed to the atmosphere in an open vessel slowly absorbs at its surface the heat of the atmosphere, which absorption gradually raises the latent heat of the molecules to that temperature at which they change into the state of vapor. It may therefore be stated that surface evaporation depends on decreasing the attraction of the surface molecules caused by the atmosphere surrounding it. The warmer the atmosphere surrounding the molecule, the quicker is the evaporation. It has been further found that surface evaporation depends, to a considerable degree, on the density of the liquid. The less dense a liquid, the quicker does it evaporate; compounds like ether, alcohol, and others having a comparatively low density, evaporate much quicker under the influence of atmospheric conditions than does water, or other liquids of equal or greater densities. The air usually holds in suspension a certain amount of water vapor and owing to this fact has a tendency to absorb moisture from the surface of the

water. The less water vapor held in suspension by the air, or in other words the drier the air, the faster the evaporation. The action of the wind also accelerates evaporation because, as the volume of the air surrounding the liquid becomes saturated, it moves to give place to the drier air. Hot air will absorb more moisture than cold air, and the cause of **dew** is the depositing of the water held in suspension in the atmosphere by the lowering of the temperature during the night.

Experiments have shown that evaporation: (1) increases with a rise in temperature; (2) increases as the elevation above sea level increases; (3) decreases as the elevation decreases.

72. Evaporation by boiling is produced through artificial heat and consequently takes place more uniformly throughout the whole body of the liquid.

The *boiling point* of a liquid depends on three conditions; namely, purity, nature of the vessel, and pressure on the surface.

The mixture of a solid substance with a liquid usually elevates the temperature of the boiling point; for example, when salt is dissolved in water, the mixture boils at a higher temperature than does pure water.

The material of which the containing vessel is made affects the temperature of the boiling point of the liquid; for example, water will boil at a lower temperature in an iron vessel than it will in glass.

The temperature of the boiling point of a liquid varies according to the pressure on its surface. As the elevation increases, the pressure decreases and the temperature of the boiling point is lowered; as the elevation decreases, the pressure increases and the temperature of the boiling point becomes higher. For example, water boils at an elevation of 1,000 feet above the sea level at a temperature of 210° F. and at 212.4° F. at 200 feet below the sea level.

73. Distillation.—Distillation is the process of separating, by heat, the more volatile parts of a substance from

the less volatile parts, and subsequently condensing, by cooling, the vapor thus formed. The purposes of distillation vary to a great extent; it may be employed to separate liquids of different boiling points, as in the instance of alcohol and water, fermented liquors, etc. The apparatus employed varies greatly according to its usage, while the principles on which all forms of apparatus depend are, of course, the same.

One form of apparatus, called a **still**, is shown in Fig. 45. It consists essentially of four parts: the body *a*, a copper

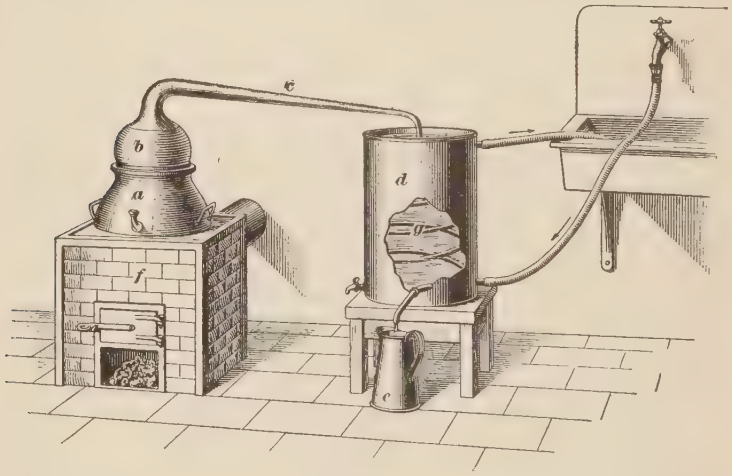


FIG. 45

vessel containing the liquid, the lower part of which fits in the furnace *f*; the head *b*, which fits on the body and from which a lateral tube *c* leads to the worm *g*, a long spiral tin or copper tube placed in a cistern, or tank *d*, kept constantly full of cold water. The worm connects the boiler vessel *a* with the receiving vessel *e*; in the worm, the vapor condenses again to the liquid form and the distilled liquid passes over into the receiving vessel.

SPIRITUOUS LIQUORS

ALCOHOL

74. **Alcohol,** as the term is generally understood, signifies vinous spirits of various strengths and is commonly generated in vegetable juices and infusions by a fermentation. Thus the fermented juice of the grape is called wine; of the apple, cider; and the fermented infusion of malt, beer. The nature of the liquids admitting of vinous fermentation, no matter how they differ in other respects, are all similar, in the respect that they contain sugar in some form or other.

Experience has demonstrated that the sugar contained in these liquids disappears wholly or in part after vinous fermentations and until recently it was believed that the only new products resulting during the progress of fermentation were alcohol and carbonic acid, the latter of which passed off.

Sugar, alone, will not undergo vinous fermentation, but must be dissolved in water subjected to the influence of a ferment and the mixture kept at a constant temperature. The water gives fluidity to the mixture and the ferment and heat begin and maintain the chemical change.

75. The alcohol is obtained from vinous liquors by successive distillations, each distillation giving a purer product. These products form the different *ardent spirits* of commerce.

The spirits obtained from the distillation of wine is called *brandy;* from fermented molasses, *rum;* from cider, corn, or rye, *whisky;* from malted barley, rye, potatoes, and rectified from turpentine, *common gin;* and from fermented rice, *arrack.*

76. The **rectification of alcohol** necessitates three series of distillations. The first comprises the distillation of the alcohol at a temperature of 154° F. This first distillation

gets rid of the ethers and the more volatile alcohols and a part of the other impurities, called *aldehydes*. The second distillation, which is conducted at a temperature of from 154° F. to 212° F., gives a good quality of alcohol, which, however, still contains some of the impurities (aldehydes). The third distillation is conducted at a temperature of from 212° F. to 216° F. The alcohol obtained from this distillation is much better than that from the second and is comparatively free from all impurities.

77. Absolute alcohol is alcohol without any water whatever, and as it absorbs water with great energy from the atmosphere it can scarcely be obtained in commerce. What is sold as absolute alcohol rarely contains more than 98 per cent. pure alcohol. The specific gravity of absolute alcohol at 60° F. is .7939.

78. Spirits of wine is the stronger alcohol that is commonly found in commerce. It contains about 90 per cent. of alcohol and 10 per cent. of water. This name (spirits of wine) was given owing to its being first obtained from the distillation of wine.

WHISKY

79. Whisky, in the strict sense of the word, is the distilled spirits from different grains, such as Indian corn, wheat, rye, and barley. The whisky of this country is commonly made of corn and rye. The term whisky, however, is sometimes extended to other forms of ardent spirits, such as the result of the distillation of cider, which is frequently, although incorrectly, called *apple whisky*.

80. Manufacture of Whisky.—In the manufacture of whisky, the rye or other grain is steeped in water and heated until fermentation takes place, by which the saccharine matter and, indirectly, the starch are converted into alcohol. In this state the resulting mixture is called the **mash.**

The mash is heated in a still and the alcohol contained in the mash becomes vaporized and is carried over to the worm

and there condensed. The condensed liquid contains, besides alcohol, other substances considered impurities, and is commonly called the *low wine*. This condensed liquid is then subjected to a second distillation, the product of which is purer and stronger and now takes the name of *raw corn spirit*, *whisky*, or *high wine*. Sometimes, a third distillation is made to get a still purer product. When this product is permitted to stand for a period of time, chemical changes take place by which the natural impurities still remaining in the liquor are destroyed and the whisky now becomes mellow and loses its disagreeable odor. Whisky, when newly prepared, is nearly colorless, but when kept in casks it gradually acquires a brownish color, which deepens with age.

WINES

81. Most **wines** are made from the juice of the grape. The juice is pressed from the ripe grape by various devices and then runs into vats. This juice is called the **must.** The temperature of the air in the building in which the vats are located should be kept at about 60° F.

Fermentation will now gradually take place in the must and will later become fully established. In the meantime, the must becomes warmer and emits a large quantity of carbonic acid, which causes the more solid parts to be forced to the surface in a mass of froth having a hemispherical shape called the *head*. The liquor, from being sweet, becomes vinous, owing to the conversion of the grape sugar into alcohol. After a while, the fermentation slackens, when it becomes necessary to accelerate it by thoroughly mixing the contents of the vat. After the liquor has acquired a strong vinous taste and becomes perfectly clear, the wine is considered formed and is racked off into casks. Fermentation continues for several months after racking off; during all this period a frothy matter is formed, which for the first few days collects around the bung but afterwards precipitates along with coloring matter and tartar, forming a deposit which constitutes the *wine lees*.

82. Wines are said to be **wet** when they contain sugar and have less of the vinous taste, and they are said to be **dry** when they have a decidedly vinous taste and are free from sugar.

LAGER BEER

83. **Lager beer** is made by a slow process of fermentation from strong steeping of malt, barley and hops and grape sugar or glucose. Beer is usually fermented in winter, as it requires a temperature not higher than 50° F.; and in summer the rooms are cooled by means of ice or ice machines. This kind of fermentation is called sedimentary or under fermentation, in contradistinction to ordinary or surface fermentation, as in wines. The scum, or yeast, collects at the bottom instead of at the surface, giving the air free access and converting the sugar more completely into alcohol.

84. **Method of Manufacture.**—The barley is placed in wooden vats, or tanks, covered with water, and allowed to remain for 2 or 3 days in soak, the water being changed every 24 hours. These vats are then drained and the soaked barley thrown out in heaps on stone floors, where it heats spontaneously and soon begins to germinate, throwing out rootlets and shoots and sweating. After this, the barley is spread out and the germination allowed to proceed for from 6 to 10 days, until the rootlets become brownish; it is then tossed about to cool and check the fermentation. Next, it is put into large brick drying ovens or kilns heated to a temperature of about 125° F., and after it is thoroughly dry, it is called *malt*.

The malt is first crushed by being passed between a series of large rollers and from there is conveyed to the mash tubs where it is stirred with water at 120° to 140° F. and boiling is then gradually incited until all is heated to 170° F. The mixture is now called *wort* and is allowed to stand until the suspended matters have settled, when it is drawn off, and a second wort is obtained by treating the residue with hot water. The first wort is boiled with the hops, the

second wort is then let in and the whole is boiled for about 4 hours. The wort is then run into a cooler, where it is quickly chilled to between 44° and 50° F. by running over small pipes through which cold water is continually flowing. After it reaches this temperature, it is run into the fermenting tuns or large casks, where it is mixed with 1 gallon of yeast to every 20 or 25 barrels. Fermentation will continue for about 20 days, a heavy froth first appearing and then subsiding, leaving the surface clear. At the end of this period, it is racked off into hogsheads, the yeast remaining at the bottom of the tuns. These hogsheads are allowed to stand with the bungs open until a few days before the beer is put into barrels for use, when the bungs are driven in to accumulate carbonic acid for *life*.

RECTIFIED AND PROOF SPIRITS

85. **Rectified spirits** are the ardent spirits rendered purer and stronger by redistillation. Cologne spirits is the highest grade of alcohol, being so purified as to be free of color or odor.

86. **Proof Spirits.**—The Revised Statues of the Treasury Department define *proof spirits* as follows: "**Proof spirits** shall be held to be that alcoholic liquid which contains one-half of the volume of alcohol of a specific gravity of .7939 at 60° Fahrenheit." For example, a mixture of 50 per cent. water and 50 per cent. alcohol is 100-per-cent. proof spirits. A mixture of 75 per cent. water and 25 per cent. alcohol is 50-per-cent. proof spirits. A mixture of 25 per cent. water and 75 per cent. alcohol is 150-per-cent. proof spirits. A mixture of 0 per cent. water and 100 per cent. alcohol is 200-per-cent. proof spirits. A mixture of 100 per cent. water and 0 per cent. alcohol is 0-per-cent. proof spirits. From the foregoing, the following rule is derived:

Rule.—*To obtain the percentage of proof of any alcoholic liquid, multiply the percentage of alcohol by 2 and the result will be the percentage of proof.*

87. A **proof gallon** contains 231 cubic inches of a mixture of equal parts of alcohol and water; as the water, however, is not considered in this case, a proof gallon of 231 cubic inches of an equal mixture of alcohol and water would be called $\frac{1}{2}$ gallon of alcohol. To make this point more clear, assume that a barrel of liquid having a capacity of 15 gallons, contains actually 5 gallons of alcohol and 10 gallons of water; it would be said the barrel contains 10 proof gallons of alcohol (composed of 50 per cent. of alcohol and 50 per cent. of water). In other words, *the number of proof gallons is always twice the number of wine gallons of alcohol the barrel or cask contains.*

To ascertain the proof gallons in a container, the following rule should be used:

Rule.—*Find, by measurement, the number of wine gallons that a container holds; also find, by means of a hydrometer, the percentage of proof of the liquor. Multiply the wine gallons found by the per cent. of the proof and the product will be the actual amount of proof gallons present.*

Let a = amount of wine gallons;
b = percentage of proof in the liquor.

Then, $\frac{a\,b}{100}$ = true amount of proof gallons

EXAMPLE.—A cask contains 59 wine gallons of liquor and by means of a hydrometer it is found that the per cent. of proof is 42; what is the amount of proof gallons in the liquor?

SOLUTION.—Substituting the proper values,

$$\frac{59 \times 42}{100} = 24.78 \text{ proof gal.} \quad \text{Ans.}$$

88. Arts. **86** and **87** should be carefully studied so that the difference between percentage of proof and proof gallons will be clearly understood. In the example in Art. **87**, the percentage of proof is 42; and according to the rule in Art. **86**, the percentage of alcohol contained in the mixture will be $42 \div 2 = 21$. The actual number of wine gallons of alcohol will then be $59 \times .21 = 12.39$. The number of proof gallons, according to Art. **87**, will be $12.39 \times 2 = 24.78$, the same as the answer given.

89. The internal-revenue law requires that a tax of $1.10 per gallon be paid on each proof gallon produced when the spirits are at or above 100-per-cent. proof, and on each wine gallon when the spirits are below proof.

EXAMPLE.—What is the total tax on two casks of spirits, one containing 160 gallons of 90-per-cent. proof spirits and the other containing 70 gallons of 125-per-cent. spirits, the rate of tax being $1.10 per gallon?

SOLUTION.—Since the contents of the first cask is below 100-per-cent. proof, the tax is on the entire contents and is equal to $160 \times \$1.10 = \$176$, and as the contents of the second cask is above 100-per-cent. proof, the tax is on the number of proof gallons, which from the formula of Art. **87** is equal to

$$\frac{70 \times 125}{100} = 87.5$$

The tax on the second cask will then be

$$87.5 \times \$1.10 = \$96.25$$

The total tax on the two casks will be

$$\$176 + \$96.25 = \$272.25. \quad \text{Ans.}$$

90. In computing tax, fractions of a gallon less than .1 are not considered. For example, either 34.49 gallons or 34.41 gallons would be considered as 34.4 gallons.

PRACTICAL EXAMPLES

EXAMPLE 1.—(*a*) How many proof gallons of alcoholic liquid, 108-per-cent. proof, is contained in a tub that measures 4 feet 4 inches at the top, 7 feet 6 inches at the bottom, and is 8 feet 2 inches high? (*b*) How many wine gallons of alcohol does the liquid contain?

SOLUTION.—(*a*)

Diameter, 4 ft. 4 in. = 52 in.
Bottom diameter, 7 ft. 6 in. = 90 in.
$2)\overline{142}$
Midway diameter, 71 in.
Height, 8 ft. 2 in. = 98 in.

According to Art. **13**,

$52 \times 52 = 2704$
$90 \times 90 = 8100$
$71 \times 71 \times 4 = 20164$
$30968 \div 6 = 5{,}161.33$
$5{,}161.33 \times 98 \div 294 = 1{,}720+$ gal.

According to Art. **87**,

$1{,}720 \times 1.08 = 1{,}858-$ proof gal. Ans.

(*b*) According to Art. **86**,

$2)108$

$.54$, percentage of alcohol;

$1{,}720 \times .54 = 929-$ gal. of alcohol

or, $1{,}858 \div 2 = 929$ gal. of alcohol. Ans.

EXAMPLE 2.—What will be the tax on the contents of a cylindrical metal tank made of $\frac{1}{2}$-inch material, containing a 176-per-cent. proof liquid, the outside circumference of the tank being 25 feet 2 inches and the height 11 feet 7 inches?

SOLUTION.— 25 ft. 2 in. = 302 in.; $302 \div 3.1416 = 96.13$ diameter of cylinder.* $96.13 - 1 = 95.13$ in., inside diameter; 11 ft. 7 in. = 139 in., outside height; $139 - 1 = 138$ in., inside height.

$95.13 \times 95.13 \times .7854 = 7{,}108$ in., area of base.*

$7{,}258 \times 138 = 980{,}904$ cu. in.

According to Art. **87**,

$980{,}904 \div 231 = 4{,}246$ gal.; $4{,}246 \times 1.76 = 7{,}472.9$ proof gal.

According to Art. **89**,

$7{,}472.9 \times 1.10 = \$8{,}220.19$. Ans.

EXAMPLE 3.—A cask of spirits weighs 464 pounds, its tare is 39 pounds; the specific gravity of the spirit is .94256, which indicates 42 per cent. alcohol. (*a*) What would be the height of a 4-foot square cistern to contain the contents of five casks of same size? (*b*) How many proof gallons would be contained in the cistern?

SOLUTION.—(*a*) $464 - 39 = 425$ lb., weight of spirits. According to Art. **38**, $425 \div .94256 = 450.90$ lb., equivalent weight of water of same volume. $450.90 \div 8.355 = 53.97$ gal., the volume of one cask; $53.97 \times 5 = 269.85$ gal., the volume in five casks. According to Art. **87**, $269.85 \times 231 = 62{,}335+$ cu. in. 4 ft. 0 in. = 48 in., $48 \times 48 = 2{,}304$ sq. in.

$62{,}335 \div 2{,}304 = 27$ in., height of cistern. Ans.

(*b*) According to Art. **86**, $42 \times 2 = 84$-per-cent. proof; from Art. **87**,

$269.85 \times .84 = 227 -$ proof gal. Ans.

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*The methods obtaining the diameter of a circle and the area of a circle, when given the circumference, are explained in *Arithmetic*, Section 6.

IMMIGRATION LAW

INTRODUCTION

To prepare for a civil service examination for the position of inspector in the Immigration Bureau it is necessary to be thoroughly familiar with the Immigration Laws and Regulations. This subject is an important one in the examination before the Civil Service Commission, and in addition, the knowledge obtained from studying it will be of material benefit to you after entering the service.

This Instruction Paper contains a copy of the act passed by the Congress of the United States, March 31, 1903, governing the admittance of aliens to the United States. Also subsequent acts pertaining to that subject as well as the Regulations of the Bureau of Immigration formulated for the instruction of employes in the conduct of their duties.

Study one section, at a time, of the Act and of the Regulations herein contained, so that you thoroughly understand their intent and meaning, and then take up the next section and so on until you have completed the study of the entire Instruction Paper.

Terms Used.—You will find in the text following some few terms with which you may not be familiar. In order that you may have no difficulty in thoroughly understanding the meaning and intent of these terms a list of the more uncommon ones, with their meaning, is given. Following each word in the list is given the page and section in which the word is used.

affidavit (p. 19, sec. 39), a voluntary sworn declaration in writing made before competent authority.

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alien (p. 3, sec. 1), one who owes allegiance to a foreign government.
appeal (p. 15, sec. 25), to take a cause to a higher court for a rehearing.
bond (p. 16, sec. 26), a surety given to effect the release of an immigrant to cover the expense of maintenance in case he becomes a public charge.
circuit court (p. 16, sec. 29), a court the sessions of which are held at different places in a given circuit.
concurrent (p. 16, sec. 29), united in action.
connive (p. 18, sec. 38), to permit by pretending not to see.
consignee (p. 4, sec. 1), a person to whom goods or other property is delivered into the care of.
conspire (p. 18, sec. 38), to combine secretly in an illegal enterprise.
contiguous (p. 3, sec. 1), adjoining the boundary.
deported (p. 12, sec. 18), carried off or away forcibly.
district court (p. 16, sec. 29), a court presided over by a single judge, having original jurisdiction in civil, criminal, and admiralty cases.
emergency (p. 11, sec. 17), a sudden or unexpected condition calling for immediate action.
epileptic (p. 4, sec. 2), a person affected with a chronic nervous disease characterized in its more violent forms by convulsions attended by loss of consciousness and sensation, facial distortion and foaming at the mouth, convulsions of the limbs, and difficult hard breathing.
guaranty (p. 16, sec. 26), a collateral undertaking by one person to be answerable for the payment of some debt or the performance of some contract or duty by another person, who stands first bound to pay or perform.
head-tax (p. 4, sec. 1), an assessment on each alien entering the United States.
immigrant (p. 4, sec. 1), a person (a foreigner) who enters a country to settle there.
inconsistent (p. 18, sec. 36), disagreeing, contradictory.
jurisdiction (p. 18, sec. 38), lawful power or right to exercise official authority.
lien (p. 4, sec. 1), a legal claim of hold on property as security.
maintenance (p. 12, sec. 19), support, as the support of a family.
manifest (p. 10, sec. 14), a document required of the master of a vessel for the information of custom officers, giving the separate items of the vessel's cargo, the ports of lading and destination, the names of consignees, a list of the passengers and their baggage.
misdemeanor (p. 7, sec. 8), any offense less than a felony, crime of a lesser grade.
moral turpitude (p. 5, sec. 2), inherent moral depravity.
naturalize (p. 19, sec. 39), to confer the rights and privileges of citizenship upon an alien.
parole (p. 6, sec. 5), a verbal pledge, word of honor.

penal institution (p. 14, sec. 22), criminal institution to expiate punishment for criminal offense.

penalty (p. 13, sec. 21), a sum of money fixed by a statute as a fine for a violation of its provisions.

polygamist (p. 5, sec. 2), one who has more than one wife or husband at one time.

preliminary declaration (p. 18, sec. 37), signifying the intention of becoming a citizen before the proper authorities.

proceeding (p. 16, sec. 27), any action at law or in equity instituted in a court.

pursuant (p. 19, sec. 38), done in accordance with.

quarantine (p. 12, sec. 19), the prevention for 40 days of all communications with persons, ships, or goods arriving from ports or places infected with contagious disease or suspected of having such disease on board.

reformatory (p. 13, sec. 22), having a tendency to reform or improve offenders.

suit (p. 16, sec. 27), a judicial action for the recovery of a right or the redress of a wrong.

tribunal (p. 19, sec. 39), a court of justice.

testimony (p. 15, sec. 24), a statement or declaration on oath or affirmation before a judicial tribunal or an officer of the law.

LAWS AND REGULATIONS

ACT OF MARCH 3, 1903

AN ACT TO REGULATE THE IMMIGRATION OF ALIENS INTO THE UNITED STATES

Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled, That there shall be levied, collected, and paid a duty of two dollars for each and every passenger not a citizen of the United States, or of the Dominion of Canada,* the Republic of Cuba, or of the Republic of Mexico, who shall come by steam, sail, or other vessel from any foreign port to any port within the United States, or by any railway or any other mode of transportation, from foreign contiguous territory to the United States. The said duty shall be paid to the collector of customs of the port or customs district to which said alien

*See act approved March 22, 1904.

passenger shall come, or, if there be no collector at such port or district, then to the collector nearest thereto, by the master, agent, owner, or consignee of every such vessel or transportation line. The money thus collected shall be paid into the United States Treasury and shall constitute a permanent appropriation to be called the "immigrant fund," to be used under the direction of the Secretary of Commerce and Labor to defray the expense of regulating the immigration of aliens into the United States under this Act, including the cost of reports of decisions of the Federal courts, and digests thereof, for the use of the Commissioner-General of Immigration, and the salaries and expenses of all officers, clerks, and employes appointed for the purpose of enforcing the provisions of this Act. The duty imposed by this section shall be a lien upon the vessel which shall bring such aliens to ports of the United States, and shall be a debt in favor of the United States against the owner or owners of such vessels, and the payment of such duty may be enforced by any legal or equitable remedy; the head-tax herein provided for shall not be levied upon aliens in transit through the United States nor upon aliens who have once been admitted into the United States and have paid the head-tax who later shall go in transit from one part of the United States to another through foreign contiguous territory: *Provided*, That the Commissioner-General of Immigration, under the direction or with the approval of the Secretary of Commerce and Labor, by agreement with transportation lines, as provided in section thirty-two of this Act, may arrange in some other manner for the payment of the duty imposed by this section upon aliens seeking admission overland, either as to all or as to any such aliens.

SEC. 2. That the following classes of aliens shall be excluded from admission into the United States: All idiots, insane persons, epileptics, and persons who have been insane within five years previous; persons who have had two or more attacks of insanity at any time previously; paupers; persons likely to become a public charge; professional beggars; persons afflicted with a loathsome or with a dangerous

contagious disease; persons who have been convicted of a felony or other crime or misdemeanor involving moral turpitude; polygamists, anarchists, or persons who believe in or advocate the overthrow by force or violence of the Government of the United States or of all government or of all forms of law, or the assassination of public officials; prostitutes, and persons who procure or attempt to bring in prostitutes or women for the purpose of prostitution; those who have been, within one year from the date of the application for admission to the United States, deported as being under offers, solicitations, promises or agreements to perform labor or service of some kind therein; and also any person whose ticket or passage is paid for with the money of another, or who is assisted by others to come, unless it is affirmatively and satisfactorily shown that such person does not belong to one of the foregoing excluded classes; but this section shall not be held to prevent persons living in the United States from sending for a relative or friend who is not of the foregoing excluded classes: *Provided*, That nothing in this Act shall exclude persons convicted of an offense purely political, not involving moral turpitude: *And provided further*, That skilled labor may be imported, if labor of like kind unemployed cannot be found in this country: *And provided further*, That the provisions of this law applicable to contract labor shall not be held to exclude professional actors, artists, lecturers, singers, ministers of any religious denomination, professors for colleges or seminaries, persons belonging to any recognized learned profession, or persons employed strictly as personal or domestic servants.

SEC. 3. That the importation into the United States of any woman or girl for the purposes of prostitution is hereby forbidden; and whoever shall import or attempt to import any woman or girl into the United States for the purposes of prostitution, or shall hold or attempt to hold, any woman or girl for such purposes in pursuance of such illegal importation shall be deemed guilty of a felony, and, on conviction thereof, shall be imprisoned not less than one nor more than five years and pay a fine not exceeding five thousand dollars.

SEC. 4. That it shall be unlawful for any person, company, partnership, or corporation, in any manner whatsoever, to prepay the transportation or in any way to assist or encourage the importation or migration of any alien into the United States, in pursuance of any offer, solicitation, promise, or agreement, parole or special, expressed or implied, made previous to the importation of such alien to perform labor or service of any kind, skilled or unskilled, in the United States.

SEC. 5. That for every violation of any of the provisions of section four of this Act the person, partnership, company, or corporation violating the same, by knowingly assisting, encouraging, or soliciting the migration or importation of any alien to the United States to perform labor or service of any kind by reason of any offer, solicitation, promise, or agreement, express or implied, parole or special, to or with such alien shall forfeit and pay for every such offense the sum of one thousand dollars, which may be sued for and recovered by the United States, or by any person who shall first bring his action therefor in his own name and for his own benefit, including any such alien thus promised labor or service of any kind as aforesaid, as debts of like amount are now recovered in the courts of the United States; and separate suits may be brought for each alien thus promised labor or service of any kind as aforesaid. And it shall be the duty of the district attorney of the proper district to prosecute every such suit when brought by the United States.

SEC. 6. That it shall be unlawful and be deemed a violation of section four of this Act to assist or encourage the importation or migration of any alien by a promise of employment through advertisements printed and published in any foreign country; and any alien coming to this country in consequence of such an advertisment shall be treated as coming under a promise or agreement as contemplated in section two of this Act, and the penalties imposed by section five of this Act shall be applicable to such a case: *Provided*, That this section shall not apply to States or Territories, the District of Columbia, or places subject to the jurisdiction

of the United States advertising the inducements they offer for immigration thereto, respectively.

SEC. 7. That no transportation company or owner or owners of vessels or others engaged in transporting aliens into the United States, shall, directly or through agents, either by writing, printing, or oral representations, solicit, invite, or encourage the immigration of any aliens into the United States except by ordinary commercial letters, circulars, advertisements, or oral representations, stating the sailings of their vessels and terms and facilities of transportation therein; and for a violation of this provision any such transportation company and any such owner or owners of vessels, and all others engaged in transporting aliens to the United States, and the agents by them employed, shall be subjected to the penalties imposed by section five of this Act.

SEC. 8. That any person, including the master, agent, owner, or consignee of any vessel, who shall bring into or land in the United States, by vessel or otherwise, or who shall attempt, by himself or through another, to bring into or land in the United States, by vessel or otherwise, any alien not duly admitted by an immigrant inspector, or not lawfully entitled to enter the United States, shall be deemed guilty of a misdemeanor, and shall, on conviction, be punished by a fine not exceeding one thousand dollars for each and every alien so landed or attempted to be landed, or by imprisonment for a term not less than three months nor more than two years, or by both such fine and imprisonment.

SEC. 9. That it shall be unlawful for any person, including any transportation company other than railway lines entering the United States from foreign contiguous territory, or the owner, master, agent, or consignee of any vessel to bring to the United States any alien afflicted with a loathsome or with a dangerous contagious disease; and if it shall appear to the satisfaction of the Secretary of Commerce and Labor that any alien so brought to the United States was afflicted with such a disease at the time of foreign embarkation, and that the existence of such disease might have been detected by means of a competent medical examination

at such time, such person or transportation company or the master, agent, owner, or consignee of any such vessel shall pay to the collector of customs of the customs district in which the port of arrival is located the sum of one hundred dollars for each and every violation of the provisions of this section; and no vessel shall be granted clearance papers while any such fine imposed upon it remains unpaid, nor shall such fine be remitted.

SEC. 10. That the decision of the board of special inquiry, hereinafter provided for, based upon the certificate of the examining medical officer, shall be final as to the rejection of aliens afflicted with a loathsome or with a dangerous contagious disease, or with any mental or physical disability which would bring such aliens within any of the classes excluded from admission to the United States under section two of this Act.

SEC. 11. That upon the certificate of a medical officer of the United States Marine-Hospital Service to the effect that a rejected alien is helpless from sickness, physical disability, or infancy, if such alien is accompanied by another alien whose protection or guardianship is required by such rejected alien, the master, agent, owner, or consignee of the vessel in which such alien and accompanying alien are brought shall be required to return said alien and accompanying alien in the same manner as vessels are required to return other rejected aliens.

SEC. 12. That upon the arrival of any alien by water at any port within the United States it shall be the duty of the master or commanding officer of the steamer, sailing or other vessel, having said alien on board to deliver to the immigration officers at the port of arrival lists or manifests made at the time and place of embarkation of such alien on board such steamer or vessel, which shall, in answer to questions at the top of said lists, state as to each alien the full name, age, and sex; whether married or single; the calling or occupation; whether able to read or write; nationality; the race; the last residence; the seaport for landing in the United States; the final destination, if any, beyond the port of landing;

whether having a ticket through to such final destination; whether the alien has paid his own passage, or whether it has been paid by any other person or by any corporation, society, municipality, or government, and if so, by whom; whether in possession of fifty dollars, and if less, how much; whether going to join a relative or friend, and if so, what relative or friend and his name and complete address; whether ever before in the United States, and if so, when and where; whether ever in prison or almshouse or an institution or hospital for the care and treatment of the insane or supported by charity; whether a polygamist; whether an anarchist; whether coming by reason of any offer, solicitation, promise or agreement, expressed or implied, to perform labor in the United States, and what is the alien's condition of health, mental and physical, and whether deformed or crippled, and if so, for how long and from what cause.

SEC. 13. That all aliens arriving by water at the ports of the United States shall be listed in convenient groups, and no one list or manifest shall contain more than thirty names. To each alien or head of a family shall be given a ticket on which shall be written his name, a number or letter designating the list in which his name, and so forth, is contained, and his number on said list, for convenience of identification on arrival. Each list or manifest shall be verified by the signature and the oath or affirmation of the master or commanding officer or the first or second below him in command, taken before an immigration officer at the port of arrival, to the effect that he has caused the surgeon of said vessel sailing therewith to make a physical and oral examination of each of said aliens, and that from the report of said surgeon and from his own investigation he believes that no one of said aliens is an idiot, or insane person, or a pauper, or is likely to become a public charge, or is suffering from a loathsome or a dangerous contagious disease, or is a person who has been convicted of a felony or other crime or misdemeanor involving moral turpitude, or a polygamist, or an anarchist, or under promise or agreement, express or implied, to perform labor in the United States, or a prostitute,

and that also, according to the best of his knowledge and belief, the information in said lists or manifests concerning each of said aliens named therein is correct and true in every respect.

SEC. 14. That the surgeon of said vessel sailing therewith shall also sign each of said lists or manifests and make oath or affirmation in like manner before an immigration officer at the port of arrival, stating his professional experience and qualifications as a physician and surgeon, and that he has made a personal examination of each of the said aliens named therein, and that the said list or manifest, according to the best of his knowledge and belief, is full, correct, and true in all particulars relative to the mental and physical condition of said aliens. If no surgeon sails with any vessel bringing aliens the mental and physical examinations and the verifications of the lists or manifests shall be made by some competent surgeon employed by the owners of the said vessel.

SEC. 15. That in the case of the failure of the master or commanding officer of any vessel to deliver to the said immigration officers lists or manifests of all aliens on board thereof as required in sections twelve, thirteen, and fourteen of this Act, he shall pay to the collector of customs at the port of arrival the sum of ten dollars for each alien concerning whom the above information is not contained in any list as aforesaid.

SEC. 16. That upon the receipt by the immigration officers at any port of arrival of the lists or manifests of aliens provided for in sections twelve, thirteen, and fourteen of this Act it shall be the duty of said officers to go or send competent assistants to the vessels to which said lists or manifests refer and there inspect all such aliens, or said immigration officers may order a temporary removal of such aliens for examination at a designated time and place, but such temporary removal shall not be considered a landing, nor shall it relieve the transportation lines, masters, agents, owners, or consignees of the vessel upon which such aliens are brought to any port of the United States from any of the obligations which, in case such aliens remain on board, would, under

the provisions of this Act, bind the said transportation lines, masters, agents, owners, or consignees: *Provided*, That where a suitable building is used for the detention and examination of aliens the immigration officials shall there take charge of such aliens, and the transportation companies, masters, agents, owners, and consignees of the vessels bringing such aliens shall be relieved of the responsibility for their detention thereafter until the return of such aliens to their care.

SEC. 17. That the physical and mental examination of all arriving aliens shall be made by medical officers of the United States Marine-Hospital Service, who shall have had at least two years' experience in the practice of their profession since receiving the degree of doctor of medicine and who shall certify for the information of the immigration officers and the boards of special inquiry hereinafter provided for, any and all physical and mental defects or diseases observed by said medical officers in any such alien, or, should medical officers of the United States Marine-Hospital Service be not available, civil surgeons of not less than four years' professional experience may be employed in such emergencies for the said service, upon such terms as may be prescribed by the Commissioner-General of Immigration, under the direction or with the approval of the Secretary of Commerce and Labor. The United States Public Health and Marine-Hospital Service shall be reimbursed by the Immigration Service for all expenditures incurred in carrying out the medical inspection of aliens under regulations of the Secretary of Commerce and Labor.

SEC. 18. That it shall be the duty of the owners, officers, and agents of any vessel bringing an alien to the United States to adopt due precautions to prevent the landing of any such alien from such vessel at any time or place other than that designated by the immigration officers, and any such owner, officer, agent, or person in charge of such vessel who shall land or permit to land any alien at any time or place other than that designated by the immigration officers, shall be deemed guilty of a misdemeanor, and shall on conviction be punished by a fine for each alien so permitted to

land of not less than one hundred nor more than one thousand dollars, or by imprisonment for a term not exceeding one year, or by both such fine and imprisonment, and every such alien so landed shall be deemed to be unlawfully in the United States and shall be deported, as provided by law.

SEC. 19. That all aliens brought into this country in violation of law shall, if practicable, be immediately sent back to the countries whence they respectively came on the vessels bringing them. The cost of their maintenance while on land, as well as the expense of the return of such aliens, shall be borne by the owner or owners of the vessels on which they respectively came; and if any master, person in charge, agent, owner, or consignee, of any such vessels shall refuse to receive back on board thereof, or of any other vessel owned by the same interest, such aliens, or shall neglect to detain them thereon, or shall refuse or neglect to return them to the foreign port from which they came, or to pay the cost of their maintenance while on land, such master, person in charge, agent, owner, or consignee shall be deemed guilty of a misdemeanor and shall, on conviction, be punished by a fine not less than three hundred dollars for each and every such offense; and no vessel shall have clearance from any port of the United States while any such fine is unpaid: *Provided*, That the Commissioner-General of Immigration, under the direction or with the approval of the Secretary of Commerce and Labor may suspend, upon conditions to be prescribed by the Commissioner-General, the deportation of any alien found to have come under promise or agreement of labor or service of any kind if, in his judgment, the testimony of such alien is necessary on behalf of the United States Government in the prosecution of offenders against the provisions of sections four and five of this Act: *Provided*, That the cost of maintenance of any person so detained resulting from such suspension of deportation shall be paid from the "immigrant fund," but no alien certified, as provided in section seventeen of this Act, to be suffering with a loathsome or with a dangerous contagious disease other than one of a quarantinable nature, shall be permitted to

land for medical treatment thereof in the hospitals of the United States.

SEC. 20. That any alien who shall come into the United States in violation of law, or who shall be found a public charge therein, from causes existing prior to landing, shall be deported as hereinafter provided to the country whence he came at any time within two years after arrival at the expense, including one-half of the cost of inland transportation to the port of deportation, of the person bringing such alien into the United States, or, if that cannot be done, then at the expense of the "immigrant fund" referred to in section one of this Act.

SEC. 21. That in case the Secretary of Commerce and Labor shall be satisfied that an alien has been found in the United States in violation of this Act he shall cause such alien, within the period of three years after landing or entry therein, to be taken into custody and returned to the country whence he came, as provided in section twenty of this Act, or, if that cannot be so done, at the expense of the "immigrant fund" provided for in section one of this Act; and neglect or refusal on the part of the masters, agents, owners, or consignees of vessels to comply with the order of the Secretary of Commerce and Labor to take on board, guard safely, and return to the country whence he came any alien ordered to be deported under the provisions of this section shall be punished by the imposition of the penalties prescribed in section nineteen of this Act.

SEC. 22. That the Commissioner-General of Immigration, in addition to such other duties as may by law be assigned to him, shall, under the direction of the Secretary of Commerce and Labor, have charge of the administration of all laws relating to the immigration of aliens into the United States, and shall have the control, direction, and supervision of all officers, clerks, and employes appointed thereunder. He shall establish such rules and regulations, prescribe such forms of bonds, reports, entries, and other papers, and shall issue from time to time such instructions, not inconsistent with law, as he shall deem best calculated for carrying out

the provisions of this Act and for protecting the United States and aliens migrating thereto from fraud and loss, and shall have authority to enter into contracts for the support and relief of such aliens as may fall into distress or need public aid; all under the direction or with the approval of the Secretary of Commerce and Labor. And it shall be the duty of the Commissioner-General of Immigration to detail officers of the immigration service from time to time as may be necessary, in his judgment, to secure information as to the number of aliens detained in the penal, reformatory, and charitable institutions (public and private) of the several States and Territories, the District of Columbia, and other territory of the United States, and to inform the officers of such institutions of the provisions of law in relation to the deportation of aliens who have become public charges: *Provided*, That the Commissioner-General of Immigration may, with the approval of the Secretary of Commerce and Labor whenever in his judgment such action may be necessary to accomplish the purposes of this Act, detail immigration officers for temporary service in foreign countries.

Sec. 23. That the duties of the commissioners of immigration shall be of an administrative character, to be prescribed in detail by regulations prepared, under the direction or with the approval of the Secretary of Commerce and Labor.

Sec. 24. That immigrant inspectors and other immigration officers, clerks, and employes shall hereafter be appointed, and their compensation fixed and raised or decreased from time to time, by the Secretary of Commerce and Labor upon the recommendation of the Commissioner-General of Immigration and in accordance with the provisions of the civil service Act of January sixteenth, eighteen hundred and eighty-three: *Provided*, That nothing herein contained shall be construed to alter the mode of appointing commissioners of immigration at the several ports of the United States as provided by the sundry civil appropriation Act approved August eighteenth, eighteen hundred and ninety-four, or the official status of such commissioners heretofore appointed. Immigration officers shall have power to administer oaths and to

take and consider testimony touching the right of any alien to enter the United States, and, where such action may be necessary, to make a written record of such testimony, and any person to whom such an oath has been administered under the provisions of this Act who shall knowingly or wilfully give false testimony or swear to any false statement in any way affecting or in relation to the right of an alien to admission to the United States shall be deemed guilty of perjury and be punished as provided by section fifty-three hundred and ninety-two, United States Revised Statutes. The decision of any such officer, if favorable to the admission of any alien, shall be subject to challenge by any other immigration officer, and such challenge shall operate to take the alien whose right to land is so challenged before a board of special inquiry for its investigation. Every alien who may not appear to the examining immigrant inspector at the port of arrival to be clearly and beyond a doubt entitled to land shall be detained for examination in relation thereto by a board of special inquiry.

Sec. 25. That such boards of special inquiry shall be appointed by the commissioners of immigration at the various ports of arrival as may be necessary for the prompt determination of all cases of aliens detained at such ports under the provisions of law. Such boards shall consist of three members, who shall be selected from such of the immigrant officials in the service as the Commissioner-General of Immigration, with the approval of the Secretary of Commerce and Labor, shall from time to time designate as qualified to serve on such boards: *Provided*, That at ports where there are fewer than three immigrant inspectors, the Secretary of Commerce and Labor upon recommendation of the Commissioner-General of Immigration, may designate other United States officials for service on such boards of special inquiry. Such boards shall have authority to determine whether an alien who has been duly held shall be allowed to land or be deported. All hearings before boards shall be separate and apart from the public, but the said boards shall keep complete permanent records of their proceedings

and of all such testimony as may be produced before them; and the decision of any two members of a board shall prevail and be final, but either the alien or any dissenting member of said board may appeal, through the commissioner of immigration at the port of arrival and the Commissioner-General of Immigration, to the Secretary of Commerce and Labor whose decisions shall then be final; and the taking of such appeal shall operate to stay any action in regard to the final disposal of the alien whose case is so appealed until the receipt by the commissioner of immigration at the port of arrival of such decision.

Sec. 26. That no bond or guaranty, written or oral, that an alien shall not become a public charge shall be received from any person, company, corporation, charitable or benevolent society or association unless authority to receive the same shall in each special case be given by the Commissioner-General of Immigration, with the written approval of the Secretary of Commerce and Labor.

Sec. 27. That no suit or proceeding for a violation of the provisions of this Act shall be settled, compromised, or discontinued without the consent of the court in which it is pending, entered of record, with the reasons therefor.

Sec. 28. That nothing contained in this Act shall be construed to affect any prosecution or other proceeding, criminal or civil, begun under any existing Act or any Acts hereby amended, but such prosecutions or other proceedings, criminal or civil, shall proceed as if this Act had not been passed.

Sec. 29. That the Circuit and district courts of the United States are hereby invested with full and concurrent jurisdiction of all causes, civil and criminal, arising under any of the provisions of this Act.

Sec. 30. That after the first day of January, nineteen hundred and three, all exclusive privileges of exchanging money, transporting passengers or baggage, or keeping eating houses, and all other like privileges in connection with any United States immigration station, shall be disposed of after public competition, subject to such conditions and limitations

as the Commissioner-General of Immigration, under the direction or with the approval of the Secretary of Commerce and Labor may prescribe: *Provided*, That no intoxicating liquors shall be sold in any such immigrant station; that all receipts accruing from the disposal of such exclusive privileges as herein provided shall be paid into the United States Treasury to the credit of the "immigrant fund" provided for in section one of this Act.

SEC. 31. That for the preservation of the peace, and in order that arrests may be made for crimes under the laws of the States and Territories of the United States where the various immigrant stations are located, the officers in charge of such stations, as occasion may require, shall admit therein the proper State and municipal officers charged with the enforcement of such laws, and for the purposes of this section the jurisdiction of such officers and of the local courts shall extend over such stations.

SEC. 32. That the Commissioner-General of Immigration, under the direction or with the approval of the Secretary of Commerce and Labor shall prescribe rules for the entry and inspection of aliens along the borders of Canada and Mexico, so as not to unnecessarily delay, impede, or annoy passengers in ordinary travel between the United States and said countries, and shall have power to enter into contracts with foreign transportation lines for the same purpose.

SEC. 33. That for the purposes of this Act the words "United States" as used in the title as well as in the various sections of this Act shall be construed to mean the United States and any waters, territory, or other place now subject to the jurisdiction thereof.

SEC. 34. That no intoxicating liquors of any character shall be sold within the limits of the Capitol building of the United States.

SEC. 35. That the deportation of aliens arrested within the United States after entry and found to be illegally therein, provided for in this Act, shall be to the trans-Atlantic or trans-Pacific ports from which said aliens embarked for the United States; or, if such embarkation was for foreign

contiguous territory, to the foreign port at which said aliens embarked for such territory.

SEC. 36. That all Acts and parts of Acts inconsistent with this Act are hereby repealed: *Provided*, That this Act shall not be construed to repeal, alter, or amend existing laws relating to the immigration, or exclusion of Chinese persons or persons of Chinese descent.

SEC. 37. That whenever an alien shall have taken up his permanent residence in this country, and shall have filed his preliminary declaration to become a citizen, and thereafter shall send for his wife or minor children to join him, if said wife, or either of said children, shall be found to be affected with any contagious disorder, and if it is proved that said disorder was contracted on board the ship in which they came, and is so certified by the examining surgeon at the port of arrival, such wife or children shall be held, under such regulations as the Secretary of Commerce and Labor shall prescribe, until it shall be determined whether the disorder will be easily curable, or whether they can be permitted to land without danger to other persons; and they shall not be deported until such facts have been ascertained.

SEC. 38. That no person who disbelieves in or who is opposed to all organized government, or who is a member of or affiliated with any organization entertaining and teaching such disbelief in or opposition to all organized government, or who advocates or teaches the duty, necessity, or propriety of the unlawful assaulting or killing of any officer or officers, either of specific individuals or of officers generally, of the Government of the United States or of any other organized government, because of his or their official character, shall be permitted to enter the United States or any Territory or place subject to the jurisdiction thereof. This section shall be enforced by the Secretary of Commerce and Labor under such rules and regulations as he shall prescribe.

That any person who knowingly aids or assists any such person to enter the United States or any Territory or place subject to the jurisdiction thereof, or who connives or conspires with any person or persons to allow, procure, or

permit any such person to enter therein, except pursuant to such rules and regulations made by the Secretary of Commerce and Labor, shall be fined not more than five thousand dollars, or imprisoned for not less than one nor more than five years, or both.

SEC. 39. That no person who disbelieves in or who is opposed to all organized government, or who is a member of or affiliated with any organization entertaining and teaching such disbelief in or opposition to all organized government, or who advocates or teaches the duty, necessity, or propriety of the unlawful assaulting or killing of any officer or officers, either of specific individuals or of officers generally, of the Government of the United States or of any other organized government, because of his or their official character, or who has violated any of the provisions of this Act, shall be naturalized or be made a citizen of the United States. All courts and tribunals and all judges and officers thereof having jurisdiction of naturalization proceedings or duties to perform in regard thereto shall, on the final application for naturalization, make careful inquiry into such matters, and before issuing the final order or certificate of naturalization cause to be entered of record the affidavit of the applicant and of his witnesses so far as applicable, reciting and affirming the truth of every material fact requisite for naturalization. All final orders and certificates of naturalization hereafter made shall show on their face specifically that said affidavits were duly made and recorded, and all orders and certificates that fail to show such facts shall be null and void.

That any person who purposely procures naturalization in violation of the provisions of this section shall be fined not more than five thousand dollars, or shall be imprisoned not less than one nor more than ten years, or both, and the court in which such conviction is had shall thereupon adjudge and declare the order or decree and all certificates admitting such person to citizenship null and void. Jurisdiction is hereby conferred on the courts having jurisdiction of the trial of such offense to make such adjudication.

That any person who knowingly aids, advises, or encourages any such person to apply for or to secure naturalization or to file the preliminary papers declaring an intent to become a citizen of the United States, or who in any naturalization proceeding knowingly procures or gives false testimony as to any material fact, or who knowingly makes an affidavit false as to any material fact required to be proved in such proceeding, shall be fined not more than five thousand dollars, or imprisoned not less than one nor more than ten years, or both.

The foregoing provisions concerning naturalization shall not be enforced until ninety days after the approval hereof.

Approved, March 3, 1903.

ACT OF MARCH 22, 1904

AN ACT TO EXTEND THE EXEMPTION FROM HEAD-TAX TO CITIZENS OF NEWFOUNDLAND ENTERING THE UNITED STATES

Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled, That section one, chapter ten hundred and twelve, of the Statutes at Large of the United States of America (Fifty-seventh Congress, second session), is hereby amended by inserting in line four, after the word "Canada," the word "Newfoundland," so as to read as follows:

"That there shall be levied, collected, and paid a duty of two dollars for each and every passenger not a citizen of the United States, or of the Dominion of Canada, Newfoundland, the Republic of Cuba, or the Republic of Mexico, who shall come by steam, sail, or other vessel from any foreign port to any port within the United States, or by any railway or any other mode of transportation from foreign contiguous territory to the United States. The said duty shall be paid to the collector of customs of the port or customs district to which said alien passenger shall come, or, if there be no collector at such port or district, then to the collector nearest thereto by the master, agent, owner, or consignee of every

such vessel or transportation line. The money thus collected shall be paid into the United States Treasury and shall constitute a permanent appropriation to be called the 'immigrant fund,' to be used under the direction of the Secretary of Commerce and Labor to defray the expense of regulating the immigration of aliens into the United States under this Act, including the cost of reports of decisions of the Federal courts, and digests thereof, for the use of the Commissioner-General of Immigration, and the salaries and expenses of all officers, clerks, and employes appointed for the purpose of enforcing the provisions of this Act. The duty imposed by this section shall be a lien upon the vessels which shall bring such aliens to ports of the United States, and shall be a debt in favor of the United States against the owner or owners of such vessels, and the payment of such duty may be enforced by any legal or equitable remedy. The head-tax herein provided for shall not be levied upon aliens in transit through the United States nor upon aliens who have once been admitted into the United States and have paid the head-tax who later shall go in transit from one part of the United States to another through foreign contiguous territory: *Provided*, That the Commissioner-General of Immigration, under the direction or with the approval of the Secretary of Commerce and Labor, by agreement with transportation lines, as provided in section thirty-two of this Act, may arrange in some other manner for the payment of the duty imposed by this section upon aliens seeking admission overland, either as to all or as to any such aliens."

APPROVED, March 22, 1904 (33 Stat., part 1, page 144).

IMMIGRATION REGULATIONS

RULE 1. Collectors of customs shall collect, as provided in section one of the Act approved March third, nineteen hundred and three, a duty of four dollars for each and every passenger, not a citizen of the United States, or of Canada, or of Newfoundland, or of Cuba, or of Mexico, who shall

come by steam or sail vessel from any foreign port to any port within the United States; but no duty shall be paid for such passengers proceeding in continuous transit through the United States to foreign territory, nor for those who have once been admitted to the United States and have paid the head-tax who shall later go in transit from one part of the United States to another through foreign contiguous territory.

Rule 2. Under the authority contained in the proviso to section one of the Act approved March third, nineteen hundred and three, the duty of two dollars levied upon railways or other transportation agencies for passengers brought by them overland from foreign contiguous territory shall be collected, until otherwise provided, in accordance with the terms of an agreement between the Commissioner-General of Immigration and certain transportation companies of the Dominion of Canada, of date November first, nineteen hundred and one.

Rule 3. All such moneys so collected, as well as all moneys collected for rentals of exclusive privileges at United States immigrant stations, shall be deposited to the credit of the Treasurer of the United States on account of "immigrant fund," with an assistant treasurer of the United States, or national-bank depositary, in the same manner as other miscellaneous collections are reported. Separate accounts of the receipts and expenditures of money under the act shall be rendered monthly to the Secretary of the Treasury through the Department of Commerce and Labor, on forms to be furnished by the Government for the purpose.

Rule 4. Every railway company or other transportation agency claiming exemption from payment of the head-tax for any passenger brought by it to a point on the land boundary of the United States for entry thereat, under the exemption from such payment provided in section one of the Act approved March third, nineteen hundred and three, shall produce to the appropriate officer at such point a letter from an immigration officer at, or nearest to, the alleged residence of such passenger, stating that after investigation said immigration officer has ascertained that said passenger had already

been lawfully admitted to the United States and that the head-tax for his admission had been paid at the time of such admission, and said company or other transportation agency shall also show, as evidence that the transit of such passenger was continuous, the dated ticket issued to such passenger.

Rule 5. The head-tax provided for in section one of the Act approved March third, nineteen hundred and three, shall be collected and paid for every alien passenger not specifically excepted therein.

Rule 6. The provisions of the immigration laws, and of regulations passed thereunder, except those of section one of the Act of March third, nineteen hundred and three, in relation to the payment of the head-tax, extend to all persons not citizens of the United States either by birth or by naturalization therein.

Rule 7. Every alien arriving at a port of the United States shall be promptly examined, as by law provided, either on shipboard or at some other place designated for that purpose. If found admissible, he shall be at once landed, but if upon special inquiry he is denied admission, he shall be informed that he has a right of appeal therefrom, and the fact that he has been so informed shall be entered of record in the minutes of the board's proceedings, but no appeal will be considered after any such alien has, in consequence of an adverse decision of a board of special inquiry, been transferred from an immigrant station to be deported.

Rule 8. Every alien detained for special inquiry, which shall be conducted separate and apart from the public, shall have a speedy hearing and, upon the conclusion thereof, be either at once landed or ordered deported. If he elects to appeal from said order of deportation, he must, to enable officers to comply with the provisions of section nineteen of the Act of March third, nineteen hundred and three, file notice of such appeal promptly, and a like right of appeal may be exercised by any member of a board of special inquiry who dissents from the decision rendered by such board.

Rule 9. Every such notice of appeal shall act as a stay upon the disposal of the alien whose rights are thereby affected

until a final decision is rendered by the department; and, within thirty-six hours after the filing of such notice, the record of the case, together with such briefs, affidavits, and statements as are to be considered in connection therewith, should be forwarded to the Commissioner-General of Immigration by the chief immigration officer at the port of arrival, accompanied by his views thereon in writing; but on such appeal of any case to the Department no evidence will be considered which has not already been passed upon in said case by a board of special inquiry. If additional time is granted to the friends or counsel of an appealing alien to prevent a miscarriage of justice, the said chief officer may require the deposit of a sum of money sufficient to defray the cost of maintaining appellant during the additional time thus allowed.

Rule 10. The commissioner of immigration or the chief immigration officer at the port of landing shall enter of record the name of every alien found upon examination to be within any of the prohibited classes, with a statement of the decision in each case; and if such decision be appealed from, immediately upon the receipt from the Department of its conclusions thereupon the alien shall be at once landed or deported in accordance with such conclusion. If a landing is refused on appeal, the master, agent, consignee, or owner of the vessel by which the said alien arrived shall be notified thereof by the commissioner or chief immigration officer, and that the said alien will be placed on the said vessel to be returned as aforesaid.

Rule 11. Attorneys and persons appearing in behalf of detained aliens shall not be permitted to charge a sum exceeding ten dollars ($10) in each case, unless the commissioner shall, in writing, allow an additional compensation, which fee shall be payable through the commissioner. Any one charging an alien a fee prior to his detention, or charging or receiving from an alien or his relatives or friends a fee, gift, or compensation for his services in excess of above rates, or who shall deprive an alien of any part of his chattels or effects in lieu of, or as security for, said fee, shall not be

permitted to practice at any immigrant station of the United States.

Rule 12. The expenses incurred for the keeping and maintenance of every alien temporarily removed from a vessel, as provided by law, shall be borne by the owner or owners of the vessel upon which he came until he is lawfully landed or delivered on board such vessel for deportation.

Rule 13. The master, agent, owner, or consignee, of any vessel on which aliens are brought to the United States, shall at least twenty-four hours in advance thereof, notify the commissioner of immigration of the intended time of sailing of such vessel, in order that said officer may place on board thereof every alien brought thereon who has been refused a landing.

Rule 14. Every alien professing to seek a landing for the purpose of proceeding directly through the United States to a foreign country shall be examined, and, if found to be a member of any one of the excluded classes, shall be refused permission to land in the same manner as though he intended to remain in the United States.

Rule 15. No alien desiring admission at a port of the United States for the professed purpose of proceeding directly therefrom to foreign territory shall be permitted to land thereat except after deposit with the collector of customs at said port by the master or owner of the vessel on which such alien is brought of the amount of the head-tax (two dollars) prescribed by section one of the Act approved March third, nineteen hundred and three, said amount to be refunded upon proof satisfactory to the immigration officer in charge at the port of arrival that said alien has passed by direct and continuous journey through and out of the United States.

Rule 16. Every alien who left the port of embarkation in good physical condition, and who would be qualified to land but for some sickness or disability other than a loathsome or a dangerous contagious disease, which was contracted or which developed during the voyage, may either remain on shipboard or be removed for hospital treatment at the expense of the owners of the vessel on which he came, and

while detained in hospital he shall not be considered as landed; *provided* that if the sickness or disability with which such alien is afflicted is so slight, in the judgment of the examining medical officer, as not to require treatment in hospital, and said alien is able to pay for the necessary medical care he needs, he shall be landed. Requests by such owners for reimbursement for hospital expenses may be made only in respect of aliens detained under the provisions of this rule, and then only in a proper manner.

Rule 17. Any alien who has been lawfully landed, but who has become a public charge from subsequently arising physical inability to earn a living which is likely to be of a permanent nature, may, with the approval of the Bureau of Immigration, be deported within one year from date of landing, at the expense of the "immigrant fund," provided that such alien is delivered to the immigration officers at a designated port free of charge; and the charges incurred for the care and treatment of any such alien in any public or charitable institution from the date of notification to the Bureau of Immigration until the expiration of one year after landing may be paid from the "immigrant fund" at fixed rates agreed upon.

Rule 18. Any alien who has been finally determined to be admissible may be permitted to wait for friends or remittances upon payment by him of expenses incurred by reason of such delay. In case such an alien is unable, from accident or other unavoidable circumstances, to immediately continue his journey, and is without sufficient means to defray the expense of his enforced delay, the commissioner of immigration may, in his discretion, pay said expense, reporting said case to the Bureau of Immigration with reasons for his action, and request that such expense be repaid out of the "immigrant fund."

Rule 19.—Not over one accompanying alien (preferably the natural guardian or a relative) shall be detained at the expense of the transportation company for the purpose of caring for an alien who, for any reason, is detained for further inquiry, if the latter requires such care. (See section eleven, Act approved March third, nineteen hundred and three.)

Rule 20.—No application for the admission under bond of a debarred alien will be considered except in cases which in deportation of the alien in whose behalf such application is made would involve the separation of immediate members of a family, and in that case only when a deposit of money is made sufficient to defray the expense of maintaining such alien while awaiting a decision upon such application.

Rule 21. The cost of returning aliens under the provisions of sections nineteen and twenty of the Act approved March third, eighteen hundred ninety three, shall include all expenses incurred for maintenance of such aliens after such cases are brought to the attention of the Bureau of Immigration, provided said Bureau, upon investigation, has ascertained the case to be one for deportation and has so ordered.

Rule 22. Every immigration officer deporting any alien under the provisions of section twenty of the Act of March third, eighteen hundred ninety three, shall submit to the Bureau of Immigration for approval bills for one-half the cost of inland transportation of such alien to the seaport of deportation against the vessel or transportation line by which such alien was brought to the United States, in order that the amount so expended by any such officer may be collected and returned to him.

Rule 23. In case of the failure of the master or commanding officer of any vessel bringing aliens to any port within the United States to deliver to the immigration officers at said port lists or manifests of such aliens, as required in sections twelve, thirteen, and fourteen of the Act approved March third, eighteen hundred ninety three, there shall be paid to the collector of customs at the port of arrival the sum of ten dollars for each alien concerning whom the information required by such sections is not contained in any list or manifest as aforesaid. Under an opinion of the Solicitor of the Treasury, the fine mentioned in this rule cannot be remitted.

Rule 24. The certificate required by section eight of the Act approved March third, eighteen hundred ninety three, shall be filed with the Secretary of Commerce and Labor upon the first days of January and July of each year.

RULE 25. No charge for food, lodging, or maintenance, or for hospital attendance, medicines, or other hospital expenses shall be made in excess of the actual cost of furnishing the same, the intention being to make the Service self-sustaining without profit.

RULE 26. Every officer of the United States Public Health and Marine-Hospital Service detailed under section seventeen of the Act approved March third, nineteen hundred and three, for duty to any point in the United States shall, during the continuance of such detail, be under the direction (subject to the Public Health and Marine-Hospital Service regulations governing the medical inspection of aliens, as approved by the Secretary of the Treasury, November eighteenth, nineteen hundred and two) of the immigration officer in charge of the said port.

RULE 27. There shall be made by the commissioners of immigration at the various ports of the United States and Canada weekly reports to the Bureau of Immigration containing a list of all aliens detained from Sunday morning until Saturday night, inclusive, of each week, in which shall be embodied the following information: Name of alien; age and sex; date of arrival; vessel on which alien arrived; date of detention; cause of detention; whether case has been disposed of, and if so, what disposition has been made of it.

RULE 28. Officers employed in the administration of the immigration and Chinese-exclusion laws are notified that all communications to the Department upon official matters must be addressed to the Commissioner-General of Immigration or to the Secretary of Commerce and Labor.

EXAMINATION OF ALIEN SEAMEN UNDER THE PROVISIONS OF THE LAWS REGULATING IMMIGRATION

To commissioners of immigration and other officers charged with the administration of the various laws pertaining to the immigration of aliens:

In view of the opinion of the Attorney-General of the United States, of date September tenth, nineteen hundred and one, you are hereby instructed that it is your duty to

make such an investigation of every vessel arriving at a port of the United States from any other than a mainland or continental port thereof, as will enable you to ascertain the members of their respective crews who are aliens, as well as the intention of such alien members in seeking a landing.

1. Alien seamen who seek to land, in the regular course of their pursuit, with the bona fide intention of departing as soon as practicable upon some outward-bound vessel, are not to be held for examination touching their right to land under the various acts regulating immigration into the United States, nor shall the masters of the vessels upon which they come to a port of the United States be charged on their account with the head-tax prescribed by section one of the Act approved March third, nineteen hundred and three.

2. Alien seamen who are discharged, or are to be discharged, or who have deserted their vessel at a port of the United States with any other object in view than departing as described in the next preceding paragraph (1) are in no respect to be distinguished on account of their prior calling or occupation as seamen from other aliens seeking admission to this country, either as regards collection of head-tax on their account or as respects the examination and determination of their right to remain under the various acts regulating immigration.

3. All other aliens signed on the ship's articles of any vessel arriving at a port of the United States shall be examined, and in all respects regarded, as alien passengers, on account of whom the head-tax should be collected and who should be admitted, or ordered returned to the countries whence they came, in accordance with the laws and regulations governing immigration.

GEO. B. CORTELYOU, *Secretary.*

REGULATIONS GOVERNING WARRANTS–IMMIGRATION SERVICE

To all officers charged with the enforcement of the immigration laws:

You are hereby directed to conform in future, in making application for warrants for the arrest and deportation of aliens found unlawfully in the United States, to the following regulations:

1. All such applications must be made, if possible, upon blank forms (No. 1565), which will be furnished upon written request to the Commissioner-General of Immigration, Department of Commerce and Labor, and which must be filled out in accordance with the printed lines contained therein, and be accompanied by the certificate of landing or entry (Form No. 1564), hereinafter prescribed, or if not so accompanied the reasons for the absence of such certificate must be given, and in that case all the facts called for in the blank form of said certificate shall be set forth in the application, so far as the facts are ascertainable.

2. A full statement must be made in every such application of the facts, supported if practicable by affidavits, which show the presence in the United States of the alien, whose arrest and deportation is sought, to be in violation of law.

3. The certificate of landing in, or entry into, the United States must contain a complete statement in detail of all the facts disposed as to any such alien by the manifest or list containing his name, with an attached certificate by the officer in charge of such manifest that the information given agrees in all particulars with the record of such alien in said list or manifest.

4. If the circumstances of any particular case necessitate resort to request by wire, such request must state that the foregoing regulations have been complied with, and that the form of application and certificate hereinbefore mentioned have been forwarded to the Department, and must give the substance of the statement of facts contained in the said application and certificate.

5. If, upon the receipt of any such application and certificate, or of the request by wire provided for in paragraph four, either completely in conformity with these regulations or accompanied by a satisfactory explanation of inability to comply therewith, it appears to the Secretary that the alien whose arrest and deportation is sought is in the United States unlawfully and that the time within which he can be deported has not expired, a warrant for his arrest will be issued directing that he be taken before an officer, or officers, named therein, and there be given full opportunity to show cause, if there be any, why he should not be deported, and as soon as arrested said alien shall be apprised of his right to be represented by counsel, and he and his counsel shall have the right to inspect all the evidence upon which the Secretary has acted in directing said alien's arrest, and be given an opportunity to offer evidence and submit an argument in his behalf, and be given an opportunity to inspect and make a copy of the report of the hearing and of the findings of the officers before whom it is held. In case said alien is unable to understand or to speak the English language, an interpreter shall, if possible, be employed at the hearing.

6. If, after the receipt of the report of such hearing, accompanied by the findings of the officers before whom it was held, it shall appear to the satisfaction of the Secretary, from all the evidence, that such alien is in the United States in violation of law and that the time within which he can be deported has not expired, a warrant will be issued for his deportation.

7. Officers are warned that their duties are not of an inquisitorial nature; but they are directed to make thorough investigation of all cases where they are credibly informed, or have reason to believe, that a specified alien is in the United States in violation of law. It is not permissible for officers to resort to intimidation by threats or violence in order to extort from any suspected alien or from any other person the information to be embodied in the application for the warrant of arrest.

MEDICAL CERTIFICATES UPON MANIFESTS OF ALIENS BROUGHT TO THE UNITED STATES ON VESSELS SAILING WITHOUT SURGEON

The Immigration Act approved March third, nineteen hundred and three (section fourteen), requires that when no surgeon sails with a vessel bringing aliens to the United States the mental and physical examination of such aliens shall be made by "some competent surgeon employed by the owners of the said vessel."

The certificate (unverified) of a responsible surgeon at the point of embarkation or at the last port of call, in the form appearing upon the reverse side of the manifest (Form 1500), will, until further notice, be accepted by immigration officers as a compliance with the statute.

Section fifteen of the Act approved March third, nineteen hundred and three, reads as follows:

"That in the case of the failure of the master or commanding officer of any vessel to deliver to the said immigration officers lists or manifests of all aliens on board thereof as required in sections twelve, thirteen, and fourteen of this Act, he shall pay to the collector of customs at the port of arrival the sum of ten dollars for each alien concerning whom the above information is not contained in any list as aforesaid."

PROCEDURE FOR IMPOSITION OF FINES ON ACCOUNT OF BRINGING DISEASED ALIENS TO THE UNITED STATES

As a means of enforcing the collection of any fine imposed under the provisions of section nine of the Immigration Act approved March third, nineteen hundred and three, the said section directs the refusal of clearance papers to any vessel bringing any alien, diseased as described therein, to a port of the United States.

To avoid, on the one hand, the denial of reasonable time to the master, agent, owner, or consignee to show cause why such fine should not be imposed, and, on the other hand, the loss of the summary and effective means provided for the collection of such fines, the following regulations are hereby promulgated:

1. *Medical Certificates.*—The certificate of the medical examiner in the case of an alien afflicted with a loathsome or dangerous contagious disease shall state in terms whether, in his judgment, the "existence of such disease might have been detected by means of a competent medical examination at the port of foreign embarkation."

2. *Notification.*—Upon the receipt of a medical certificate in compliance with the preceding paragraph hereof, the commissioner of immigration or inspector in charge at the port of arrival shall *at once* serve notice upon the master, agent, owner, or consignee of the vessel upon which such alien arrived, in the following form, printed blanks for that purpose to be procured from the Department, viz:

DEPARTMENT OF COMMERCE AND LABOR,
IMMIGRATION SERVICE,
OFFICE OF_____
PORT OF_____
_____, 190__

To_____

_____of the steamship_____
[*Master, agent, owner, or consignee*]

In conformity with the requirements of Department circular number fifty-eight, you are hereby notified that a certificate of the examining surgeon, based upon a physical examination of the alien whose name is shown herein, indicates that a fine should be imposed under the provisions of section nine of the Immigration Act approved March third, nineteen hundred and three.

If you desire to submit evidence to show that the said fine should not be imposed in this instance, you will be allowed fourteen days from the date of this notice for that purpose, and the vessel on which the said alien arrived will be granted clearance papers when she is ready to sail and allowed to proceed upon her outward-bound voyage, upon condition that you deposit with the collector of customs at this port within twenty-four hours after the receipt of this, or, if said vessel shall sail sooner, prior to her sailing, the sum of one hundred dollars as security for the payment of the said fine, should it be imposed.

NAME OF ALIEN	STEAMSHIP	DISEASE

(Signature)_____
(Title)_____
Received the above notice_____, 190__, at_____M.
[*Time*]
(Witness:) _____
_____

The notification shall be prepared *in triplicate*, the original to be delivered by an employe of the Immigration Service at the office of the master, agent, owner, or consignee to whom it is addressed, said employe to witness the signature of the recipient. Receipt of service shall be indorsed upon the duplicate and triplicate, the duplicate to be returned to the office of the commissioner of immigration or inspector in charge and preserved as proof of delivery, and the triplicate to be delivered to the collector of customs, who will withhold clearance papers until the deposit is made.

3. *Deposit.*—The special deposit of one hundred dollars required to stay action for the period of fourteen days shall be made to the collector of customs for the district wherein the port of arrival is located within twenty-four hours after delivery of the above-described notice, or, if said vessel sails sooner, before such sailing, and in default thereof all further proceedings shall be discontinued and the facts certified to the Bureau of Immigration by first mail, together with the medical certificate and duplicate notice, in order that such action may be taken as the evidence requires.

4. *Stay of Action.*—If, upon the expiration of twenty-four hours after service of the notice as provided in paragraph two of this circular, the deposit of one hundred dollars has been made in conformity with the said notice, the commissioner of immigration or inspector in charge shall suspend further proceedings until the submission of the evidence offered to show why the said fine should not be imposed, or until the lapse of the specified period of fourteen days thereafter. When the said evidence has been submitted it shall be forwarded, together with the certificate of the examining surgeon, to the Commissioner-General of Immigration, for presentation to the Secretary of Commerce and Labor, by the said commissioner or inspector in charge, who shall at the same time present his written views as to whether the said fine should be imposed. If no evidence is submitted prior to the expiration of the said fourteen days, then said commissioner or inspector in charge shall report the case without such evidence, for action by the Secretary of Commerce and Labor.

5. *Final Proceedings.*—Upon receipt of the decision of the Secretary of Commerce and Labor, a copy thereof shall be forwarded to the collector of customs, together with such data as may enable him to identify the special deposit made in that particular case. If the said decision imposes the fine, the one hundred dollars deposited as security shall be accounted for by the said collector in the usual manner, as a fine; if the decision holds that the penalty has not been incurred, the collector of customs shall return to the depositor the amount deposited as security.

CHINESE EXCLUSION LAW

TREATY, LAWS, AND REGULATIONS

CHINESE EXCLUSION TREATY

TREATY OF NOVEMBER 17, 1880

ARTICLE I

Preamble.—Whenever in the opinion of the Government of the United States the coming of Chinese laborers to the United States, or their residence therein, affects or threatens to affect the interests of that country, or to endanger the good order of the said country, or of any locality within the territory thereof, the Government of China agrees that the Government of the United States may regulate, limit, or suspend such coming or residence, but may not absolutely prohibit it. The limitation or suspension shall be reasonable, and shall apply only to Chinese who may go to the United States as laborers, other classes not being included in the limitations. Legislation taken in regard to Chinese laborers will be of such a character only as is necessary to enforce the regulation, limitation, or suspension of immigration, and immigrants shall not be subject to personal maltreatment or abuse.

ARTICLE II

Privileged Classes.—Chinese subjects, whether proceeding to the United States as teachers, students, merchants, or from curiosity, together with their body and household servants, and Chinese laborers who are now in the United

States shall be allowed to go and come of their own free will and accord, and shall be accorded all the rights, privileges, immunities, and exemptions which are accorded to the citizens and subjects of the most favored nation.

ARTICLE III

Ill Treatment of Chinese.—If Chinese laborers, or Chinese of any other class, now either permanently or temporarily residing in the territory of the United States, meet with ill treatment at the hands of any other persons, the Government of the United States will exert all its power to devise measures for their protection and to secure to them the same rights, privileges, immunities, and exemptions as may be enjoyed by the citizens or subjects of the most favored nation, and to which they are entitled by treaty.

ARTICLE IV

Correction of Hardships.—The high contracting powers having agreed upon the foregoing articles, whenever the Government of the United States shall adopt legislative measures in accordance therewith, such measures will be communicated to the Government of China. If the measures as enacted are found to work hardship upon the subjects of China, the Chinese minister at Washington may bring the matter to the notice of the Secretary of State of the United States, who will consider the subject with him; and the Chinese Foreign Office may also bring the matter to the notice of the United States minister at Peking and consider the subject with him, to the end that mutual and unqualified benefit may result.

ACT OF JULY 5, 1884

Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled, That section one of the Act entitled "An Act to execute certain treaty stipulations relating to Chinese," approved May sixth, eighteen hundred and eighty-two, is hereby amended so as to read as follows:

Preamble.—"Whereas in the opinion of the Government of the United States the coming of Chinese laborers to this country endangers the good order of certain localities within the territory thereof; Therefore

Period of Exclusion.—"*Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled*, That from and after the passage of this Act, and until the expiration of ten years next after the passage of this Act, the coming of Chinese laborers to the United States be, and the same is hereby suspended, and during such suspension it shall not be lawful for any Chinese laborer to come from any foreign port or place, or having so come to remain within the United States."

Liability of Master of Vessel.—Section two of said Act is hereby amended so as to read as follows:

"SECTION 2. That the master of any vessel who shall knowingly bring within the United States on such vessel, and land, or attempt to land, or permit to be landed any Chinese laborer, from any foreign port or place, shall be deemed guilty of a misdemeanor, and on conviction thereof shall be punished by a fine of not more than five hundred dollars for each and every such Chinese laborer so brought, and may also be imprisoned for a term not exceeding one year."

Exceptions.—Section three of said Act is hereby amended so as to read as follows:

"SECTION 3. That the two foregoing sections shall not apply to Chinese laborers who were in the United States on the seventeenth day of November, eighteen hundred and eighty, or who shall have come into the same before the expiration of ninety days next after the passage of the Act to which this Act is amendatory, nor shall said sections apply to Chinese laborers, who shall produce to such master before going on board such vessel, and shall produce to the collector of the port in the United States at which such vessel shall arrive, the evidence hereinafter in this Act required of his being one of the laborers in this section mentioned; nor shall the two foregoing sections apply to the

case of any master whose vessel, being bound to a port not within the United States, shall come within the jurisdiction of the United States by reason of being in distress or in stress of weather, or touching at any port of the United States on its voyage to any foreign port or place: *Provided*, That all Chinese laborers brought on such vessel shall not be permitted to land except in case of absolute necessity, and must depart with the vessel on leaving port."

List of Departing Laborers.—Section four of said Act is hereby amended so as to read as follows:

"SECTION 4. That for the purpose of properly identifying Chinese laborers who were in the United States on the seventeenth day of November, eighteen hundred and eighty, or who shall have come into the same before the expiration of ninety days next after the passage of the Act to which this Act is amendatory, and in order to furnish them with the proper evidence of their right to go from and come to the United States as provided by the said Act and treaty between the United States and China dated November seventeenth, eighteen hundred and eighty, the collector of customs of the district from which any such Chinese laborers shall depart from the United States shall, in person or by deputy, go on board each vessel having on board any such Chinese laborer, and cleared or about to sail from his district for a foreign port, and on such vessel make a list of all such Chinese laborers, which shall be entered in registry books, to be kept for that purpose in which shall be stated the individual, family, and tribal name in full, the age, occupation, when and where followed, last place of residence, physical marks or peculiarities, and all facts necessary for the identification of each such Chinese laborer, which books shall be safely kept in the custom house; and every such Chinese laborer so departing from the United States shall be entitled to and shall receive, free of any charge or cost upon application therefor, from the collector or his deputy, in the name of said collector and attested by said collector's seal of office, at the time such list is taken, a certificate, signed by the collector or his deputy and attested by his seal of office, in

such form as the Secretary of the Treasury shall prescribe, which certificate shall contain a statement of the individual, family, and tribal name in full, age, occupation, when and where followed, of the Chinese laborer to whom the certificate is issued, corresponding with the said list and registry in all particulars.

Certificate to be Given Laborer.—In case any Chinese laborer, after having received such certificate, shall leave such vessel before her departure, he shall deliver his certificate to the master of the vessel; and if such Chinese laborer shall fail to return to such vessel before her departure from port, the certificate shall be delivered by the master to the collector of customs for cancelation.

Certificate to be Sole Evidence of Right to Enter.—"The certificate herein provided for shall entitle the Chinese laborer to whom the same is issued to return to and reenter the United States upon producing and delivering the same to the collector of customs of the district at which such Chinese laborer shall seek to reenter, and said certificate shall be the only evidence permissible to establish his right of reentry; and upon delivering of such certificate by such Chinese laborer to the collector of customs at the time of reentry in the United States, said collector shall cause the same to be filed in the custom house and duly canceled."

Permission and Identification by Chinese Government.—Section six of said Act is hereby amended so as to read as follows:

"SECTION 6. That in order to the faithful execution of the provisions of this Act, every Chinese person, other than a laborer, who may be entitled by said treaty or this Act to come within the United States, and who shall be about to come to the United States, shall obtain the permission of and be identified as so entitled by the Chinese Government, or of such other foreign government of which at the time such Chinese person shall be a subject, in each case to be evidenced by a certificate issued by such Government, which certificate shall be in the English language, and shall show such permission, with the name of the permitted person

in his or her proper signature, and which certificate shall state the individual, family, and tribal name in full, title or official rank, if any, the age, height, and all physical peculiarities, former and present occupation or profession, when and where and how long pursued, and place of residence of the person to whom the certificate is issued, and that such person is entitled by this Act to come within the United States.

Merchants.—"If the person so applying for a certificate shall be a merchant said certificate shall, in addition to above requirements, state the nature, character, and estimated value of the business carried on by him prior to and at the time of his application as aforesaid: *Provided*, That nothing in this Act nor in said treaty shall be construed as embracing within the meaning of the word 'merchant,' hucksters, peddlers, or those engaged in taking, drying, or otherwise preserving shell or other fish for home consumption or exportation.

Travelers.—"If the certificate be sought for the purpose of travel for curiosity, it shall also state whether the applicant intends to pass through or travel within the United States, together with his financial standing in the country from which such certificate is desired.

Consular Vise.—"The certificate provided for in this Act, and the identity of the person named therein, shall, before such person goes on board any vessel to proceed to the United States, be viséed by the indorsement of the diplomatic representatives of the United States in the foreign country from which such certificate issues, or of the consular representative of the United States at the port or place from which the person named in the certificate is about to depart; and such diplomatic representative or consular representative whose indorsement is so required is hereby empowered, and it shall be his duty, before indorsing such certificate as aforesaid, to examine into the truth of the statements set forth in said certificate, and if he shall find upon examination that said or any of the statements contained therein are untrue it shall be his duty to refuse to indorse the same.

Certificate Sole Evidence for Holder.—"Such certificate viséed as aforesaid shall be prima facie evidence of the facts set forth therein, and shall be produced to the collector of customs of the port in the district in the United States at which the person named therein shall arrive, and afterward produced to the proper authorities of the United States whenever lawfully demanded, and shall be the sole evidence permissible on the part of the persons so producing the same to establish a right of entry into the United States; but said certificate may be controverted and the facts therein stated disproved by the United States authorities."

Master of Vessel to Furnish List of Passengers.—Section eight of said Act is hereby amended so as to read as follows:

"SECTION 8. That the master of any vessel arriving in the United States from any foreign port or place shall, at the same time he delivers a manifest of the cargo, and if there be no cargo, then at the time of making a report of the entry of the vessel pursuant to law, in addition to the other matter required to be reported, and before landing, or permitting to land, any Chinese passengers, deliver and report to the collector of customs of the district in which such vessels shall have arrived a separate list of all Chinese passengers taken on board his vessel at any foreign port or place, and all such passengers on board the vessel at that time. Such list shall show the names of such passengers (and if accredited officers of the Chinese or of any other foreign Government, traveling on the business of that Government, or their servants, with a note of such facts), and the names and other particulars as shown by their respective certificates; and such list shall be sworn to by the master in the manner required by law in relation to the manifest of the cargo.

Penalty.—"Any refusal or wilful neglect of any such master to comply with the provisions of this section shall incur the same penalties and forfeiture as are provided for a refusal or neglect to report and deliver a manifest of the cargo."

Liability of Vessel.—Section ten of said Act is hereby amended so as to read as follows:

"SECTION 10. That every vessel whose master shall knowingly violate any of the provisions of this Act shall be deemed forfeited to the United States, and shall be liable to seizure and condemnation in any district of the United States into which such vessel may enter or in which she may be found."

Persons Aiding or Abetting Unlawful Entry.—Section eleven of said Act is hereby amended so as to read as follows:

"SECTION 11. That any person who shall knowingly bring into or cause to be brought into the United States by land, or who shall aid or abet the same, or aid or abet the landing in the United States from any vessel, of any Chinese person not lawfully entitled to enter the United States, shall be deemed guilty of a misdemeanor, and shall on conviction thereof, be fined a sum not exceeding one thousand dollars, and imprisoned for a term not exceeding one year."

Entry by Land.—Section twelve of said Act is amended so as to read as follows:

"SECTION 12. That no Chinese person shall be permitted to enter the United States by land without producing to the proper officer of customs the certificate in this Act required of Chinese persons seeking to land from a vessel.

Deportation.—"And any Chinese person found unlawfully within the United States shall be caused to be removed therefrom to the country from whence he came, and at the cost of the United States, after being brought before some justice, judge, or commissioner of a court of the United States and found to be one not lawfully entitled to be or to remain in the United States; and in all such cases the person who brought or aided in bringing such person to the United States shall be liable to the Government of the United States for all necessary expenses incurred in such investigation and removal; and all peace officers of the several States and Territories of the United States are hereby invested with the same authority as a marshal or United States marshal in reference to carrying out the provisions of this Act or the Act of which this is amendatory, as a marshal or deputy marshal of the United States, and shall be entitled to like compensation to be audited and paid by the same officers.

Maintenance and Return of Person Having Certificate. "And the United States shall pay all costs and charges for the maintenance and return of any Chinese person having the certificate prescribed by law as entitling such Chinese person to come into the United States who may not have been permitted to land from any vessel by reason of any of the provisions of this Act."

Diplomatic Officers Exempt.—Section thirteen of said Act is hereby amended so as to read as follows:

"SECTION 13. That this Act shall not apply to diplomatic and other officers of the Chinese or other Governments traveling upon the business of that Government, whose credentials shall be taken as equivalent to the certificate in this Act mentioned, and shall exempt them and their body and household servants from the provisions of this Act as to other Chinese persons."

Application of Act.—Section fifteen of said Act is hereby amended so as to read as follows:

"SECTION 15. That the provisions of this Act shall apply to all subjects of China and Chinese, whether subjects of China or any other foreign power; and the words Chinese laborers, wherever used in this Act shall be construed to mean both skilled and unskilled laborers and Chinese employed in mining."

SECTION 16. *Penalties.*—That any violation of any of the provisions of this Act, or of the Act of which this is amendatory, the punishment of which is not otherwise herein provided for shall be deemed a misdemeanor, and shall be punishable by a fine not exceeding one thousand dollars, or by imprisonment for not more than one year, or both such fine and imprisonment.

SECTION 17. *Not to Affect Proceedings Under Former Act.* That nothing contained in this Act shall be construed to affect any prosecution or other proceeding, criminal or civil, begun under the Act of which this is amendatory; but such prosecution or other proceeding, criminal or civil, shall proceed as if this Act had not been passed.

Approved, July 5, 1884.

ACT OF SEPTEMBER 13, 1888

AN ACT TO PROHIBIT THE COMING OF CHINESE LABORERS TO THE UNITED STATES

Immigration of Chinese Laborers Prohibited.—Be it enacted by the Senate and House of Representatives of the United States of America in Congress Assembled, That from and after the date of the exchange of ratifications of the pending treaty between the United States of America and His Imperial Majesty the Emperor of China, signed on the twelfth day of March, anno Domini eighteen hundred and eighty-eight, it shall be unlawful for any Chinese person, whether a subject of China or of any other power, to enter the United States, except as hereinafter provided.

SECTION 2. *Privileged Classes.*—That Chinese officials, teachers, students, merchants, or travelers for pleasure or curiosity, shall be permitted to enter the United States, but in order to entitle themselves to do so, they shall first obtain the permission of the Chinese Government, or other Government of which they may at the time be citizens or subjects.

Certificate of United States Consul.—Such permission, and also their personal identity shall in such case be evidenced by a certificate to be made out by the diplomatic representative of the United States in the country, or of the consular representative of the United States at the port or place from which the person named therein comes. The certificate shall contain a full description of such person, of his age, height, and general physical features, and shall state his former and present occupation or profession and place of residence, and shall be made out in duplicate.

One copy shall be delivered open to the person named and described, and the other copy shall be sealed up and delivered by the diplomatic or consular officer as aforesaid to the captain of the vessel on which the person named in the certificate sets sail for the United States, together with the sealed certificate, which shall be addressed to the collector of customs at the port where such person is to land.

Letter to Collector.—There shall be delivered to the aforesaid captain a letter from the consular officer addressed to the collector of customs aforesaid, and stating that said consular officer has on a certain day delivered to the said captain a certificate of the right of the person named therein to enter the United States as a Chinese official, or other exempted person, as the case may be.

Penalties.—And any captain who lands or attempts to land a Chinese person in the United States, without having in his possession a sealed certificate as required in this section, shall be liable to the penalties prescribed in section nine of this Act.

SECTION 3. *Applicable to Chinese Generally.*—That the provisions of this Act shall apply to all persons of the Chinese race, whether subjects of China or other foreign power, excepting Chinese diplomatic or consular officers and their attendants; and the words "Chinese laborers" whenever used in this Act, shall be construed to mean both skilled and unskilled laborers and Chinese employed in mining.

SECTION 4. *Master of Vessel to Deliver Certificate, Letter, and List of Passengers.*—That the master of any vessel arriving in the United States from any foreign port or place with any Chinese passengers on board shall, when he delivers his manifest of cargo, and if there be no cargo, when he makes legal entry of his vessel, and before landing or permitting to land any Chinese person (unless a diplomatic or consular officer, or attendant of such officer) deliver to the collector of customs of the district in which the vessel shall have arrived, the sealed certificate and letters as aforesaid, and a separate list of all Chinese persons taken on board of his vessel at any foreign port or place, and of all such persons on board at the time of arrival as aforesaid. Such list shall show the names of such persons and other particulars as shown by their open certificates, or other evidences required by this Act, and such list shall be sworn to by the master in the manner required by law in relation to the manifest of the cargo.

Diplomatic Officers.—The master of any vessel, as aforesaid, shall not permit any Chinese diplomatic or consular

officer or attendant of such officer to land without having first been informed by the collector of customs of the official character of such officer or attendant.

Penalties.—Any refusal or wilful neglect of the master of any vessel to comply with the provisions of this section shall incur the same penalties and forfeitures as are provided for a refusal or neglect to report and deliver a manifest of the cargo.

SECTION 5. *Return of Laborers Prohibited.*—That from and after the passage of this Act, no Chinese laborer in the United States shall be permitted, after having left, to return thereto, except under the conditions stated in the following sections.

SECTION 6. *Exceptions.*—That no Chinese laborer within the purview of the preceding section shall be permitted to return to the United States, unless he has a lawful wife, child, or parent in the United States, or property therein of the value of one thousand dollars, or debts of like amount due him and pending settlement.

The marriage to such wife must have taken place at least a year prior to the application of the laborer for a permit to return to the United States, and must have been followed by the continuous cohabitation of the parties as man and wife.

If the right to return be claimed on the ground of property or of debts, it must appear that the property is bona fide and not colorably acquired for the purpose of evading this Act, or that the debts are unascertained and unsettled, and not promissory notes or other similar acknowledgments of ascertained liability.

SECTION 7. *Identification of Returning Laborer.*—That a Chinese person claiming the right to be permitted to leave the United States and return thereto on any of the grounds stated in the foregoing section, shall apply to the collector of customs of the district from which he wishes to depart at least a month prior to the time of his departure, and shall make on oath before the said collector a full statement descriptive of his family, or property, or debts, as the case may be, and shall furnish to said collector such proofs of the

facts entitling him to return as shall be required by the rules and regulations prescribed from time to time by the Secretary of the Treasury, and for any false swearing in relation thereto he shall incur the penalties of perjury.

He shall also permit the collector to take a full description of his person, which description the collector shall retain and mark with a number.

Certificate to be Sole Evidence of Right to Return.—And if the collector, after hearing the proofs and investigating all the circumstances of the case, shall decide to issue a certificate of return, he shall at such time and place as he may designate, sign and give to the person applying a certificate containing the number of the description last aforesaid, which shall be the sole evidence given to such person of his right to return.

If this last-named certificate be transferred, it shall become void, and the person to whom it was given shall forfeit his right to return to the United States.

Limitation.—The right to return under the said certificate shall be limited to one year; but it may be extended for an additional period, not to exceed a year, in cases where, by reason of sickness or other cause of disability beyond his control, the holder thereof shall be rendered unable sooner to return, which facts shall be fully reported to and investigated by the consular representative of the United States at the port or place from which such laborer departs for the United States, and certified by such representative of the United States to the satisfaction of the collector of customs at the port where such Chinese person shall seek to land in the United States, such certificate to be delivered by said representative to the master of the vessel on which he departs for the United States.

Return Certificate Indispensable.—And no Chinese laborer shall be permitted to reenter the United States without producing to the proper officer of the customs at the port of such entry the return certificate herein required. A Chinese laborer possessing a certificate under this section shall be admitted to the United States only at the port from which

he departed therefrom, and no Chinese person, except Chinese diplomatic or consular officers and their attendants, shall be permitted to enter the United States except at the ports of San Francisco, Portland, Oregon, Boston, New York, New Orleans, Port Townsend, or such other ports as may be designated by the Secretary of the Treasury.

SECTION 8. *Secretary of the Treasury to Prescribe Regulations.*—That the Secretary of the Treasury shall be, and he hereby is, authorized and empowered to make and prescribe, and from time to time to change and amend such rules and regulations, not in conflict with this Act, as he may deem necessary and proper to conveniently secure to such Chinese persons as are provided for in articles second and third of the said treaty between the United States and the Empire of China, the rights therein mentioned, and such as shall also protect the United States against the coming and transit of persons not entitled to the benefit of the provisions of said articles.

Form of Certificates.—And he is hereby further authorized and empowered to prescribe the form and substance of certificates to be issued to Chinese laborers under and in pursuance of the provisions of said articles, and prescribe the form of the record of such certificate and of the proceedings for issuing the same, and he may require the deposit, as a part of such record, of the photograph of the party to whom any such certificate shall be issued.

SECTION 9. *Violation of Act by Master of Vessel.*—That the master of any vessel who shall knowingly bring within the United States on such vessel, and land, or attempt to land, or permit to be landed, any Chinese laborer or other Chinese person, in contravention of the provisions of this Act, shall be deemed guilty of a misdemeanor and, on conviction thereof, shall be punished with a fine of not less than five hundred dollars, nor more than one thousand dollars, in the discretion of the court, for every Chinese laborer or other Chinese person so brought, and may also be imprisoned for a term of not less than one year, nor more than five years, in the discretion of the court.

SECTION 10. *Vessel in Distress or Touching at Port.*—That the foregoing section shall not apply to the case of any master whose vessel shall come within the jurisdiction of the United States in distress or under stress of weather, or touching at any port of the United States on its voyage to any foreign port or place. But Chinese laborers or persons on such vessel shall not be permitted to land, except in case of necessity, and must depart with the vessel on leaving port.

SECTION 11. *Forgery of Certificate.*—That any person who shall knowingly and falsely alter or substitute any name for the name written in any certificate herein required, or forge such certificate, or knowingly utter any forged or fraudulent certificate, or falsely personate any person named in any such certificate, and any person other than the one to whom a certificate was issued who shall falsely present any such certificate, shall be deemed guilty of a misdemeanor, and upon conviction thereof shall be fined in a sum not exceeding one thousand dollars and imprisoned in a penitentiary for a term of not more than five years.

SECTION 12. *Examination of Passengers.*—That before any Chinese passengers are landed from any such vessel the collector, or his deputy, shall proceed to examine such passengers, comparing the certificates with the list and with the passengers; and no passenger shall be allowed to land in the United States from such vessel in violation of law; and the collector shall in person decide all questions in dispute with regard to the right of any Chinese passenger to enter the United States, and his decision shall be subject to review by the Secretary of the Treasury and not otherwise.

SECTION 13. *Arrest of Chinese Unlawfully in United States.* That any Chinese person or person of Chinese descent found unlawfully in the United States or its Territories may be arrested upon a warrant issued upon a complaint, under oath, filed by any party on behalf of the United States, by any justice, judge, or commissioner of any United States court, returnable before any justice, judge, or commissioner of a United States court, or before any United States court, and when convicted upon a hearing and found and adjudged

to be one not lawfully entitled to be or remain in the United States, such person shall be removed from the United States to the country whence he came.

Appeal to Judge of District Court.—But any such Chinese person convicted before a commissioner of a United States court may, within ten days from such conviction, appeal to the judge of the district court for the district.

A certified copy of the judgment shall be the process upon which said removal shall be made, and it may be executed by the marshal of the district or any officer having authority of a marshal under the provisions of this section.

Punishment of Person Aiding.—And in all such cases the person who brought or aided in bringing such person into the United States shall be liable to the Government of the United States for all necessary expenses incurred in such investigation and removal; and all peace officers of the several States and Territories of the United States are hereby invested with the same authority in reference to carrying out the provisions of this Act, as a marshal or deputy marshal of the United States, and shall be entitled to like compensation, to be audited and paid by the same officers.

SECTION 14. *Diplomatic and Consular Officers.*—That the preceding sections shall not apply to Chinese diplomatic or consular officers or their attendants, who shall be admitted to the United States under special instructions of the Treasury Department, without production of other evidence than that of personal identity.

SECTION 15. *Repeal of Prior Acts.*—That the Act entitled "An Act to execute certain treaty stipulations relating to Chinese," approved May sixth, eighteen hundred and eighty-two, and an Act to amend said Act, approved July fifth, eighteen hundred and eighty-four, are hereby repealed, to take effect upon the ratification of the pending treaty as provided in section one of this Act.

Approved, September 13, 1888.

ACT OF MAY 5, 1892

AN ACT TO PROHIBIT THE COMING OF CHINESE PERSONS INTO THE UNITED STATES

Extension of Period of Exclusion.—Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled, That all laws now in force prohibiting and regulating the coming into this country of Chinese persons and persons of Chinese descent are hereby continued in force for a period of ten years from the passage of this Act.

SECTION 2. *Deportation.*—That any Chinese person or person of Chinese descent, when convicted and adjudged under any of said laws to be not lawfully entitled to be or remain in the United States, shall be removed from the United States to China, unless he or they shall make it appear to the justice, judge, or commissioner before whom he or they are tried that he or they are subjects or citizens of some other country, in which case he or they shall be removed from the United States to such country: *Provided*, That in any case where such other country of which such Chinese person shall claim to be a citizen or subject shall demand any tax as a condition of the removal of such person to that country, he or she shall be removed to China.

SECTION 3. *Burden of Proof.*—That any Chinese person or person of Chinese descent arrested under the provisions of this Act or the Acts hereby extended shall be adjudged to be unlawfully within the United States unless such person shall establish, by affirmative proof, to the satisfaction of such justice, judge, or commissioner, his lawful right to remain in the United States.

SECTION 4. *Imprisonment.*—That any such Chinese person or person of Chinese descent convicted and adjudged to be not lawfully entitled to be or remain in the United States shall be imprisoned at hard labor for a period of not exceeding one year and thereafter removed from the United States, as hereinbefore provided.

SECTION 5. *Writ of Habeas Corpus.*—That after the passage of this Act, on an application to any judge or court

of the United States in the first instance for a writ of habeas corpus, by a Chinese person seeking to land in the United States, to whom that privilege has been denied, no bail shall be allowed and such application shall be heard and determined promptly without unnecessary delay.

SECTION 6. *Registration of Resident Laborers.*—And it shall be the duty of all Chinese laborers within the limits of the United States at the time of the passage of this Act, and who are entitled to remain in the United States, to apply to the collector of internal revenue of their repective districts, within one year after the passage of this Act, for a certificate of residence, and any Chinese laborer within the limits of the United States who shall neglect, fail, or refuse to comply with the provisions of this Act, or who, after one year from the passage hereof, shall be found within the jurisdiction of the United States without such certificate of residence, shall be deemed and adjudged to be unlawfully within the United States, and may be arrested by any United States customs official, collector of internal revenue or his deputies, United States marshal or his deputies, and taken before a United States judge, whose duty it shall be to order that he be deported from the United States, as hereinbefore provided, unless he shall establish clearly to the satisfaction of said judge that by reason of accident, sickness or other unavoidable cause, he has been unable to procure his certificate, and to the satisfaction of the court, and by at least one credible white witness, that he was a resident of the United States at the time of the passage of this Act; and if upon the hearing it shall appear that he is so entitled to a certificate, it shall be granted upon his paying the cost.

Loss of Certificate.—Should it appear that said Chinaman had procured a certificate which has been lost or destroyed, he shall be detained and judgment suspended a reasonable time to enable him to procure a duplicate from the officer granting it, and in such cases the cost of said arrest and trial shall be in the discretion of the court.

Persons Not Laborers.—And any Chinese person, other than a Chinese laborer, having a right to be and remain in

the United States, desiring such certificate as evidence of such right, may apply for and receive the same without charge.

SECTION 7. *Secretary of Treasury to Prescribe Rules and Forms.*—That immediately after the passage of this Act the Secretary of the Treasury shall make such rules and regulations as may be necessary for the efficient execution of this Act, and shall prescribe the necessary forms and furnish the necessary blanks to enable collectors of internal revenue to issue the certificates required hereby, and make such provisions that certificates may be procured in localities convenient to the applicants.

Contents of Certificate.—Such certificates shall be issued without charge to the applicant, and shall contain the name, age, local residence, and occupation of the applicant, and such other description of the applicant as shall be prescribed by the Secretary of the Treasury, and a duplicate thereof shall be filed in the office of the collector of internal revenue for the district within which such Chinaman makes application.

SECTION 8. *Forgery.*—That any person who shall knowingly and falsely alter or substitute any name for the name written in such certificate or forge such certificate, or knowingly utter any forged or fraudulent certificate, or falsely personate any person named in such certificate, shall be guilty of a misdemeanor, and upon conviction thereof shall be fined in a sum not exceeding one thousand dollars or imprisoned in the penitentiary for a term of not more than five years.

SECTION 9. *Fees for Services.*—The Secretary of the Treasury may authorize the payment of such compensation in the nature of fees to the collectors of internal revenue, for services performed under the provisions of this Act, in addition to salaries now allowed by law, as he shall deem necessary, not exceeding the sum of one dollar for each certificate issued.

Approved, May 5, 1892.

NOTE.—The Act of March 3, 1893, for the enforcement of the immigration and contract labor laws, does not apply to Chinese persons.

ACT OF FEBRUARY 15, 1893

AN ACT GRANTING ADDITIONAL QUARANTINE POWERS AND IMPOSING ADDITIONAL DUTIES UPON THE MARINE-HOSPITAL SERVICE

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SECTION 7. That whenever it shall be shown to the satisfaction of the President that by reason of the existence of cholera, or other infectious or contagious diseases, in a foreign country there is serious danger of the introduction of the same into the United States, and that notwithstanding the quarantine defense this danger is so increased by the introduction of persons or property from such country that a suspension of the right to introduce the same is demanded, in the interest of the public health, the President shall have power to prohibit, in whole or in part, the introduction of persons and property from such countries or places as he shall designate and for such period of time as he may deem necessary.

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Approved, February 15, 1893.

ACT OF NOVEMBER 3, 1893

Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled, That section six of an Act entitled "An Act to prohibit the coming of Chinese persons into the United States" approved May fifth, eighteen hundred and ninety-two, is hereby amended so as to read as follows:

SECTION 6. *Resident Laborers Must Register.*—And it shall be the duty of all Chinese laborers within the limits of the United States who are entitled to remain in the United States before the passage of the Act to which this is an amendment to apply to the collector of internal revenue of their respective districts within six months after the passage of this Act for a certificate of residence; and any Chinese laborer within the limits of the United States who shall

neglect, fail, or refuse to comply with the provisions of this Act and the Act to which this is an amendment, or who, after the expiration of said six months, shall be found within the jurisdiction of the United States without such certificate of residence, shall be deemed and adjudged to be unlawfully within the United States, and may be arrested by any United States customs official, collector of internal revenue or his deputies, United States marshal or his deputies, and taken before a United States judge, whose duty it shall be to order that he be deported from the United States, as provided in this Act and in the Act to which this is an amendment, unless he shall establish clearly to the satisfaction of said judge that by reason of accident, sickness, or other unavoidable cause he has been unable to procure his certificate, and to the satisfaction of said United States judge, and by at least one credible witness other than Chinese, that he was a resident of the United States on the fifth of May, eighteen hundred and ninety-two; and if, upon the hearing, it shall appear that he is so entitled to a certificate, it shall be granted upon his paying the cost.

Loss of Certificates.—"Should it appear that said Chinaman had procured a certificate which has been lost or destroyed, he shall be detained and judgment suspended a reasonable time to enable him to procure a duplicate from the officer granting it, and in such cases the cost of said arrest and trial shall be in the discretion of the court; and any Chinese person, other than a Chinese laborer, having a right to be and remain in the United States, desiring such certificate as evidence of such right, may apply for and receive the same without charge; and that no proceedings for a violation of the provisions of said section six of said Act of May fifth, eighteen hundred and ninety-two, as originally enacted, shall hereafter be instituted, and that all proceedings for said violation now pending are hereby discontinued:"

Felons Cannot Register.—Provided, That no Chinese person heretofore convicted in any court of the States or Territories or of the United States of a felony shall be permitted to register under the provisions of this Act; but all such persons

who are now subject to deportation for failure or refusal to comply with the Act to which this is an amendment shall be deported from the United States as in said Act and in this Act provided, upon any appropriate proceedings now pending or which may be hereafter instituted.

SECTION 2. *Laborer Defined.*—The words "laborer" or "laborers," wherever used in this Act, or in the Act to which this is an amendment, shall be construed to mean both skilled and unskilled manual laborers, including Chinese employed in mining, fishing, huckstering, peddling, laundrymen, or those engaged in taking, drying, or otherwise preserving shell or other fish for home consumption or exportation.

The term "merchant," as employed herein and in the Acts of which this amendatory, shall have the following meaning and none other: A merchant is a person engaged in buying and selling merchandise, at a fixed place of business, which business is conducted in his name, and who during the time he claims to be engaged as a merchant, does not engage in the performance of any manual labor, except such as is necessary in the conduct of his business as such merchant.

Establishing Status of Merchant.—Where an application is made by a Chinaman for entrance into the United States on the ground that he was formerly engaged in this country as a merchant, he shall establish by the testimony of two credible witnesses other than Chinese the fact that he conducted such business as hereinbefore defined for at least one year before his departure from the United States, and that during such year he was not engaged in the performance of any manual labor, except such as was necessary in the conduct of his business as such merchant, and in default of such proof shall be refused landing.

Execution of Order of Deportation.—Such order of deportation shall be executed by the United States marshal of the district within which such order is made, and he shall execute the same with all convenient dispatch; and pending the execution of such order such Chinese person shall remain

in the custody of the United States marshal, and shall not be admitted to bail.

Photograph of Resident.—The certificate herein provided for shall contain the photograph of the applicant, together with his name, local residence, and occupation, and a copy of such certificate, with a duplicate of such photograph attached, shall be filed in the office of the United States collector of internal revenue of the district in which such Chinaman makes application.

Such photograph in duplicate shall be furnished by each applicant in such form as may be prescribed by the Secretary of the Treasury.

Approved, November 3, 1893.

NOTE.—By Act of December 7, 1893, collectors are not entitled to fees for registering Chinese. 28 Stat., p 575.

ACT OF AUGUST 18, 1894

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Finality of Decision.—In every case where an alien is excluded from admission into the United States under any law or treaty now existing or hereafter made, the decision of the appropriate immigration or customs officers, if adverse to the admission of such alien, shall be final, unless reversed on appeal to the Secretary of the Treasury.

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Approved, August 18, 1894.

JOINT RESOLUTION OF JULY 7, 1898

Hawaiian Islands.—There shall be no further immigration of Chinese into the Hawaiian Islands, except upon such conditions as are now or may hereafter be allowed by the laws of the United States; and no Chinese, by reason of anything herein contained, shall be allowed to enter the United States from the Hawaiian Islands.

ACT OF APRIL 30, 1900

AN ACT TO PROVIDE A GOVERNMENT FOR THE TERRITORY OF HAWAII

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SECTION 101. *Registration of Chinese in Hawaii.*—That Chinese in the Hawaiian Islands when this Act takes effect may within one year thereafter obtain certificates of residence as required by "An Act to prohibit the coming of Chinese persons into the United States," approved May fifth, eighteen hundred and ninety-two, as amended by an Act approved November third, eighteen hundred and ninety-three, entitled "An Act to amend an Act entitled An Act to prohibit the coming of Chinese persons into the United States,' approved May fifth, eighteen hundred and ninety-two," and until the expiration of said year shall not be deemed to be unlawfully in the United States if found therein without such certificates: *Provided, however*, That no Chinese laborer, whether he shall hold such certificate or not, shall be allowed to enter any State, Territory, or District of the United States from the Hawaiian Islands.

Approved, April 30, 1900.

ACT OF JUNE 6, 1900

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Hereafter the Commissioner-General of Immigration, in addition to his other duties, shall have charge of the administration of the Chinese exclusion law and of the various Acts regulating immigration into the United States, its Territories, and the District of Columbia, under the supervision and direction of the Secretary of the Treasury.

Approved, June 6, 1900.

ACT OF MARCH 3, 1901

Commissioners Who Shall Try Chinese.—Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled, That it shall be lawful for the district attorney of the district in which any Chinese

person may be arrested for being found unlawfully within the United States, or having unlawfully entered the United States, to designate the United States commissioner within such district before whom such Chinese person shall be taken for a hearing.

SECTION 2. *United States Commissioners' Fees.*—That a United States commissioner shall be entitled to receive a fee of five dollars for hearing and deciding a case arising under the Chinese Exclusion Laws.

SECTION 3. *Who May Make Complaint.*—That no warrant of arrest for violations of the Chinese Exclusion Laws shall be issued by United States commissioners excepting upon the sworn complaint of a United States district attorney, assistant United States district attorney, collector, deputy collector, or inspector of customs, immigration inspector, United States marshal, or deputy United States marshal, or Chinese inspector, unless the issuing of such warrant of arrest shall first be approved or requested in writing by the United States district attorney of the district in which issued.*

SECTION 4. That this Act shall take effect immediately.

Approved, March 3, 1901.

ACT OF APRIL 29, 1902

AN ACT TO PROHIBIT THE COMING INTO AND TO REGULATE THE RESIDENCE WITHIN THE UNITED STATES, ITS TERRITORIES, AND ALL TERRITORY UNDER ITS JURISDICTION, AND THE DISTRICT OF COLUMBIA, OF CHINESE AND PERSONS OF CHINESE DESCENT

Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled, That all laws now in force prohibiting and regulating the coming of Chinese persons, and persons of Chinese descent, into the United States, and the residence of such persons therein, including sections five, six, seven, eight, nine, ten, eleven, thirteen, and fourteen of the Act entitled "An Act to prohibit the coming of Chinese laborers into the United States,"

*See Act of April 27, 1904, reenacting section 13, Act of September 13, 1888.

approved September thirteenth, eighteen hundred and eighty-eight, be, and the same are hereby, reenacted, extended, and continued so far as the same are not inconsistent with treaty obligations,* until otherwise provided by law, and said laws shall also apply to the island territory under the jurisdiction of the United States, and prohibit the immigration of Chinese laborers, not citizens of the United States, from such island territory to the mainland territory of the United States, whether in such island territory at the time of cession or not, and from one portion of the island territory of the United States to another portion of said island territory: *Provided, however*, That said laws shall not apply to the transit of Chinese laborers from one island to another island of the same group; and any islands within the jurisdiction of any State or the District of Alaska shall be considered a part of the mainland under this section.

SECTION 2. *Rules and Regulations and Appointment of Agents.*—That the Secretary of the Treasury is hereby authorized and empowered to make and prescribe, and from time to time change, such rules and regulations not inconsistent with the laws of the land as he may deem necessary and proper to execute the provisions of this Act and of the Acts hereby extended and continued and of the treaty of December eighth, eighteen hundred and ninety-four, between the United States and China, and with the approval of the President to appoint such agents as he may deem necessary for the efficient execution of said treaty and said Acts.

SECTION 3. *Chinese Taking Part in Exhibitions.*—That nothing in the provisions of this Act or any other Act shall be construed to prevent, hinder, or restrict any foreign exhibitor, representative, or citizen of any foreign nation, or the holder, who is a citizen of any foreign nation, of any concession or privilege from any fair or exposition authorized by Act of Congress from bringing into the United States, under contract, such mechanics, artisans, agents, or other employes, natives of their respective foreign countries, as

*See Act of April 27, 1904, post.

they or any of them may deem necessary for the purpose of making preparation for installing or conducting their exhibits or of preparing for installing or conducting any business authorized or permitted under or by virtue of or pertaining to any concession or privilege which may have been or may be granted by any said fair or exposition in connection with such exposition, under such rules and regulations as the Secretary of the Treasury may prescribe, both as to the admission and return of such person or persons.

SECTION 4. *Registration in Philippine Islands.*—That it shall be the duty of every Chinese laborer, other than a citizen, rightfully in, and entitled to remain in any of the insular territory of the United States (Hawaii excepted) at the time of the passage of this Act, to obtain within one year thereafter a certificate of residence in the insular territory wherein he resides, which certificate shall entitle him to residence therein, and upon failure to obtain such certificate as herein provided he shall be deported from such insular territory; and the Philippine Commission is authorized and required to make all regulations and provisions necessary for the enforcement of this section in the Philippine Islands, including the form and substance of the certificate of residence so that the same shall clearly and sufficiently identify the holder thereof and enable officials to prevent fraud in the transfer of the same: *Provided, however*, That if said Philippine Commission shall find that it is impossible to complete the registration herein provided for within one year from the passage of this Act, said Commission is hereby authorized and empowered to extend the time for such registration for a further period not exceeding one year.

Approved, April 29, 1902.

ACT OF FEBRUARY 14, 1903

AN ACT TO ESTABLISH THE DEPARTMENT OF COMMERCE AND LABOR

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SECTION 2. *Examination of Accounts.*—That there shall be in said Department an Assistant Secretary of Commerce and Labor, to be appointed by the President, who shall receive a salary of five thousand dollars a year. He shall perform such duties as shall be prescribed by the Secretary or required by law. There shall also be one chief clerk and a disbursing clerk and such other clerical assistants as may from time to time be authorized by Congress; and the Auditor for the State and other Departments shall receive and examine all accounts of salaries and incidental expenses of the office of the Secretary of Commerce and Labor, and of all bureaus and offices under his direction, all accounts relating to the Light-House Board, Steamboat-Inspection Service, Immigration, Navigation, Alaskan fur-seal fisheries, the National Bureau of Standards, Coast and Geodetic Survey, Census, Department of Labor, Fish Commission, and to all other business within the jurisdiction of the Department of Commerce and Labor, and certify the balances arising thereon to the Division of Bookkeeping and Warrants and send forthwith a copy of each certificate to the Secretary of Commerce and Labor.

* * * * * * * * * *

SECTION 4. *Transfer of Bureau of Immigration.*—That the following-named offices, bureaus, divisions, and branches of the public service, now and heretofore under the jurisdiction of the Department of the Treasury, and all that pertains to the same, known as the Light-House Board, the Light-House Establishment, the Steamboat-Inspection Service, the Bureau of Navigation, the United States Shipping Commissioners, the National Bureau of Standards, the Coast and Geodetic Survey, the Commissioner-General of Immigration, the commissioners of immigration, the Bureau of Immigration, the immigration service at large, and the Bureau of Statistics, be, and the same hereby are, transferred from the

Department of the Treasury to the Department of Commerce and Labor, and the same shall hereafter remain under the jurisdiction and supervision of the last-named Department;

* * * * * * * * * *

SECTION 7. *Authority to Enforce Chinese Exclusion Laws.* That the jurisdiction, supervision, and control now possessed and exercised by the Department of the Treasury over the fur-seal, salmon, and other fisheries of Alaska and over the immigration of aliens into the United States, its waters, territories, and any place subject to the jurisdiction thereof, are hereby transferred and vested in the Department of Commerce and Labor: *Provided*, That nothing contained in this Act shall be construed to alter the method of collecting and accounting for the head-tax prescribed by section one of the Act entitled "An Act to regulate immigration," approved August third, eighteen hundred and eighty-two. That the authority, power, and jurisdiction now possessed and exercised by the Secretary of the Treasury by virtue of any law in relation to the exclusion from and the residence within the United States, its territories and the District of Columbia, of Chinese and persons of Chinese descent, are hereby transferred to and conferred upon the Secretary of Commerce and Labor, and the authority, power, and jurisdiction in relation thereto now vested by law or treaty in the collectors of customs and the collectors of internal revenue, are hereby conferred upon and vested in such officers under the control of the Commissioner-General of Immigration, as the Secretary of Commerce and Labor may designate therefor.

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SECTION 13. That this Act shall take effect and be in force from and after its passage: *Provided, however*, That the provisions of this Act other than those of section twelve in relation to the transfer of any existing office, bureau, division, officer or other branch of the public service or authority now conferred thereon, to the Department of Commerce and Labor shall take effect and be in force on the first day of July, nineteen hundred and three, and not before.

Approved, February 14, 1903.

ACT OF APRIL 27, 1904

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That section one of the Act of Congress approved April twenty-ninth, nineteen hundred and two, entitled "An Act to prohibit the coming into and to regulate the residence within the United States, its Territories, and all territory under its jurisdiction, and the District of Columbia, of Chinese and persons of Chinese descent," is hereby amended so as to read as follows:

Laws Reenacted Without Limitation.—"All laws in force on the twenty-ninth day of April, nineteen hundred and two, regulating, suspending, or prohibiting the coming of Chinese persons or persons of Chinese descent into the United States, and the residence of such persons therein, including sections five, six, seven, eight, nine, ten, eleven, thirteen, and fourteen of the Act entitled 'An Act to prohibit the coming of Chinese laborers into the United States,' approved September thirteen, eighteen hundred and eighty-eight, be, and the same are hereby, reenacted, extended, and continued, without modification, limitation, or condition; and said laws shall also apply to the island territory under the jurisdiction of the United States, and prohibit the immigration of Chinese laborers, not citizens of the United States, from such island territory to the mainland territory of the United States, whether in such island territory at the time of cession or not, and from one portion of the island territory of the United States to another portion of said island territory: *Provided, however*, That said laws shall not apply to the transit of Chinese laborers from one island to another island of the same group; and any islands within the jurisdiction of any State or the district of Alaska shall be considered a part of the mainland under this section."

Approved, April 27, 1904.

ACT OF FEBRUARY 6, 1905

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Administration of Immigration Laws in Philippine Islands. That the immigration laws of the United States in force in the Philippine Islands shall be administered by the officers of the general government thereof designated by appropriate legislation of said government, and all moneys collected under said laws as duty or head-tax on alien immigrants coming into said islands shall not be covered into the general fund of the Treasury of the United States, but shall be paid into the treasury of said islands to be used and expended for the government and benefit of said islands.

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Approved, February 6, 1905.

ACT OF MARCH 3, 1905

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Expenses of Aliens Detained as Witnesses.—Provided, That the necessary expenses incident to the detention of aliens ordered deported, whose attendance as witnesses is required in behalf of the United States in prosecutions arising under the immigration laws, may be paid from the permanent appropriation for Expenses of Regulating Immigration.

(Supplementing section nineteen, Immigration Act of March 3, 1903.)

CHINESE EXCLUSION REGULATIONS

(Approved May 3, 1905)

RULE 1. *Admissible Classes.*—Under the provisions of the treaty and laws in relation to the exclusion of Chinese persons, only such persons as are registered Chinese laborers, officials of the Chinese Government, teachers, students, travelers for curiosity or pleasure, merchants and their lawful wives and minor children, seamen as provided in Rule 38, laborers seeking in good faith to pass through the

country to foreign territory as provided in Rule 39, and persons whose physical condition necessitates immediate hospital treatment shall be permitted to land at any port of the United States.

RULE 2. *Application to Persons of American Birth.*—Only those Chinese persons who are expressly declared by the treaty and laws relating to the exclusion of Chinese to be admissible shall be allowed to enter the United States, and those only upon compliance with the requirements of said treaty and laws and of regulations issued thereunder. If the Chinese person has been born in the United States, neither the immigration Acts nor the Chinese Exclusion Acts prohibiting persons of the Chinese race, and especially Chinese laborers, from coming into the United States apply to such person.

RULE 3. *Application of Immigration Laws.*—Chinese aliens shall be examined as to their right to admission to the United States under the provisions of the laws regulating immigration as well as under the laws in relation to the exclusion of Chinese persons.

RULE 4. *Ports of Entry.*—No Chinese person, other than a Chinese diplomatic or consular officer and attendants, shall be permitted to enter the United States except at the ports of San Francisco, California; Portland, Oregon; Boston, Massachusetts; New York, New York; New Orleans, Louisiana; Port Townsend, Washington; Richford, Vermont; Malone, New York; Portal, North Dakota; Sumas, Washington; Honolulu, Hawaii; San Juan and Ponce, Porto Rico; San Diego, California; and Tampa, Florida.

RULE 5. *Isolation of Applicants for Admission.*—Immediately upon the arrival of Chinese persons at any port mentioned in Rule 4, it shall be the duty of the officer in charge of the administration of the Chinese Exclusion Laws to adopt suitable means to prevent communication with them by any persons other than officials under his control, to have said Chinese persons examined promptly, as by law provided, touching their right to admission and to permit those proving such right to land.

RULE 6. *Examination.*—The examination prescribed in Rule 5 should be separate and apart from the public, in the presence of Government officials and such witness or witnesses only as the examining officer shall designate, and, if, upon the conclusion thereof, the Chinese applicant for admission is adjudged to be inadmissible, he should be advised of his right of appeal and his counsel should be permitted, after duly filing notice of appeal, to examine, but not make copies of, the evidence upon which the excluding decision is based.

RULE 7. *Burden of Proof.*—The burden of proof in all cases rests upon Chinese persons claiming the right of admission to, or residence within, the United States, to establish such right affirmatively and satisfactorily to the appropriate Government officers, and in no case in which the law prescribes the nature of the evidence to establish such right shall other evidence be accepted in lieu thereof, and in every doubtful case the benefit of the doubt shall be given by administrative officers to the United States Government.

RULE 8. *Lawful Landing Defined.*—Every Chinese person permitted to land from a vessel for examination at some designated place shall be still considered as on shipboard until finally and lawfully landed, so far as relates to his admission to the United States and so far as relates to the responsibility of the master, agent, or owner of such vessel for his safe guarding, maintenance, and hospital expenses.

RULE 9. *Steamships to Give Notice of Sailings.*—The master, agent, or owner of any vessel or other means of transportation by which Chinese persons are brought to any port of entry named in Rule 4 shall, at least twenty-four hours in advance thereof, notify the chief officer in charge of the administration of the Chinese Exclusion Laws at said port of the intended time of sailing of said vessel or departure of other vehicle therefrom, in order that the said officer may cause to be placed on board thereof every Chinese person whose application for permission to land has been finally denied.

RULE 10. *Chinese to be Returned to Country Whence Came.*—Every Chinese person refused admission to the United States, being actually or constructively on the vessel or other conveyance by which he was brought to a port of entry, must necessarily be returned to the country whence he came, at the expense of the transportation agency owning such vessel or conveyance, since such vessel or conveyance returns to the port or place at which such Chinese person embarked for the United States.

RULE 11. *Records to be Kept at Ports of Entry.*—Every officer in charge of the enforcement of the Chinese Exclusion Laws at a port of entry shall cause to be kept in books, to be furnished by the Bureau of Immigration, complete records of every Chinese person arriving thereat, synopses of which shall be forwarded upon appropriate blanks, not later than the tenth day of each calendar month for the next preceding month, to the Commissioner-General of Immigration, as also weekly reports upon forms to be furnished by the Bureau of Immigration.

RULE 12. *Certificates to be Taken Up at Ports of Entry.* All certificates, or other evidence, offered by Chinese persons to establish their right of admission to the United States, other than laborers' registration certificates, shall be retained by the officers in charge of the administration of the Chinese Exclusion Laws at ports of entry; the immunity from arrest of the Chinese persons admitted thereon resting upon their exclusive occupation in the pursuits for which their certificates, or other evidence, claim that they respectively seek admission to the United States. (See Rule 59.)

RULE 13. *Appeals.*—Every Chinese person refused admission under the provisions of the exclusion laws by the decision of the officer in charge at the port of entry must, if he shall elect to take an appeal to the Secretary, give written notice thereof to said officer within two days after such decision is rendered.

RULE 14. *Evidence Considered on Appeals.*—Notice of appeal provided for in Rule 13 shall act as a stay upon the disposal of the Chinese person whose case is thereby affected

until a final decision is rendered by the Secretary; and, within three days after the filing of such notice, unless further delay is required to investigate and report upon new evidence, the complete record of the case, together with such briefs, affidavits, and statements as are to be considered in connection therewith, shall be forwarded to the Commissioner-General of Immigration by the officer in charge at the port of arrival, accompanied by his views thereon in writing; but on such appeal no evidence will be considered that has not been made the subject of investigation and report by the said officer in charge.

RULE 15. *Time for Perfecting Cases.*—Additional time for the preparation of cases after the expiration of three days next succeeding the filing of notice of appeal will be allowed only in those instances in which, in the judgment of said officer in charge, a literal compliance with Rule 14 would occasion injustice to the appellant or the risk of defeat of the purposes of the law, and the reasons for delay beyond the time prescribed shall in every instance be stated in writing in the papers forwarded to the Commissioner-General of Immigration.

RULE 16. *Papers to be Forwarded With Appeal.*—With the papers on appeal from a decision of the officer in charge at a port of entry there shall be transmitted to the Commissioner-General of Immigration all the evidence bearing upon the case appealed, including the evidence offered in accordance with the laws by returning Chinese laborers who have remained out of the United States for a period of time in excess of one year from the date of their departure to show that the delay was occasioned by their sickness or some other cause of disability beyond their control.

RULE 17. *Return Certificates.*—Only persons registered as Chinese laborers are entitled to the return certificate provided for by section six, Act approved May fifth, eighteen hundred and ninety-two.

RULE 18. *Evidence on Which Return Certificates Are Issued.*—Administrative officers are hereby required, in accordance with section seven of the Act of September

thirteenth, eighteen hundred and eighty-eight, to exact of every registered Chinese laborer applying for a return certificate, not merely a statement of the ground, or grounds, on which such laborer claims a right thereto, but also proofs of the existence of such ground or grounds, either in the form of oral or of written testimony in detail by not less than two credible witnesses, who shall satisfy such officers that they have had opportunity to know the circumstances to which they testify. Until such proof is supplied, the investigation preliminary to granting or refusing the return certificate sought shall not be made.

Rule 19. *Laborer's Return Certificate.*—A Chinese person claiming the right to be permitted to leave the United States and return thereto as a duly registered laborer shall apply in person to the immigration officer whose official station is most conveniently reached from his place of residence, at least a month prior to the time of his departure; shall deposit with said officer a certificate of registration from the internal-revenue collector for the district in which he resided at the time of registration, and shall make, on oath before the officer, in writing, a full statement descriptive of his family, or property or debts, as the case may be, and fully describing himself, giving his age, name, local residence, occupation, color of eyes and complexion, and distinguishing marks, if any, and naming the port from which he expects to depart from the United States, which shall be one of those designated in Rule 4. Such written description shall be filed in duplicate, and to each shall be permanently attached a photograph of the Chinese person referred to therein. The officer with whom such certificate of registration and written description are filed will make a thorough examination to ascertain whether the applicant is registered, and as to the accuracy of the descriptive statement; that the photograph accompanying the latter for the purpose of identification is that of the person described in such certificate and statements, and that his height and descriptive physical marks are accurately given, and will then write his official signature in part across such photograph and in part upon the adjoining portion of the

written descriptive statement, to prevent substitution. The said officer will then transmit the certificate of registration to the Commissioner-General of Immigration for comparsion with the record thereof in his office, in respect not only to name and date therein, but in all other particulars. At the same time the said officer will in person or through an immigrant or Chinese inspector make thorough investigation as to the facts stated therein. As soon as practicable thereafter the said officer will transmit such registration certificate, one copy of the sworn statement, and the reports of investigation to the officer in charge at the port from which such Chinese laborer intends to depart from the United States, and at the same time will transmit to said Chinese laborer the duplicate copy of such sworn statement, with instructions to present the same in person to the officer in charge at the port of departure. Upon the receipt of such certificate of registration, the duplicate copies of said sworn statement, and the reports of investigation, the officer in charge, or his deputy at said port of departure, after one month from the date of filing of the original application with the officer who investigated the case, if he finds that the person presenting such duplicate statement is the Chinese person therein described and is entitled thereto, may sign and give to such person on his departure from said port a certificate containing the number of the description referred to, in the following form:

FORM OF LABORER'S RETURN CERTIFICATE

No. _____ UNITED STATES OF AMERICA

Certificate issued to Chinese laborer departing from the United States with the intention of returning thereto

This is to certify that_____, a Chinese laborer, described in identification paper numbered_____, port of_____, departed from this port for_____ on this_____day of_____, 190_, with the avowed intention of returning to the United States via this port within twelve months from said date.

Given under my hand and seal this_____day of_____, 190_, at_____, State of_____.

[SEAL] _____

Officer in Charge, Port of_____, District of_____

Certificate Void if Transferred.—If the last-named certificate be transferred it shall become void, and the person to whom it was given by the officer in charge shall forfeit his right to return to the United States.

The certified description shall be carefully preserved by the officer in charge at the port of exit as a means of identification of the Chinese person therein mentioned, who must return via the port of departure within one year from the date of his leaving the United States, unless prevented by sickness or other disability beyond his control.

Report of Investigation.—The officer who conducts the above-mentioned investigation will furnish the officer in charge of the district in which the Chinese laborer resides with a copy of the report forwarded by him to the officer in charge at the port of departure.

RULE 20. *Laborers Must Be Advised of Conditions.*—Every departing Chinese laborer who has been furnished with a return certificate under the provisions of section seven of the Act approved September thirteenth, eighteen hundred and eighty-eight, must be informed by the officer by whom such certificate is granted that upon his return to the port of departure, as a prerequisite to admission, there must exist one of the grounds recited in section six of said Act to entitle him to admission after a temporary absence from the United States.

RULE 21. *Investigating Status of Laborers.*—The officer issuing a return certificate to a departing laborer shall at the time of such issuance furnish said laborer with the following notice (properly filled out) and attached form of letter for his use in sending information of his intended return:

OFFICE OF_____
PORT OF_____
_____, 19___

SIR: In accordance with Rule 21 of regulations issued by the Secretary of Commerce and Labor, you are hereby informed that, in order to avoid the risk of a possibly fruitless voyage, you should, at least ninety days and not exceeding four months prior to your intended return to this port, notify the undersigned officer of the approximate date of such return, giving your name and foreign

address on the subjoined blank form, and stating the ground or grounds on which you claim the right to admission to the United States, as well as the occasion of your delay, if there has been such delay in returning, beyond one year from the time of your departure. An investigation of your case will, on receipt of such notification, be promptly made and you will be formally advised of the result thereof by a letter sent to the foreign address given.

Respectfully,

(Name of officer)_____

(Title of officer)_____

(Address)_____

(Date)_____, 19__

OFFICER IN CHARGE ENFORCEMENT CHINESE EXCLUSION LAWS,

Port of_____

SIR: In compliance with the above notification, delivered to me prior to my departure from the United States, you are informed that I intend to return thereto through your port, per steamship _____, on or about_____, 19__

I was granted return certificate No._____, dated_____, 19__, and departed from your port per steamship_____on _____, 19__

I claim the right of admission because I have in the United States (either):

A father, named_____; or mother, named_____, residing at_____; a child (or children), named_____, a lawful wife, named_____, residing at_____, to whom I was married on the_____day of_____, and with whom I have continuously cohabited since the said marriage.

The following property of the clear, unencumbered value of $_____, _____, _____

The following bona-fide debts due me and pending settlement, and not evidenced by promissory notes or other similar acknowledgments of ascertained liability, and not colorably acquired for the purpose of evading the law:

Due from_____, $_____

Due from_____, $_____

Address of debtors, _____ _____

I respectfully request that my claims be preinvestigated and that I may be notified at the above address before_____, 19__, whether I will be permitted to reenter the United States.

I was delayed_____days_____months beyond the year after my departure by_____, and will submit the evidence thereof required by the law and regulations.

Respectfully,

(Name)_____

RULE 22. *Disposition of Duplicates and Stubs of Return Certificates.*—The officer in charge at the port of departure shall send a copy of the certified description, with photograph of the person therein named attached, and also one of the stubs hereinafter referred to, to the Commissioner-General of Immigration; said stub and said copy of the certified description shall be filed together.

Certificates as above described, with a serial number attached, will be issued to officers in charge at ports from which Chinese depart upon application therefor to the Commissioner-General of Immigration. In all instances, officers in charge will fill out the blanks on the stubs of the certificates. Officers in charge will submit reports to the Commissioner-General of Immigration weekly of Chinese persons departing from and returning to their respective ports, debiting themselves with the number of certificates received from the Department, crediting themselves with the number used and reporting the number remaining on hand. The officers in charge shall cancel all return certificates presented on admission of returning Chinese and forward said certificates so canceled to the Commissioner-General of Immigration.

RULE 23. *Bertillon Record of Laborers.*—It shall be the duty of the officer in charge at the port of entry to register and make a Bertillon record of all Chinese laborers arriving at or departing from said port, which record shall show the individual, family, and tribal name, age, occupation, when and where followed, last place of residence, physical peculiarities, and such other pertinent information as may assist in the identification or investigation of such person in case of his returning to the United States. A duplicate of the record of such registry shall be transmitted to the Commissioner-General of Immigration for each such vessel or railway train.

RULE 24. *Readmission of Laborers.*—Upon the arrival of any registered Chinese laborer at a port of the United States designated in Rule 4, he shall exhibit his return certificate to the officer in charge of the enforcement of the Chinese Exclusion Laws at said port, and if the evidence already secured in such a case establishes the right of such a returning Chinese

laborer to admission and is not controverted, then he should be admitted without delay, but otherwise he shall be refused admission until he establishes his right thereto.

RULE 25. *Return of Registration Certificates to Laborers.* If a returning Chinese laborer is admitted, his certificate of residence, which must have been left with the officer who granted him a return certificate, shall be returned to him and the said return certificate be taken up. After indorsing upon such return certificate his action in the case and the date thereof, the officer in charge shall forward it to the Commissioner-General of Immigration, accompanied, if the Chinese applicant is denied admission, by said laborer's certificate of residence.

RULE 26. *Indorsement of Consular Officers.*—Whenever a Chinese laborer holding a return certificate is detained by his sickness or by other disability beyond his control for a time in excess of one year after the date of his departure from the United States, the facts shall be fully reported to and investigated by the consular representative of the United States at the port or place from which such laborer departs for the United States, and such consular representative shall certify, to the satisfaction of the officer in charge at the port of return, which must be the port from which such laborer departed, that he has fully investigated the statements of such laborer and believes that he was unavoidably detained for the time specified and for the reason stated, such certificate to be delivered by such consular representative to the master of the vessel on which the Chinese laborer departs for the United States and by the master delivered to the officer in charge at the port of return.

RULE 27. *Investigation of Status of Merchants Prior to Departure.*—Every Chinese merchant resident within the United States who desires to go abroad temporarily, to avoid unnecessary delay in securing admission upon his return, may transmit to the officer in charge at the intended port of departure, not less than thirty days prior to the date on which he intends to depart, affidavits of two credible witnesses, other than Chinese, showing said merchant's actual

ownership, in whole or in part, of an established mercantile business in this country, the amount of his interest therein, the nature of his personal employment in conducting said business, and that during the entire year antecedent to the date of such affidavits he engaged in no manual labor except such as was necessarily incident to the conduct of said business. If after a thorough investigation said officer is satisfied of the truth of the statements contained in the above-mentioned affidavits, he may indorse thereon over his signature his finding to that effect, coupled with a statement that the said Chinese merchant will, upon his return to the port of departure and proper identification, be permitted to enter the United States, unless some of said statements should subsequently be shown to be false; provided, however, that said merchant still retains, unchanged, the mercantile interest owned by him at the time of his departure.

RULE 28. Chinese persons applying for admission upon the ground of having been domiciled in the United States as merchants shall be required to establish, to a reasonable certainty, that they are actually owners of the business claimed or members of the firm owning such business, with proof of the amounts actually paid for their respective interests, the times at which such payments were made, and, if residents of the United States during the period of registration, whether they failed to register, and, if so, the reason therefor.

RULE 29. *Character of Evidence Required of Returning Merchants.*—In the enforcement of the provisions of section two of the Act of November third, eighteen hundred and ninety-three, relating to the application for admission of alleged returning Chinese merchants, it will not be enough to have witnesses testify that an applicant, for at least one year before his departure from the United States, was not engaged in the performance of any manual labor except such as was necessary in the conduct of his business as such merchant, but the testimony should show specifically the kind of work the Chinaman has done during the entire year, and, after detailing the character of such work, should say that he has

not performed any other labor than that specifically set forth.

RULE 30. *Photographing Establishments of Returning Merchants.*—With a view to assisting in the decision of the cases of alleged Chinese merchants applying for admission to the United States as such, the following instructions are promulgated, for the guidance of all officers called upon to investigate the character of establishments in which such applicants claim to have an interest, to wit:

(*a*) In every case when investigation shows that the alleged mercantile establishment has operated in connection with it a laundry, restaurant, gambling establishment, or other nonmercantile business, or where there is anything to indicate that such establishment is in fact different from that which is claimed, it is directed that photographs be taken of the room or rooms occupied by the firm, and, if necessary, a view of the front or rear of the building, or both.

(*b*) These photographs should be of a size not less than 5 inches by 7 inches nor more than 8 inches by 10 inches, and should be taken in such manner as to clearly show the real character of the business reported upon.

(*c*) The cost of each photograph shall include three unmounted prints thereof, together with the original negative. The latter should be taken in charge by the officer making the investigation, and by him carefully preserved, subject to the orders of the Department. If subsequently ordered sent elsewhere, negatives should be carefully packed in wooden boxes with excelsior filling, and should be properly labeled and shipped by express—never by mail—with suitable marks upon the exterior of package to insure care of the contents.

(*d*) One print of each photograph should be retained with the negative, the remaining two prints to accompany the report of investigation sent to the official who originally desired the inquiry made.

(*e*) Care should be taken to avoid excessive charges by photographers, but reasonable expense for this service will be allowed upon request made of the Bureau of Immigration,

Department of Commerce and Labor, such items to be paid from the appropriation Enforcement Chinese Exclusion Acts. Receipt of photographer will be required as a subvoucher in each instance.

RULE 31. *Investigation of Status of Returning Merchants.* As an additional means of securing prompt action upon the case of every Chinese person seeking admission as a returning merchant, as well as to save him the risk of a possibly fruitless voyage, the officer shall, at the time of departure of every such Chinese person, whose case has not already been investigated under Rule 27, furnish him with the following notice (properly filled out) and attached form of letter for his use in sending notification of his intended return:

OFFICE OF_____

PORT OF_____

_____ (Date)_____, 19__

SIR: In accordance with Rule 31 of the Chinese Regulations of the Department of Commerce and Labor, you are hereby notified that, in order to avoid as far as possible the risk of making an unnecessary trip which would result from a failure to establish your right to admission, you should, at least ninety days and not exceeding four months prior to your intended arrival at this port, notify the undersigned officer of your intention to return, the approximate date thereof, giving your name and foreign address, and a statement of your claims to the right of admission, on the subjoined blank. You may now, or within the time above mentioned, submit the evidence required by law to establish your mercantile status. The said claims will, on receipt of such notice, be promptly investigated, and a formal notification of the result of such investigation will be sent to you at the address given. This decision is not final, and you are not precluded, if such result is adverse, from obtaining upon your arrival at a port of the United States a further investigation of your claims.

Respectfully,

(Name of officer)_____

(Title of officer)_____

(Address)_____

(Date)_____, 19__

OFFICER IN CHARGE ENFORCEMENT CHINESE EXCLUSION LAWS.

Port of_____.

SIR: In compliance with the above notification, delivered to me prior to my departure from the United States, you are informed that I intend to return thereto, through your port, per_____on or about_____, 19__

I departed from the United States on the steamship_____, which sailed from your port on_____, 19__

I have filed with you the evidence required by law to support my claim of right to readmission. I am a merchant and member (or proprietor) of the store of_____, at_____, in which my interest was, and still remains, $_____

I respectfully request that my claim be preinvestigated, and that I may be notified before_____, 19__, at the above address, whether I will be permitted to reenter the United States, and, if so, upon what condition. Respectfully,

(Name)_____

Upon the receipt of such notification, the officer in charge shall make, or cause to be made, a thorough investigation of the claim therein advanced and send due notice of his conclusions in regard thereto to the applicant, who, upon arrival, shall establish his identity with the sender of the notification and submit such additional evidence as may be necessary to prove his right to admission. If the evidence previously secured shall establish the right of such applicant to admission, and is not controverted, then he shall be admitted without delay, but otherwise he shall be refused admission until he shall establish his right thereto.

RULE 32. *Admission of Wives and Minor Children of Domiciled Merchants.*—The Supreme Court having decided that the lawful wife and minor children of a Chinese merchant may be admitted to the United States without presenting the certificate prescribed by section six of the Act approved July fifth, eighteen hundred and eighty-four, the attention of administrative officers is directed to the nature of the evidence required by section two of the Act approved November third, eighteen hundred and ninety-three, to establish the right of a domiciled Chinese merchant to readmission after temporary absence from the United States, and the

said officers are directed to require such evidence as will give reasonable assurance that every Chinese applicant for admission as the lawful wife or minor child of a resident Chinese merchant sustains the actual relationship claimed to such merchant, and, in the case of a child, that such child is beyond question a minor.

RULE 33. *Officers Designated to Issue Certificates.*—The officers whose titles are given below have been authorized by their respective Governments to issue to Chinese *subjects*, or *citizens*, of such Governments the certificates prescribed by section six of the Act approved July fifth, eighteen hundred and eighty-four. Until other officers are designated by foreign Governments only certificates issued by those named shall be accepted as conforming to the requirements of law; and Chinese persons of the exempt classes coming from countries which have not designated officers to issue such certificates to Chinese *subjects*, or *citizens*, thereof cannot be admitted to the United States.

Brazil: Chiefs of Police, or corresponding officers, in the municipalities and civil subdivisions.

Canada:

Vancouver—Collector of Customs.
Victoria—Collector of Customs.
New Westminster—Collector of Customs.
Ottawa—Chief Controller of Chinese, or Chief Clerk in the Department of Trade and Commerce.

China:

Acting Viceroy of Hu Kuang (Hunan and Hupeh).
Acting Viceroy of Sze Ch'uen.
Acting Viceroy of Liang Kuang (Kuangtung and Kunaghsi).
Tartar general of Fu-chou and Customs Superintendent of Fu-k'ien.
Governor of Anhui.
Governor of Hunan.
Governor of Shantung.
Governor of Kiangsi.
Customs Taot'ai of Tientsin.
Taot'ai of the Hui-Ning-Ch'ih-T'ai-Kwang Circuit.

Taot'ai of the Hang-chia-hu Circuit.
Acting Taot'ai of the Ning-Shao-T'ai Circuit.
Taot'ai of the Wen-Ch'u Circuit.
Taot'ai of the Yue-Ch'ang-Li Circuit.
Taot'ai of the Teng-Lai-Ch'ing Circuit.
Taot'ai of the Su-Sung-T'ai Circuit.

Cuba: Chief of Immigration Department.

German Protectorate of Kiautschou: Commissioner for Chinese Affairs to the Government, Civil Commissioner, or Oberrichter.

Guatemala: Minister of Foreign Affairs, or Sub-Secretary of State.

Hawaii: Secretary of the Territory.

Hongkong: Registrar-General.

Japan:
Governor of any Fu (district) or Ken (prefecture).
Hokkaido—Governor-General.
Formosa—Chief of Prefecture having jurisdiction.

Mexico: Department for Foreign Affairs.

Philippine Islands: Collector of Customs.

Portugese Province of Macau: Secretary-General.

Society Islands: Commissioner of Police of the Municipality of Papeete, Tahiti.

Straits Settlements: Colonial Secretary.
Federated Malay States—Colonial Secretary, Federal Secretary, or Secretary for Chinese Affairs.

Trinidad: Governor.

Venezuela: Mayors of Cities or Governors of Provinces.

RULE 34. *Omission of Required Items in Certificates.*—The omission from a certificate prescribed by section six of the Act approved July fifth, eighteen hundred and eighty-four, of any of the information required by said section is fatal to the sufficiency of such certificate as evidence of the right of the person to whom issued to enter the United States.

RULE 35. *Cancelation of Certificates.*—The certificates prescribed by section six of the Act of July fifth, eighteen hundred and eighty-four, on which Chinese of the exempt class were admitted to the United States, should be canceled

by officers in charge by writing across the face thereof in red ink the fact of the admission and the date thereof.

RULE 36. *Definition of Term Student.*—In considering evidence touching the right of a Chinese student to enter the United States, administrative officers should be governed by the opinion of the solicitor of the Department of Commerce and Labor (of April seventeenth, nineteen hundred and five) that a Chinese student, within the intention of the treaty of November seventeenth, eighteen hundred and eighty, and the laws, is a person who intends to pursue some of the higher branches of study, or to become fitted by study for some profession or occupation, and for whose support and maintenance in this country, as a student, provision has been made.

RULE 37. *Temporary Landing of Sick Chinese.*—No Chinese person, other than a seaman, will be permitted to land temporarily under bond, except upon the certificate of a surgeon of the Public Health and Marine-Hospital Service that such temporary landing is necessitated by the condition of his health, and with the permission of the Secretary of Commerce and Labor, under such conditions as he shall prescribe.

RULE 38. *Seamen's Bonds.*—To prevent violations of law by Chinese seamen discharged or granted shore leave at ports of the United States, bond with approved security in the penalty of $500 for each such seaman shall be exacted for his departure from and out of the United States within thirty days.

RULE 39. *Admission of Transits.*—Every Chinese person seeking the privilege of transit through the United States to foreign territory shall, as a condition precedent to being allowed such privilege, comply with the following requirements; and if such a person is found, in the judgment of the officer in charge at the port of arrival, to be seeking the privilege of transit with an ulterior purpose of gaining an unlawful access to the United States, he shall be refused permission to land:

(*a*) The applicant shall be required to produce to the officer in charge of the enforcement of the Chinese Exclusion

Laws at the port of arrival a prepaid through ticket across the whole territory of the United States, land or water, intended to be traversed (and to his alleged foreign destination according to the manifest of the vessel on which he arrives), and such other reasonable proof as may be required of him to satisfy the said officer that a bona-fide transit only is intended and that the applicant does not seek the foreign destination named by him with an ulterior purpose of thereby gaining access to the United States in violation of law; and such ticket and other evidence in writing presented by the applicant must be so stamped or marked and dated by the said officer or his deputy as to prevent their use a second time; but no such applicant shall be considered as intending in good faith to make such transit only if he has already, on same arrival, made application for and been denied admission to the United States.

(*b*) The applicant in such case, or some responsible person in his behalf, or the transportation company whose through ticket he holds, shall furnish to the said officer in charge a good and sufficient bond in the penal sum of $500, conditioned for applicant's continuous transit through and actual departure from the United States within a reasonable time, not exceeding in any instance twenty days from the date upon which said privilege is granted; but the said bond shall not be required of any such applicant who remains on shipboard for transit through the water territory of the United States, or who is transferred from one vessel to another vessel in a port of the United States for a similar transit, unless the vessel on which said applicant departs may touch at another port of the United States on the way to its foreign port or destination.

(*c*) The applicant in every such case shall furnish to said officer in charge, to be taken as directed by said officer, a photograph of himself in triplicate, and shall submit, if he is not a member of one of the exempt classes, to the physical examination of his person required by the Bertillon system of identification.

Rule 40. *Action of Officers Concerning Transits.*—The officer in charge of the enforcement of the Chinese Exclusion

Laws at the port of arrival shall prepare a descriptive list, to which one of the photographs required by Rule 39 (*c*) shall be attached, bound in book form, for file in his office, containing as to each Chinese applicant for the privilege of passing through the United States to foreign territory, the following information: Name, age, sex, last place of residence, and the data referred to therein by file number required for his identification. To the said descriptive list there shall be attached a dated and signed statement by the said officer in charge that applicant has complied with all the provisions of Rule 39, and that being assured of applicant's good faith the privilege of transit under bond has been accorded him.

Rule 41. Two copies of the bound descriptive list, required by Rule 40, shall be prepared by the officer in charge on detached blanks corresponding in form with the said bound descriptive list, to each of which shall be attached one of the photographs required by Rule 39 (*c*), and upon both of said photographs, as well as on the one attached to said bound list, shall be stamped the seal of the said officer in charge, so placed as not to cover any part of the face. One of said copies shall be forwarded by the first mail after it is prepared to the officer in charge of the intended port of exit and the remaining one shall be given to the conductor of the train, or to the captain of the vessel, by which the Chinese person to whom they relate is carried, for delivery to the said officer at the port of exit.

Rule 42. One of the copies described in Rule 41 shall be retained by the officer in charge at the port of exit, for his files, and the other, after an indorsement has been made thereon, duly signed and dated, to the effect that the Chinese person named therein has been identified and has departed from the United States, shall be returned by mail to the officer by whom it was prepared, and its receipt by him, duly executed as herein required, shall be his authority for cancelation of the bond given on behalf of the Chinese person, as provided in Rule 39 (*b*), to whom said descriptive list refers.

Rule 43. *Chinese Dwelling in Insular Territory.*—In view of the provisions of section one of the Act approved April

twenty-ninth, nineteen hundred and two, it will be necessary for Chinese persons of the exempt classes who are *citizens*, or *subjects*, of the insular territory of the United States, to comply with the terms of section six of the Act approved July fifth, eighteen hundred and eighty-four, and for this purpose the permission of such persons to go from one insular territory to another insular territory of the United States, or from such insular territory to the mainland territory of the United States, shall be granted by an officer designated for that purpose by the chief executives of said insular territories, respectively, and the duties imposed by section six of the Act approved July fifth, eighteen hundred and eighty-four, upon United States diplomatic and consular officers in foreign countries in relation to Chinese persons of the said classes shall be discharged by the officers in charge of the enforcement of the Chinese Exclusion Acts at the ports, respectively, from which any members of such excepted classes intend to depart from any insular territory of the United States.

RULE 44. *Admission of Chinese to Take Part in Expositions.*—*Every Chinese person seeking admission to the United States under the provisions of section three of the Act approved April twenty-ninth, nineteen hundred and two, for the purpose of taking part in any fair or exhibition authorized by Act of Congress, shall, as a condition precedent to such admission, comply with the following conditions, and officers charged with the enforcement of the Chinese Exclusion Laws shall likewise, so far as action on their part is required, conform thereto:

(*a*) Every such person shall furnish to the officer in charge of the enforcement of the Chinese Exclusion Laws at the port of entry satisfactory evidence that he is a holder of a privilege from the officers of said exposition or an employe of a holder of such a privilege, engaged to take part therein.

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*The provisions of Rule 44 do not apply to Chinese persons seeking admission as members of the exempt classes. Such persons are allowed to enter upon complying with the requirements of section six of the Act of July fifth, eighteen hundred and eighty-four.

(*b*) He shall furnish to the said officer in charge at the port of entry a photograph of himself, in triplicate, and shall submit to such examination as may be deemed necessary to insure his identification.

(*c*) He shall, if admitted, proceed immediately, by direct and continuous travel, to the said exposition, wherever located; remain there during his stay in the United States; engage while there solely in the occupation for which he was admitted; return within thirty days after the close of said exposition by direct and continuous travel to the port at which he was admitted and thence depart, by the first vessel sailing thereafter, to China or to the country of which he is a citizen or subject.

(*d*) He shall furnish to the United States officer in charge, by whom he is admitted, a bond of $500, of a character satisfactory to the Department, conditioned for his immediate departure to said exposition grounds, his constant attendance and employment at said grounds, and his departure from the United States, all in accordance with the requirements mentioned in paragraph (*e*), RULE 44.

FORM OF CHINESE RECORD

Chinese Employe's Record of Admission

DEPARTMENT OF COMMERCE AND LABOR
BUREAU OF IMMIGRATION
File No.____of Record____.
Name _____
Native of_____
Employed by ____ _____, of_____, an exhibitor at the_____
When admitted, _____ —, _____
(Here attach photograph to both duplicate and original.)

No._____ U. S. IMMIGRATION SERVICE
Port of_____
_____

This is to certify, that_____, a native of_____, who is duly accredited as an employe of_____, of_____, an exhibitor at the_____, U. S. A., has been permitted to enter the United States as such employe (or exhibitor) in pursuance of an Act of Congress approved April 29, 1902.

(Name of officer) _____
(Title of officer)_____

(Duplicate to be forwarded to Commissioner-General of Immigration.)

(*e*) The foregoing form of record shall be kept in duplicate by the officer in charge at the port of admission of all Chinese persons admitted in accordance with these regulations, the original to be kept on file at said port and the duplicate to be forwarded to the Bureau of Immigration, Department of Commerce and Labor.

(*f*) An officer, or officers if necessary, will be designated to remain at the grounds of said exposition, whose duty it shall be to keep, in duplicate, a record, in the following form (unless hereafter modified), of every Chinese person admitted under these rules and regulations to take part in said exposition:

FORM OF REPORT

File No., _____

(Place to attach photograph)

Name of employe or exhibitor, _____
Sex, _____
Age, _____
Name of exhibitor by whom employed, _____
Character of work to be done in connection with exhibition, _____
What hours and how many per diem employed, _____
Time of arrival at exhibition grounds _____
Time of departure from exhibition grounds, _____

_____
(Signature of Chinese employe or exhibitor.)

_____
(Signature of officer or officers.)

(Duplicate to be forwarded to Commissioner-General of Immigration.)

Said officer or officers shall also have such supervision over every such Chinese person and such duties in relation to them as are authorized by law and these regulations and instructions issued thereunder.

(*g*) That one of the photographs provided for in paragraph (*b*) hereinbefore shall be sent by the officer at the port of arrival at which any such Chinese person is admitted to the officer or officers provided for in paragraph (*f*) of these regulations, who shall attach said photograph to the form

of report prescribed, using it as a means of identification of the person whose portrait it is.

(*h*) Should any such Chinese person desire to leave the grounds of said exposition temporarily, he must apply to the officer provided for in paragraph (*f*) hereof, who may issue to him a dated card, giving his name and other particulars for his identification, and stating that he is permitted to absent himself from said grounds not longer than forty-eight hours from the date of said card, which shall, upon his return within the specified time, be delivered to said officer and canceled.

(*i*) All Chinese persons holding such cards who fail to return within the specified period shall be deemed to be unlawfully within the United States, and the bonds given on their behalf shall be forfeited and such person shall be deported. The officer designated to issue said cards shall promptly notify the Commissioner-General of Immigration of the absence, in violation of these regulations, of any Chinese person, in order that the penalty of the bond given in his behalf shall be collected and such Chinese person deported.

(*j*) Immediately after the expiration of the thirty days following the close of said exposition the officers at the ports of arrival at which any such Chinese person is admitted are directed to report to the Commissioner-General of Immigration the total number, together with names, etc., of the Chinese persons who were admitted at their ports, respectively, for the purpose stated, and the numbers and names of those who have been properly identified and permitted to depart, in accordance with these regulations.

RULE 45. *Acceptance of Certified Copy of Court Record.* No authenticated copy of a judicial finding that a Chinese person was born in the United States shall be accepted as conclusive in favor of the person presenting it unless he be completely identified as the person to whom such authenticated copy purports to relate.

RULE 46. *Certificates Found Elsewhere Than With Persons to Whom Issued.*—Certificates of residence issued to Chinese

registered laborers, if found elsewhere than in possession of persons to whom issued, should be taken up and deposited with the officer in charge, subject to orders of the Department.

RULE 47. *Certificates of Residence Under Act of 1892.* Chinese laborers duly registered under Act of eighteen hundred and ninety-two, are not required to register under Act of eighteen hundred and ninety-three to secure admission.

RULE 48. *Officer Empowered to Issue Certificates of Residence.*—The authority, power, and jurisdiction in relation to the registration of Chinese lawfully resident in the United States, formerly vested by law in collectors of internal revenue, have been transferred to and conferred upon H. L. Pittenger, Chinese Inspector, Bureau of Immigration, Washington, District of Columbia.

RULE 49. *Method of Issuing Certificates of Residence.* Hereafter an original certificate of residence can be issued to a Chinese laborer only upon the finding of a justice, judge, or commissioner of a United States court that such Chinese laborer was a resident of the United States during the period of registration and that by reason of accident, sickness, or other unavoidable cause, he was then unable to secure such a certificate; except under section four of the Act approved April twenty-nine, nineteen hundred and two.

RULE 50. *Manner of Issuing Duplicate Certificates of Residence.*—Duplicate certificates of residence shall be issued only upon satisfactory proof to the Commissioner-General of Immigration that the Chinese person upon whose behalf application therefor is made has actually, by unavoidable accident, lost his original certificate.

RULE 51. *The Issuing of Certificates.*—Officers in charge of the enforcement of the Chinese Exclusion Laws shall not issue any certificate, letter, or other document, or any duplicate thereof, other than those provided for by law and these regulations, setting forth the status of a Chinese person as a resident of this country, or otherwise indorsing such person.

RULE 52. *Arrest of Chinese.*—Every Chinese laborer found within the United States without the certificate of registration

prescribed by law shall be arrested, after being allowed a reasonable opportunity under the close surveillance of the officer making such arrest to produce such certificate, by any officer charged with the enforcement of the Chinese Exclusion Laws; and shall be taken by such officer before a justice, judge, or commissioner of a United States court in order that a warrant may be issued upon the oath of such officer for the commitment and trial of the said Chinese laborer.

RULE 53. *Arrests Because of Wrong Admission.*—Chinese persons who gain admission to the United States as members of the admissible classes and who after admission become laborers shall be arrested as being unlawfully in this country, as provided in Rule 52.

RULE 54. *Method of Assuring Proper Identification of Prisoners.*—The Comptroller of the Treasury having, on April fourteenth, nineteen hundred and five, rendered a decision to the effect that the expense of photographing Chinese arrested within the United States may properly be paid from the appropriation for the Enforcement of the Chinese Exclusion Acts, the practice with respect to the photographing of such persons heretofore existing will be modified to the extent indicated by the following:

(*a*) Every Chinese person whose arrest is caused by an immigration or other official will be photographed immediately upon the consummation of the arrest, the photograph to be prepared in triplicate and not retouched nor mounted, one copy to be attached to the United States court or commissioner's docket, one to be furnished the officer in charge of the district in which the arrest occurs, and the other to be attached (in the event that deportation is finally ordered) to the writ of deportation.

(*b*) When arrests occur at stations the officers of which are supplied with photographer's apparatus, the photographs will be made by such officers; when in other localities, the immigration officers will have the photographs made by local photographers at the least possible expense compatible with a proper performance of the work, bills therefor to be rendered on the blank vouchers supplied for rendering

accounts, payable from the appropriation Enforcement of the Chinese Exclusion Acts.

(*c*) The copy of the photograph attached to the docket of the court or commissioner should be permanently affixed thereto and in such manner as to render as remote as possible the chance of any change or substitution being made.

(*d*) The copy furnished the officer in charge of the district will be placed in his office records, together with a short history of the case to which it relates, being filed in such manner as to furnish a comprehensive record that can be readily referred to when needed at any future time.

(*e*) The copy attached to the writ in case of deportation should be affixed permanently thereto, and in such manner as to prevent the substitution of some other photograph therefor (the best method of obtaining this result being the impression of the court or commissioner's seal over the edge of such photograph but in such manner as to not mar or deface the features represented thereby), the objects of its use being to afford a means to the marshal of identifying his prisoner as the person referred to in the writ, and to supply the immigration official at the port of deportation with a means of identifying the person delivered on board the vessel as the person to whom the writ relates.

(*f*) Inspectors should request, and will undoubtedly receive, the full cooperation of commissioners or judges and marshals or deputy marshals, so far as necessary, in carrying out the above instructions.

Rule 55. *Prosecutions.*—The Department of Justice, under date of August twenty-sixth, eighteen hundred and ninety-seven, gave the following instructions:

> United States attorneys are hereby directed, either in person or by their assistants, to appear on behalf of the Government to secure the proper enforcement of the law at all hearings held in their respective districts, to determine whether Chinese persons or persons of Chinese descent, who have been arrested under the Chinese Exclusion Laws, have a lawful right to remain in this country.

Rule 56. *Deportation.*—Orders for the deportation of Chinese persons can only be given by a justice, judge, or

commissioner of a United States court upon his decision that such Chinese persons have been found to be unlawfully within the United States, and the cost of making any such deportation is payable from the appropriation for the enforcement of the Chinese Exclusion Laws.

RULE 57. *Charges Incident to Deportation.*—The appropriation Enforcement Chinese Exclusion Acts should be charged with the following expenses:

(*a*) The cost of maintenance of Chinese persons who are taken into custody up to and including the date upon which warrant issued by a United States judge or commissioner is received by the marshal.*

(*b*) The cost of maintenance of Chinese prisoners commencing with the date writ of deportation is first received by the marshal, provided the said order of deportation is not subsequently reversed upon appeal.

(*c*) The cost of deportation, including railroad and steamship fares of prisoners and marshal or deputy, authorized expenses for guard hire, and maintenance en route.

Upon receiving writs of deportation marshals should at once make written report to the Commissioner-General of Immigration, Department of Commerce and Labor, Washington, District of Columbia, giving names of the prisoners, where confined in jail, and when the period of appeal provided by section thirteen of the Act approved September thirteen, eighteen hundred and eighty-eight, will expire. Instructions will then be issued as to the route to be followed, number of guards to be employed, and to whom accounts are to be presented or forwarded for settlement.

RULE 58. *Designation and Location of Officers in Charge.* Under the authority conferred by section seven of the Act approved February fourteen, nineteen hundred and three, entitled "An Act to establish the Department of Commerce and Labor," the authority, power, and jurisdiction in relation to the exclusion of Chinese persons and persons of Chinese descent heretofore vested by law in collectors of customs have been conferred upon and vested in officers in charge of

*Regarding cost of photographing, see Rule 54.

districts as follows, such officers being under the control of the Commissioner-General of Immigration:

Title of Officer	Location of Headquarters	States in District
Commissioner of Immigration	Boston, Mass.	Maine, New Hampshire, Massachusetts, Connecticut, Vermont, and Rhode Island.
Chinese Inspector in Charge..	New York, N. Y.	New York.
Commissioner of Immigration	Philadelphia, Pa.	Pennsylvania, New Jersey, and Delaware.
Chinese Inspector in Charge..	Washington, D. C.	Maryland, Virginia, West Virginia, and District of Columbia.
Chinese Inspector in Charge..	Charleston, S. C.	North Carolina and South Carolina.
Chinese Inspector in Charge..	Mobile, Ala.	Georgia and Alabama.
Chinese Inspector in Charge..	Tampa, Fla.	Florida.
Chinese Inspector in Charge..	New Orleans, La.	Louisiana and Mississippi.
Chinese Inspector in Charge..	Toledo, Ohio	Ohio.
Chinese Inspector in Charge..	Chicago, Ill.	Illinois, Indiana, Kentucky, and Tennessee.
Chinese Inspector in Charge..	Detroit, Mich.	Michigan.
Chinese Inspector in Charge..	Minneapolis, Minn.	Minnesota and Wisconsin.
Chinese Inspector in Charge..	St. Louis, Mo.	Missouri, Iowa, and Arkansas.
Chinese Inspector in Charge..	Portal, N. Dak.	North Dakota and South Dakota.
Chinese Inspector in Charge..	Denver, Colo.	Colorado, Nebraska, Kansas Indian Territory, and Oklahoma.
Chinese Inspector in Charge..	El Paso, Tex.	Texas.
Chinese Inspector in Charge..	Helena, Mont.	Montana and Idaho.
Chinese Inspector in Charge..	Salt Lake City, Utah	Utah, Nevada, and Wyoming.
Chinese Inspector in Charge..	Tucson, Ariz.	New Mexico and Arizona.
Chinese Inspector in Charge..	Port Townsend, Wash.	Washington.
Chinese Inspector in Charge..	Portland, Ore.	Oregon.
Commissioner of Immigration	San Francisco, Cal	California.
Chinese Inspector in Charge..	Ketchikan, Alaska	Alaska.
Commissioner of Immigration	San Juan, Porto Rico	Porto Rico.
Chinese Inspector in Charge..	Honolulu, Hawaii	Hawaiian Islands.

RULE 59 *Identification of Exempts.*—(*a*) With a view to affording a proper and efficient means of identification to Chinese persons, or persons of Chinese descent, admitted to the United States upon proof of their status as members of the classes specifically exempted from the excluding provisions of the Chinese Exclusion Laws, viz., merchants (domiciled or foreign), teachers, students, and travelers for curiosity or pleasure, a certificate of identity, of which the following is a copy, will hereafter be issued by the officer in

charge at the port of entry to each such person admitted or readmitted to this country by him, who may apply for the same; the instructions hereinafter given to be carefully observed in issuing such certificates: *Provided*, That only one such certificate shall be issued to any one Chinese person—except that duplicates may be furnished of those unavoidably lost or destroyed, under the terms of paragraph (*h*) of this rule—the method to be followed with a view to preventing a violation of this proviso being described in paragraph (*i*) hereof.

(*b*)

[FACE]

FORM OF IDENTIFICATION CERTIFICATE

No. _____ Original

UNITED STATES OF AMERICA

Certificate showing lawful admission—Issued in conformity with a regulation of the Department of Commerce and Labor adopted May 3, 1905.

This is to certify that_____, a Chinese person, or person of Chinese descent, belonging to one of the classes specifically exempted from the excluding provisions of the Chinese Exclusion Laws, has this day been admitted by me, satisfactory proof having been submitted and placed on file showing_____to be a_____

This certificate is not transferable, and is granted solely for the identification and protection of said Chinese person so long as_____ status remains unchanged; to insure the attainment of which object an accurate description of said person is written on the reverse side hereof, and photographic likenesses showing full face and profile of said person are attached, with_____name written partly across, the number above given perforated through the edge of, and my official seal impressed partly over, each of said photographs.

[BACK]

Description

[Name]

[Perforation]

Photograph (Full face)

[SEAL]

Name: _____ Age: _____

Occupation: _____

Local residence: _____

Height: ___ft.___in. Head length:_

Head width:____ Head Bi. Zi.:____

Physical marks and peculiarities:

_____

_____

_____

Officer in charge

Port of_____

_____, 19__

[Name]

[Perforation]

Photograph (Profile)

[SEAL]

(*c*) The certificates are printed from engraved plates, numbered consecutively, and bound in books containing fifty each, an original, a duplicate, and a triplicate of each number being furnished, arranged the latter two above the former for convenience in copying from one to the other, and perforated to permit of easy detachment from the book and from each other.

(*d*) In issuing said certificates care will be exercised to have the original and the duplicate correspond in every detail. All blank spaces remaining after writing in the data required to complete the identification of the person to whom the certificate is issued will be covered by ruled lines, so as to prevent the insertion of any additional word or words without detection. When placing the impression of the seal upon the certificate the three copies will be inserted in the seal at once; and in perforating the margin of the photographs, such perforation will be made simultaneously, so that if necessary at any subsequent time to verify the original certificate by comparison with the duplicate or triplicate, placing one immediately over the other will cause the perforated numbers to exactly match. Care will be exercised to have the perforations pass through the edge of the photograph without touching the face or other portion of the subject of the photograph. The copy of certificate herein given has been so printed as to furnish an illustration of the manner in which such certificate should be prepared.

(*e*) These certificates being issued as an accommodation to Chinese persons, such persons as may apply for same will be required to furnish six photographs of themselves (there showing full face and three profile view) and to submit to the four measurements necessary to obtain the items of personal description to be entered on the certificate. Care will be exercised to have the measurements exact. The height will be inserted in feet and inches and the head measurements according to the metric system, such measurements being taken with the Bertillon instruments in use at all ports of entry. In recording physical marks and peculiarities, those which are the most prominent

and the least likely to be obliterated by lapse of time should be selected.

(*f*) These certificates, as shown on their face, are issued for the protection and identification of Chinese of the exempt classes *only so long as such persons shall retain their exempt status*. Therefore, when such a certificate is found by an inspector in the possession of a person engaged in the performance of manual labor, or of a person to whom it does not relate as shown by a comparison of such person with the photograph and personal description appearing thereon, it should be taken up and forwarded to the Bureau of Immigration, Department of Commerce and Labor, for cancellation.

(*g*) The duplicates of the certificates will be forwarded to the Bureau of Immigration immediately upon the issuance of the originals, in order that such duplicates may be safely filed for future reference, and the triplicates will be retained on the files of the officer in charge.

(*h*) If such a certificate of identification shall be unavoidably lost or destroyed at any time, a certificate in lieu thereof will be issued by the officer of the Bureau of Immigration designated to issue duplicates of laborers' registration certificates, upon the applicant's furnishing to the Commissioner-General of Immigration satisfactory proof of the unavoidable loss or destruction of such certificates and of his identity as the person to whom it was originally issued.

(*i*) With a view to preventing the issuance of more than one certificate of identity to any one Chinese person of the exempt classes, officers in charge of ports of entry will render to each of the other officers in charge at such ports quarterly reports, giving the names and descriptions of all persons to whom such certificates have been issued, such reports to be typewritten on blanks furnished for that purpose.

CHRONOLOGY RELATING TO THE EXCLUSION OF CHINESE

Nov. 17, 1880—Treaty agreeing to regulation, limitation, and suspension of immigration of laborers; and defining classes not to be considered as laborers.

May 6, 1882—Act making provisions for carrying into effect the stipulations contained in the treaty of November 17, 1880.

July 5, 1884—Act amending Act of May 6, 1882, and totally suspending immigration of laborers for a period of ten years.

Sept. 13, 1888—Act prohibiting the coming of Chinese laborers to the United States from China or any other country.

May 5, 1892—Act continuing period of exclusion of laborers for additional ten years, and providing for registry of resident laborers.

Feb. 15, 1893—Act granting additional quarantine powers and imposing additional duties upon the Marine-Hospital Service.

Nov. 3, 1893—Act amending Act of May 5, 1892.

Aug. 18, 1894—Act making the decision of an immigration officer upon admissibility of alien (Chinese) final, unless reversed upon appeal to the Secretary.

July 7, 1898—Joint resolution extending the provisions of the Chinese Exclusion Laws to the Hawaiian Islands, and prohibiting emigration of Chinese from Hawaiian Islands to mainland.

Apr. 30, 1900—Act providing for registration of Chinese residents of Hawaiian Islands.

June 6, 1900—Act placing administration of Chinese Exclusion Laws under the Commissioner-General of Immigration.

Mar. 3, 1901—Act fixing manner of designating and fees to be paid United States commissioners trying Chinese arrested within the United States.

Apr. 29, 1902—Act reenacting, without limitation as to time, all Chinese Exclusion Laws not inconsistent with treaty obligations; authorizing the Secretary to prescribe rules and appoint agents, and providing for admission of Chinese to take part in expositions and for registration of Chinese in Philippine Islands.

Feb. 14, 1903—Act establishing the Department of Commerce and Labor, and transferring administration of Chinese Exclusion Laws from Secretary of Treasury to Secretary of Commerce and Labor.

Apr. 27, 1904—Act amending Act of April 29, 1902, and reenacting all Chinese Exclusion Laws without limitation.

Feb. 6, 1905—Act providing for administration of immigration laws in the Philippine Islands.

Mar. 3, 1905—Act providing for the payment of expenses of aliens detained as witnesses.

Executive Order of Governor of Philippines, of September 23, 1904, providing for the issuance of certificates under the provisions of section six, Act of July 5, 1884, to Chinese of the exempt classes who are citizens of the Philippine Islands.

Regulations of Department, approved May 3, 1905.

POSTAL INFORMATION

INTRODUCTION

1. The beginning of a postal service in the United States dates from 1639, when a house in Boston was employed for the receipt and delivery of letters for or from beyond the seas. In 1672, the government of New York colony established "a post to go monthly from New York to Boston"; in 1702, it was changed to a fortnightly one. A general post office was established and erected in Virginia in 1692, and in Philadelphia in 1693. In 1789, when the post office was transferred to the new federal government, the number of offices in the thirteen colonies was only about seventy-five.

The following are the leading events in the history of the American postal service: The negotiation of a postal treaty with England (1846); the introduction of postage stamps (1847); of stamped envelopes (1852); of the system of registering letters (1855); the establishment of the free-delivery system and of the traveling post-office system (1863); the introduction of the money-order system (1864); of postal cards (1873); and, between the last two dates, of stamped newspaper wrappers, and of envelopes bearing requests for the return of the enclosed letter to the writer in case of non-delivery; the formation of the Universal Postal Union (1873); the issue of "postal notes" payable to bearer (1883); the establishment of a special-delivery system (1885), in which letters bearing an extra 10-cent stamp are delivered by special messengers immediately on arrival and the establishment of the rural-free delivery system (1897) by which mail

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is delivered to residents living outside the limits of the regular free-delivery system.

The number of post offices in the United States is larger than in any other country; but as regards the number of persons employed, the United States takes third rank.

DOMESTIC MAIL

CLASSES OF MAIL MATTER AND RATES OF POSTAGE

2. Domestic mail includes all matter deposited for local delivery, or for transmission from one place to another within the United States, or to or from or between the possessions of the United States; mail sent from the United States to Canada, Mexico, Cuba, and the United States postal agency at Shanghai, China.

Porto Rico and Hawaii are included in the term "United States." The Philippine Archipelago, Guam, Tutuila (including all adjacent islands of the Samoan group that are possessions of the United States), and the Canal Zone are included in the term "possessions of the United States." The term "Canal Zone" includes all the territory purchased from Panama, embracing the "Canal Zone" proper and the islands in the Bay of Panama named Perico, Naos, Culebra, and Flamenco.

Domestic mail is divided into *first-class*, *second-class*, *third-class*, and *fourth-class matter*.

FIRST-CLASS MATTER

3. First-class matter includes all written matter, all matter closed against inspection, and all matter, though printed, that has the nature of personal correspondence, except that certain writing or printing may be placed upon matter of the second, third, and fourth classes without increasing the rate. The rate is 2 cents for each ounce or fraction, and the limit of weight, 4 pounds.

Typewriting, carbon, and letter-press copies are subject to the first-class rate. A printed communication having the character of personal correspondence is classified as a letter.

4. Following is a list of articles commonly dispatched in the mails and that require first-class postage:

ARTICLES REQUIRING FIRST-CLASS POSTAGE

Albums, autograph, with writing.
Architectural designs, containing writing (drawings).
Assessment notices, partly in writing.
Bills, wholly or partly in writing.
Blank books, with written entries.
Blank printed forms, with written signature.
Carbon copies of typewritten matter.
Cards, visiting, bearing written name.
Certificates, filled out in writing.
Checks, in writing, canceled or uncanceled.
Diplomas, filled out in writing.
Drawings or plans containing written words, letters, or figures indicating size, dimensions, etc.
Labels, written.
Letter-press copies of handwritten or typewritten matter.
Manuscript or typewritten copy, without proof sheets.
Postal cards, wholly or partly in writing, remailed.
Price lists, printed, containing written figures changing individual items.
Receipts, printed, with written signatures.
Receipts, partly printed, with writing.
Shorthand or stenographic notes.
Typewritten matter, original letter-press and manifold copies thereof.

SECOND-CLASS MATTER

5. Second-class matter includes all newspapers and periodicals that bear the authorized statement: "Entered at the post office as second-class matter." The rate, when sent unsealed, is 1 cent for each 4 ounces or fraction; full payment is required. There is no limit of weight. This is the rate that applies when the matter is mailed by the general public.

6. Publishers' Rate.—The postal regulations regarding second-class matter, when mailed by publishers, being lengthy, are not treated here. In brief, it may be stated that publishers who have had their publications recognized and admitted as second-class matter are entitled to a rate of

1 cent a pound under the conditions that the publication is a bona fide one with a list of bona-fide subscribers that pay a reasonable price for the publication. The second-class rate is not allowed on a house publication nor on one given away free or as a premium; and a publisher cannot mail at the rate of 1 cent a pound more than twice as many copies as he has bona-fide subscribers.

7. **On the wrapper,** in addition to the regular address, may be the name and address of the sender and the words "sample copy" or "marked copy," or both.

8. **On the matter** itself, the sender may place all that is permitted on the wrapper; correct typographical errors in the text; designate by marks (not by words) a word or passage in the text to which it is desired to call attention. Any other writing will subject the package to the first-class rate. To be entitled to the special second-class rate, copies of newspapers or periodical publications must be complete. Partial or incomplete copies are third-class matter.

THIRD-CLASS MATTER

9. **Third-class matter** includes books, newspapers, and periodicals not admitted to the second class, circulars, miscellaneous printed matter on paper—not having the nature of personal correspondence; proof sheets, corrected proof sheets, and accompanying manuscript copy. The rate, when sent unsealed, is 1 cent for each 2 ounces or fraction; full payment is required. The limit of weight is 4 pounds, unless it is a single book.

10. **Printed Matter.—Printed matter** is the reproduction on paper by any process except handwriting and typewriting of words, letters, characters, or figures, not having the character of personal correspondence. Matter produced by the photographic process (including blueprints) is treated as printed matter.

11. Circulars.—A **circular** is defined by law to be a printed letter which, according to internal evidence, is being sent in identical terms to several persons. A circular may bear a written, a typewritten, or a hand-stamped date; name and address of the person addressed and of the sender; and corrections of typographical errors.

Where a name (except that of the addressee or sender), date (other than that of the circular), figure, or anything else is written, typewritten, or hand stamped in the body of the circular for any other reason than to correct a genuine typographical error, the circular will be subject to postage at the first-class (letter) rate, whether sent sealed or unsealed. However, if such name, date, or other matter is hand stamped and is not of a personal nature, the character of the circular is not changed thereby.

12. Reproductions or **imitations** of handwriting and typewriting obtained by means of the printing press, neostyle, hectograph, electric pen, or similar process will be treated as third-class matter, provided that they are mailed at the post-office window or other place designated by the postmaster in a minimum number of twenty perfectly identical copies separately addressed. If mailed in a smaller quantity, they will be subject to first-class rate.

13. Following is a list of articles that require third-class postage:

ARTICLES REQUIRING THIRD-CLASS POSTAGE

Advertising cards combined with post cards.
Almanacs.
Architectural designs, printed.
Assessment notices, wholly in print.
(Blind.) Indented or perforated sheets of paper containing characters that can be read by the blind unsealed.
Blueprints.
Books, printed.
Calendars, printed on paper.
Cards, Christmas, Easter, etc., printed on paper.
Catalogs.
Certificates, blank.
Check books, blank.
Checks, blank.
Chromos, printed on paper.
Circulars, printed.

Copy books, school, with printed lines and instructions for use.
Designs, wholly in print, on paper.
Diplomas, blank.
Engravings and wood cuts, impressions from, printed on paper.
Invitations, printed or engraved, containing no other writing than the date and name and address of the person addressed and the sender.
Lithographs.
Manuscript copy, accompanied with proof sheets.
Maps, printed on paper, with the necessary mountings.
Music books.
Newspaper clippings, with name and date of paper stamped or written in.
Photographs, mounted or unmounted.
Plans and architectural designs, printed.
Postage stamps, canceled or uncanceled.
Post cards, printed.
Posters, printed on paper.
Price lists, wholly in print.
Printing, samples of.
Proof sheets, printed, corrected, with or without manuscript.
Sheet music.
Telegram blanks.
Valentines, printed on paper.

FOURTH-CLASS MATTER

14. Fourth-class matter includes all merchandise and all other matter not comprehended in the first, second, and third classes. The *rate*, when sent unsealed, is 1 cent for each ounce or fraction, except seed, bulbs, scions, and plants intended for propogation—the rate for which is 1 cent for each 2 ounces or fraction. Full prepayment is required, and the limit of weight is 4 pounds.

On the wrapper, envelope, tag, or label, in addition to the name and address of the addressee, there may be written or printed the name, occupation, and residence or business address of the sender, preceded by the word "From."

Written designation of contents, such as "samples," "candy," "cigars," are permissible on the wrapper.

15. Enclosures.—With a package of fourth-class matter prepaid at the proper rate for that class, the sender may enclose any mailable matter of the third class. A single card bearing the written name of the sender and such inscriptions as "Merry Christmas," "Happy New Year," "With best wishes," etc. may also be enclosed without affecting classification.

Mail matter of different classes enclosed in the same letter or parcel subjects the letter or parcel to the mail rate of that class requiring the most postage.

16. Following is a list of articles that require fourth-class postage:

ARTICLES REQUIRING FOURTH-CLASS POSTAGE

Advertising signs printed on other material than paper.
Albums, autograph, without writing.
Albums, photograph.
Animals, stuffed.
Artificial flowers.
Baggage checks, metal.
Bees, queen.
Blank account books.
Blank cards.
Botanical specimens, not susceptible of being used in propogation.
Bulbs (special rate of postage see page 27).
Calendars, printed on other material than paper.
Candies.
Card games
Cards, blank.
Cards, Christmas, Easter, etc., printed on other material than paper.
Chestnuts (for special rate of postage see page 27).
Cigars.
Cloth, samples of.
Coin.
Coin holders, card blanks.
Crayon pictures or drawings, framed or unframed.
Cuts, wood and metal.
Cuttings of plants or trees (for special rate of postage see page 27).
Daguerreotypes.
Drawings (pen or pencil), without writing, framed or unframed.
Electrotype plates.
Engravings, when framed.
Engravings and wood cuts on wood or metal base.
Envelopes, mailed in bulk.
Flour, samples of.
Flowers, cut or artificial.
Fruit, dried.
Geological specimens.
Grain, samples of (for special rate of postage see page 27).
Herbs, dried.
Honey, in comb.
Insects, dried.
Liquids.
Maps, printed on cloth.
Medals or coins.
Merchandise, samples of.
Metals.
Minerals.
Nuts, in natural state (for special rate of postage see page 27)

Paintings, framed or unframed.
Paper, blank.
Patterns, printed or unprinted.
Plants for propagating purposes (for special rate of postage see page 27).
Posters, printed on cloth.
Printed matter on other material than paper.
Roots (for special rate of postage see page 27).
Rulers, wooden or metal.
Seeds.
Soap.
Tags.
Tintypes.
Valentines, printed on other material than paper.
Wallpaper.

SPECIAL DELIVERY

17. A 10-cent special-delivery stamp, in addition to the lawful postage, secures the immediate delivery of any piece of mail matter at any United States post office within the letter-carrier limits of free-delivery offices and within 1 mile of any other post office. **Special delivery** can be effected only by the use of the special-delivery stamp.

18. Hours of Delivery.—The hours of delivery are from 7 A. M. to 11 P. M. at all free-delivery offices, and from 7 A. M. to 7 P. M. at all other offices, or until after the arrival of the last mail at night, provided that be not later than 9 P. M. Special-delivery mail must be delivered on Sundays, as well as on other days, if the post office is open.

If special-delivery matter fails of delivery because there is no person to receive it at the place of address, the matter is returned to the post office and delivered in the ordinary mail.

Special-delivery matter may be forwarded, but it is not entitled to special delivery at the second office of address unless forwarded on a general forwarding request before attempt at delivery has been made at the post office of the original address.

POSTAL MONEY ORDERS

19. Postal money orders may be obtained at or paid at 35,000 money-order offices in the United States, and may be drawn on post offices in forty-eight foreign countries. This system provides an absolutely safe and convenient means of transmitting money.

On payment of the sum to be sent, and a small fee, to the postmaster of a money-order office, a money order can be drawn for any desired amount not exceeding $100, payable at any money-order office in the United States designated by the applicant. When a larger sum than $100 is to be sent, additional orders may be obtained. For example, to send a sum of $275.60 it would be necessary to get three money orders for the following amounts: $100, $100, and $75.60. International money orders may be obtained at all of the larger post offices and at many of the smaller ones.

20. Identification.—The person who presents an order for payment must be prepared to prove his identity. In case of payment to the wrong person, the Department will see that the amount is made good to the owner, provided that the wrong payment was not brought about through fault on the part of the remitter, payee, or indorsee. A money order may be paid on a written order or power of attorney from the payee, as well as on his own indorsement. More than one indorsement on a money order is prohibited by law. The stamp impressions placed on the back of orders by banks are not regarded as indorsements.

21. Invalid Orders.—An order that has not been paid or repaid within 1 year from the last day of the month of its issue is invalid and not payable. The owner, however, may obtain payment of the amount thereof by making application through the postmaster at any money-order office, or to the Post-Office Department at Washington, District of Columbia, for a warrant for the amount of the order.

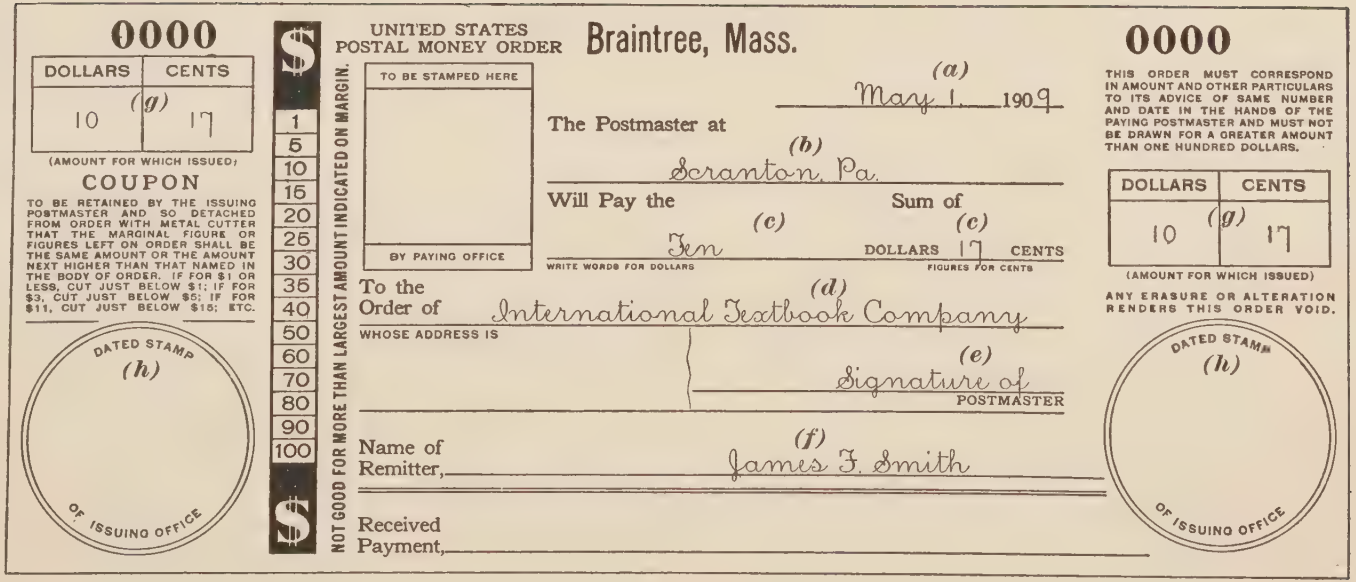
0000
DOLLARS
CENTS
10
(g)
17
(AMOUNT FOR WHICH ISSUED)
COUPON
TO BE RETAINED BY THE ISSUING POSTMASTER AND SO DETACHED FROM ORDER WITH METAL CUTTER THAT THE MARGINAL FIGURE OR FIGURES LEFT ON ORDER SHALL BE THE SAME AMOUNT OR THE AMOUNT NEXT HIGHER THAN THAT NAMED IN THE BODY OF ORDER. IF FOR $1 OR LESS, CUT JUST BELOW $1; IF FOR $3, CUT JUST BELOW $5; IF FOR $11, CUT JUST BELOW $15; ETC.
DATED STAMP
(h)
OF ISSUING OFFICE
$
1
5
10
15
20
25
30
35
40
50
60
70
80
90
100
$
NOT GOOD FOR MORE THAN LARGEST AMOUNT INDICATED ON MARGIN.
UNITED STATES
POSTAL MONEY ORDER
Braintree, Mass.
TO BE STAMPED HERE
BY PAYING OFFICE
(a)
May 1. 1909
The Postmaster at
(b)
Scranton, Pa.
Will Pay the
Sum of
(c)
Ten
WRITE WORDS FOR DOLLARS
(c)
DOLLARS 17 CENTS
FIGURES FOR CENTS
To the
Order of
(d)
International Textbook Company
WHOSE ADDRESS IS
(e)
Signature of
POSTMASTER
Name of
Remitter,
(f)
James J. Smith
Received
Payment,
0000
THIS ORDER MUST CORRESPOND IN AMOUNT AND OTHER PARTICULARS TO ITS ADVICE OF SAME NUMBER AND DATE IN THE HANDS OF THE PAYING POSTMASTER AND MUST NOT BE DRAWN FOR A GREATER AMOUNT THAN ONE HUNDRED DOLLARS.
DOLLARS
CENTS
10
(g)
17
(AMOUNT FOR WHICH ISSUED)
ANY ERASURE OR ALTERATION RENDERS THIS ORDER VOID.
DATED STAMP
(h)
OF ISSUING OFFICE

FIG. 1

(m) (j)

10 17

DETACH ADVICE HERE

Braintree, Mass. **0000**

U. S. POSTAL MONEY ORDER

May 1, 1906

ADVICE

The Postmaster at Scranton, Pa.

WILL PAY ON CORRESPONDING ORDER Ten DOLLARS 17 CENTS

To the Order of International Textbook Company

WHOSE ADDRESS IS

Signature of POSTMASTER

Name of Remitter, James J. Smith

THE MONEY ORDER ISSUED AT ABOVE NAMED OFFICE AND BEARING CORRESPONDING NUMBER AND DATE, MUST AGREE IN ALL ESSENTIAL PARTICULARS, SUCH AS AMOUNT, NAME OF PAYEE, ETC., WITH THIS ADVICE

0000

RECEIPT

FOR

U. S. POSTAL MONEY ORDER

TO BE GIVEN BY ISSUING POSTMASTER TO THE PURCHASER, WHO WILL RETAIN SAME AND PRESENT IT AT OFFICE

DOLLARS	CENTS
10	17

OF ISSUE IF NECESSARY TO MAKE INQUIRY REGARDING THE ORDER

DATED STAMP OF ISSUING OFFICE

(n) (k)

FIG. 2

22. **Lost Orders.**—In all cases of lost orders, the remitter, payee, or indorsee may make application for a duplicate through either the office at which the original order was issued or the office on which it was drawn. No charge is made for issuing a duplicate.

MONEY-ORDER RATES

Domestic Orders	Cents
Sums not exceeding $2.50	3
Sums not exceeding $5	5
Sums over $ 5 and not exceeding $10	8
Sums over $10 and not exceeding $20	10
Sums over $20 and not exceeding $30	12
Sums over $30 and not exceeding $40	15
Sums over $40 and not exceeding $50	18
Sums over $50 and not exceeding $60	20
Sums over $60 and not exceeding $75	25
Sums over $75 and not exceeding $100	30

Foreign Orders	Cents
Sums not exceeding $10	10
Sums over $10 and not exceeding $20	20
Sums over $20 and not exceeding $30	30
Sums over $30 and not exceeding $40	40
Sums over $40 and not exceeding $50	50
Sums over $50 and not exceeding $60	60
Sums over $60 and not exceeding $70	70
Sums over $70 and not exceeding $80	80
Sums over $80 and not exceeding $90	90
Sums over $90 and not exceeding $100	$1

23. **Money-Order Books.** — Money-order books are furnished to all post offices issuing postal money orders. These books contain a blank postal money order and stub, on one page and an advice card and purchaser's receipt on the next page. In Fig. 1 is shown a specimen of the postal money order and the stub, and in Fig. 2 is shown a specimen of the advice card and purchaser's receipt. The postal money order and stub are printed on a blue tinted paper and the advice card and purchaser's receipt are printed on white paper. The money order, stub, advice card, and purchaser's receipt are numbered alike, the figures being printed in red ink.

24. **How to Make Out a Money Order.**—First, place a piece of carbon paper, the size of the page in the money-order

book, between the blue and white pages with the carbon side next to the white page. Then write, with ink, in the space marked *a*, the date; in the space marked *b*, the name of the post office at which the money is to be paid; in the space marked *c*, the amount of the order, writing words for dollars and figures for cents; in the space marked *d*, the name of the person to whom the money is to be paid; in the space marked *e*, the signature of the postmaster at the issuing office; in the space marked *f*, the name of the person paying for the order; in the space marked *g*, the amount of the order in figures; and in the spaces marked *h*, stamp the date of issue with the date stamp of the issuing office. Remove the carbon paper and place it between the next blue and white pages in the book. Then detach the money order from the stub with the metal cutter, so that the marginal figure or figures remaining on the order will be the same amount or the amount next higher than the amount written in the body of the order. For example, for $1 or less, you should cut just below 1; for $3, cut just below 5; if for $11, cut just below 15. On the white sheet below the order will appear the same written words, as on the money order and stub, as illustrated in Fig. 2. Detach the purchaser's receipt from the advice card at the dotted lines *j k* and give it to the purchaser. Then detach the advice card from the stub at the dotted line *m n* and forward it to the postmaster of the post office on which the order is drawn.

REGISTERED MAIL

25. Registered mails reach every post office in the world. The system insures safe transit and correct delivery. Registered matter is handled under special conditions and by bonded employes, and such matter is the object of extraordinary care from the moment it is registered. A complete chain of records and receipts from the point of mailing to the point of delivery enables the accurate tracing of every piece of registered mail.

The registry fee is 8 cents for each separate letter or package, in addition to the postage, both of which must be fully prepaid with postage stamps attached to the letter or parcel.

26. How to Register.—In order to have a letter or parcel registered it is not only necessary to have it properly addressed and stamped, but it must have the name and address of the sender written or printed on it. It should be handed to the postmaster, clerk, or carrier (not dropped in a mailing box), who will write out a registry receipt for the sender for each piece registered. A second receipt from the addressee or his authorized agent, acknowledging delivery, is returned to the sender in every case without extra charge. This receipt is, under the law, prima facie evidence of delivery.

27. In case of loss, the sender or owner of a registered article prepaid at the letter rate of postage, mailed at and addressed to a United States post office, is indemnified for its value up to $25. When a valuable registered letter (or package prepaid at the letter rate) is lost, the sender should make application for indemnity to the postmaster at the office where the piece was mailed. There is no indemnity unless the registered matter is prepaid at the letter rate.

28. Delivery of Registered Mail.—Registered mail is delivered only to the addressee or to his written order. The sender may, however, restrict delivery to the addressee in person by indorsing on the envelope or wrapper the words "Deliver to addressee only." The words "Personal" or "Private" do not so restrict delivery. Persons applying for registered mail, if unknown to the postmaster, are required to prove their identity.

Registered mail will be forwarded on the written or telegraphic order of the addressee—first-class matter immediately and without extra charge; other matter on prepayment of the postage chargeable by law for forwarding. No additional registry fee is charged for forwarding or returning registered matter.

Undelivered registered mail is returned to the sender's address after 30 days or such other time as may be specified in a return request on the envelope or wrapper. First-class matter is returned without extra charge; other matter on prepayment of the return postage.

29. Registry Receipt Books.—Receipt books are furnished to each mail carrier so that he may register any parcel or letter at the residence of the sender. These books contain a *registry receipt* and a *carrier's permanent record* on one page and a *post-office* or *station record* on the next page. In Figs. 3 and 4 are shown specimens of these two pages. The registry receipt and the carrier's permanent record are printed with red ink on a cream-tinted paper, and the post-office record is printed with black ink on white paper.

30. How to Make Out a Registry Receipt.—First, place a piece of carbon paper the size of the page in the receipt book between the receipt and the record with the carbon side next to the white page. Then in the space marked *a*, Fig. 3, the number of the receipt is written with an indelible lead pencil. The first receipt in each book is numbered 1, and the following receipts are numbered in their numerical order as issued. In the space marked *b*, is written the name of the post office to which the carrier is attached. In the space marked *c*, the date of issue is written; in the space marked *d*, the name of the sender; in the space marked *e*, the address of the sender; in the space marked *f*, the name of the person to whom the letter is addressed; in the space marked *g*, the address of this person, mentioning only city and state; in the space marked *h*, the carrier's signature; in the space marked *j*, the number of the route; and in the space marked *k*, the amount received for postage and registry, if the letter does not have postage on it equal in value to the regular postage rate plus 8 cents for registration. Then remove the carbon paper and place it between the next cream and white pages. Next, detach the registry receipt from the carrier's record at the dotted line *m n*, and a facsimile copy of the written matter on the receipt will then appear on the office

(m)

CARRIER'S PERMANENT RECORD

Received 190

Of Carrier .....

Route No. .....

Reg. { Letter / Parcel } No. .....

Sent by .....

To .....

.....

.....

POSTMASTER

Main Office No. .....

REGISTRY RECEIPT GIVEN TO THE SENDER

~~Letter~~ / ~~Parcel~~ No. 1 (a)

First class postage prepaid.

P. O. Gouldsboro Pa. (b)

Received for registration May 1 (c), 190 9, from

(DATE)

Frank H. James (d)

(NAME OF SENDER)

Residing at R F D No 4 Gouldsboro Pa. (e)

(LOCATE PLACE OF RESIDENCE)

Addressed to James J Smith (f)

MONEY RECEIVED FOR POSTAGE AND REGISTRY FEE IN LIEU OF STAMPS (k) 10 Cents

Philadelphia Pa. (g)

(CITY AND STATE ONLY)

J G Moore (h) Rural Carrier,

FOR POSTMASTER

Route No. 4 (j)

This article will be returned if not fully prepaid and otherwise acceptable when received at the Post Office.

(n)

FIG. 3

POST OFFICE OR STATION RECORD

Received ____, 190

Of Carrier ____

Route No. ____

Reg. {Letter / Parcel} No. ____

Sent by ____

To ____

____

____
POSTMASTER

Main Office No. ____

~~Letter~~ / ~~Parcel~~ } No. 1

P. O. Gouldsboro Pa

First class postage prepaid

Received for registration May 1, (DATE) 190 9 , from

Frank H. James
(NAME OF SENDER)

Residing at R. F. D. No. 4 Gouldsboro Pa.
(LOCATE PLACE OF RESIDENCE)

Addressed to James J. Smith

Philadelphia, Pa.
(CITY AND STATE ONLY)

MONEY RECEIVED FOR POSTAGE AND REGISTRY FEE IN LIEU OF STAMPS
10 Cents

J. G. Moore Rural Carrier,
FOR POSTMASTER

Route No. 4

R. P. E. No. ____ Pkg. Rec't Ret'd ____ Reg. Bill Ret'd. ____

FIG. 4

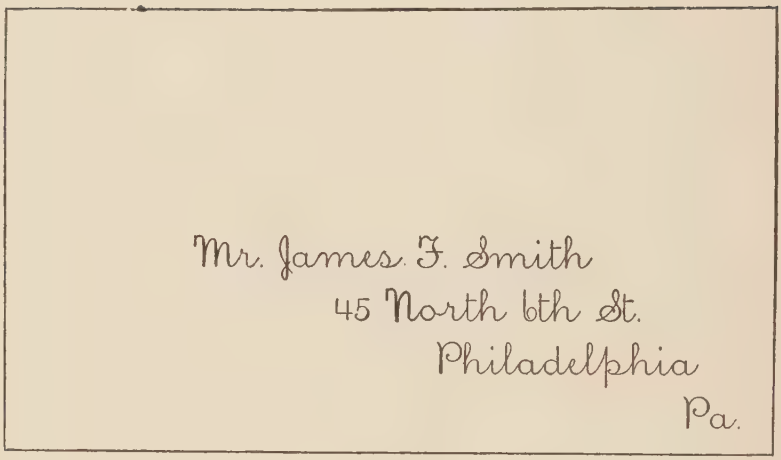
Mr. James F. Smith
45 North 6th St.
Philadelphia
Pa.

FIG. 5

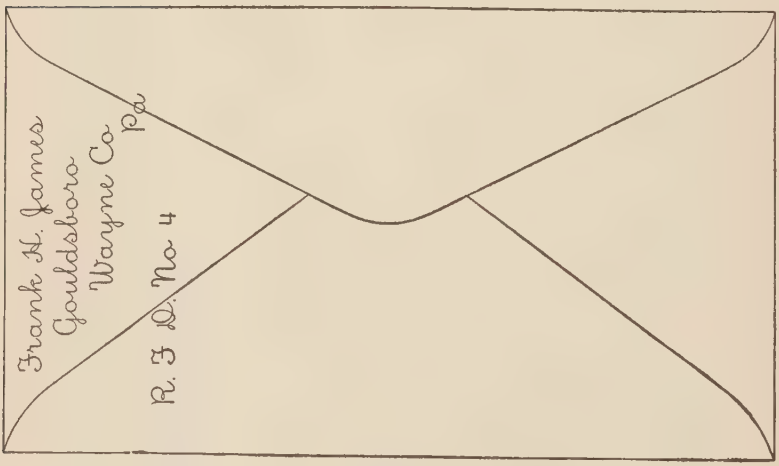
Frank H. James
Goulds boro
Wayne Co
Pa
R. F. D. No 4

FIG. 6

record. The carrier's record is made out at the post office to which the carrier is attached.

31. A registered letter or parcel should have the name and address of the sender either printed or written on it. If written, it is usually placed on the back. For example, a letter addressed as in Fig. 5 should be indorsed on the back with the name and address of sender, as shown in Fig. 6, and the registry receipt and office record for a letter addressed and indorsed as in Figs. 5 and 6 would be made out as shown.

32. The student should pay particular attention to the form of the registry receipt and try to remember it, so that he can write a duplicate of same, without referring to this Instruction Paper. The student can acquire this knowledge by continually copying the entire receipt shown in Fig. 3, substituting other names for those shown.

FOREIGN MAIL

33. Classification.—Articles for or from foreign countries (except Canada, Cuba, and Mexico) are classified as *letters*, *post cards*, *prints*, *commercial* or *business papers*, and *samples of merchandise*. This is known as tne *Postal Union classification* of mail matter. There is no provision in the Postal Union mails for other merchandise than samples (see Parcels Post, page 23). A package of merchandise sealed and prepaid in full at the letter rate is, however, transmitted in the Postal Union mails as a letter. The right of its entry into a foreign country is determined by the administration of the country of its destination. Prohibited articles, if mailed sealed against inspection, will *not* be delivered, although they reach the country of their destination.

Articles addressed for delivery at the following places in China, viz., Cheefoo (Yentia) Chin Kiang, Chung King, Foochow, Hankow, Hung Chow, Ichang, Kaiping, Kalgan, Kiukiang, Nanking, Newchwang, Ningpo, Ourga, Peking, Shanghai, Taku, Tient-sin, Wenchow, Wuchang, Wuhu, Yntai, are transmissible in the mails for the United States

postal agency at Shanghai, at domestic rates, but letters, postal cards, and prints for places other than Shanghai are subject to the postage rates named in Art. **36.**

34. Canada, Cuba, and Mexico.—Articles of every kind or nature admitted to the domestic mails of the United States will be admitted under the same conditions to the mails for Canada, Cuba, and Mexico, except that commercial papers and bona-fide trade samples (including samples of liquids and fatty substances) are transmissible at the postage rate and subject to the conditions applicable to those articles in Postal Union mails; and also that packages—other than single volumes of printed books—the weight of which exceeds 4 pounds 6 ounces, are excluded from the mails for Cuba. Sealed packages, other than letters in their usual and ordinary form, are unmailable to Canada, Cuba, and Mexico.

35. Prohibited Articles.—The transmission of the following articles is absolutely prohibited in the foreign mails: Publications that violate copyright laws; packets, except single books, that weigh more than 4 pounds 6 ounces; poisons, explosives or inflammable substances; live or dead (not dried) animals; insects (except bees) and reptiles; anything likely to decay; lottery tickets; obscene or immoral articles; articles likely to damage the mails or to injure persons handling them.

36. Rates of Postage. —The rates of postage applicable to all foreign countries other than those already named are as follows:

	Cents
Letters, for each half ounce or fraction of half ounce	5
Single postal cards (including souvenir cards), each	2
Double postal cards (including souvenir cards), each	4
Printed matter of all kinds, for each 2 ounces or fraction of 2 ounces	1
Commercial papers for the first 10 ounces or less	5
And for each additional 2 ounces or fraction of 2 ounces	1
Samples of merchandise, for the first 4 ounces or less	2
And for each additional 2 ounces or fraction of 2 ounces	1
Registration fee, in addition to postage	8

37. Letters.—The postal conventions do not define the term letter, but it is held that a package on which postage at the letter rate has been prepaid in full was intended, by the sender, to be sent as a letter, and, when it does not contain prohibited articles, is required to be considered and treated as such. Consequently, packages addressed to foreign countries, except Canada, Cuba, and Mexico, fully prepaid at the rate of postage applicable to letters for the countries to which they are addressed, are allowed to be forwarded by mail to their destinations, even though they contain articles of miscellaneous merchandise that are not sent as bona-fide trade samples.

38. Inspection of Packages.—Articles, other than letters in their usual form, will be inspected by customs officers on their arrival at the exchange post office of the country of destination, and customs and duties will be levied on any articles found to be dutiable under the laws of that country and not prohibited transmission in the mails.

39. Commercial Papers.—As commercial papers are included all instruments or documents written or drawn wholly or partly by hand that have not the character of an actual and personal correspondence, such as papers of legal procedure, deeds of all kinds drawn up by public functionaries, way bills or bills of lading, invoices, the various documents of insurance companies, copies of or extracts from deeds under private signature, written, stamped or unstamped papers, scores or sheets of manuscript music, manuscript of books or of articles for publication in periodicals, forwarded separately, corrected tasks of pupils, excluding all comment on work, etc. The limit of weight is 4 pounds, 6 ounces.

40. Samples of Merchandise.—Packages of miscellaneous merchandise for foreign countries (except Canada, Mexico, Cuba, and United States postal agency at Shanghai, China) are restricted to bona-fide samples or specimens having no salable or commercial value in excess of that actually necessary for their use as samples or specimens. The limit of weight is 12 ounces.

Goods sent for sale, in execution of an order, or as gifts, however small the quantity, are not admissible at the sample rate and conditions. Pairs of articles, such as gloves and shoes, are not transmissible to foreign countries as samples of merchandise.

Samples of merchandise must be placed in bags, boxes, or removable envelopes in such a manner as to admit of easy inspection; they must not have any salable value or bear any manuscript other than the name or profession of the sender, the address of the addressee, a manufacturer's trade-mark, numbers, prices, indications relating to weight or size, and words necessary to indicate the origin and nature of the merchandise.

Packages containing articles of merchandise may be sent to Cuba at the postage rate and subject to the conditions applicable to fourth-class matter in United States mails.

All matter to be sent at less than the letter rates of postage must be securely wrapped, so that it can be easily examined at the office of delivery, as well as at the mailing office, without damaging the wrapper.

41. Foreign Registered Mail.—The letters and parcels addressed to foreign countries, if admissible to the Postal Union mails, may be registered under the same conditions as those addressed to domestic destinations. The registry fee in every case is 8 cents in addition to lawful postage and both must be fully prepaid.

Registered articles addressed to or received from foreign countries are delivered according to the rules of the country of address. No indemnity is paid by the United States for the loss in the mails of registered letters or parcels addressed to any foreign country. No receipt from the addressee will be forwarded to the sender unless the words "Return Receipt Demanded" are written or stamped across the face of the letter or parcel.

PARCELS POST

42. Where in Use.—Unsealed packages may be sent by **Parcels Post** to the following-named countries:

Bahamas	Newfoundland
Barbados	Honduras (Republic of)
Colombia	Trinidad, including Tobago
Costa Rica	Chile
The Danish West Indies	Germany
Honduras (British)	Guatemala
Jamaica	Nicaragua
Leeward Islands	New Zealand
Mexico	Venezuela
Salvador	Bolivia
British Guiana	Hongkong
Windward Islands	

43. Size of Packages.—The maximum length of packages must not exceed 3 feet 6 inches, and in Mexico, Colombia, and Costa Rica, 2 feet. The weight of a single package is limited to 11 pounds; but parcels for Germany and Hongkong, China, must not weigh more than 4 pounds 6 ounces. The postage must be prepaid in full, by stamps affixed at the rate of 12 cents a pound or fraction of a pound; on packages to Chile and Bolivia, the rate is 20 cents a pound or fraction thereof. Letters or other communications in writing must not be enclosed with such packages.

44. Matter intended for Parcels Post must not be posted in a letter box, but must be taken to the post office for inspection.

45. In addition to the name and full address of the person to whom sent, the package must bear the words "Parcels Post" in the upper left-hand corner, with the name and address of the sender. A custom declaration, furnished by the postmaster, must be filled out properly and must be firmly attached to the cover of the package.

Customs duties cannot be prepaid by the sender of dutiable articles; they will be collected from addressees.

MISCELLANEOUS INFORMATION

46. Addressing.—When addressing mail matter, the name, post office, and state must be given; the street address, when there is one; or the post-office box, if the number is known. If the addressee resides on a rural free-delivery route, the number of the route should be given. If the matter is intended for delivery through the general delivery at the post office, the words "General Delivery" should be added. To secure return, the sender's name and address should always be written or printed in the upper left-hand corner of all mail matter.

Postage stamps should be placed in the upper right-hand corner of the address side, care being taken to see that they are securely fixed. Do not place the stamps all over a package in irregular order or have them upside down; if the package is small but heavy, purchase stamps of a large denomination.

47. Complaints.—All complaints should be addressed to the local postmaster, accompanied, whenever possible, with the envelope or wrapper about which the complaint is made. When manifestly improper to direct complaints to the local postmaster, address the Postmaster-General, at Washington, District of Columbia.

48. Concealed Matter.—For knowingly concealing or enclosing any matter of a higher class in that of a lower class and depositing it in the mails, the offender will be liable to a fine of $10.

49. Lists of Names.—Postmasters are forbidden to furnish lists of names of persons receiving mail at their post offices.

50. Mail in Care of Another.—When a letter arrives at a post office addressed to one person in care of another, in the absence of further instructions, the postmaster will deliver it to the first of the two persons named who may call for it.

51. **Parent or guardian** may control the delivery of mail addressed to minors, except when they do not depend on parent or guardian for support.

52. **Revenue and Cut Stamps.**—Postage-due stamps, internal-revenue stamps, or embossed stamps cut from stamped envelopes, or stamps cut from postal cards will not be accepted in payment for postage. Revenue stamps are neither good for postage nor redeemable by the Post-Office Department.

53. **Permissible Additions.**—The words "Personal," "To be called for," and other directions as to delivery and requests for forwarding or return on prepayment of new postage, are permissible as a part of the address on second-, third-, and fourth-class matter.

54. **Redemption of Unused Envelopes, Etc.**—Unused stamped envelopes and newspaper wrappers, when presented in a substantially whole condition, will be redeemed by postmasters at their face value, either in postage stamps, stamped envelopes, or postal cards, but stamped envelopes with printed return card will be redeemed only from original purchasers. Postmasters will redeem unused, uncanceled, and unserviceable postal cards at 75 per cent. of their face value. Redemption will be made only to original purchasers and value given only in stamps, stamped envelopes, or other stamped paper.

55. **Government Printed Envelopes.**— When Government stamped envelopes are purchased in lots of five hundred, or its multiple, of a single size, quality, and denomination, the Department will, on request, print the purchaser's return card on them without extra charge. Return cards will not be printed on newspaper wrappers.

56. **Forwarding Mail.**—The only kind of domestic mail matter that is returnable to the sender without additional postage for such service when undeliverable is letters and other first-class matter prepaid one full rate (2 cents), official matter mailed under penalty envelope or frank, and double postal cards; but not single postal cards nor post cards.

First-class matter indorsed "After——days, return to——" if not delivered will be returned at the expiration of the time indicated on the envelope or wrapper. If no time be set, the matter will be returned at the end of 30 days. The sender has the right to lengthen or shorten the time set, by subsequent direction to the postmaster, but mail matter in any event must remain in the post office for delivery at least 3 days.

57. Postage Due.—Matter of the first class prepaid one full rate—2 cents—will be dispatched with the amount of deficient postage rated thereon to be collected on delivery; other matter will not be forwarded unless fully prepaid. The weight of matter at the mailing post office determines the amount of postage chargeable thereon.

58. Postal Cards.—United States Government postal cards are entitled to all the privileges of letters except that of return to the sender when undeliverable. They must not bear any printing or writing on the address side other than the name and address and such ordinary index marks as the sender may employ to identify the correspondent.

A postal card with a statement of account written thereon, or a legal notice that taxes are due or about to become due, may be transmitted in the mails when such statement or notice does not contain anything reflecting injuriously on the conduct or character of a person. Threats and like matters are prohibited by law from being written or printed on postal cards sent through the mails.

59. Unmailable Matter.—All matter concerning any lottery, or other enterprise of chance or relating to schemes for the purpose of obtaining money or property under false pretenses is unmailable.

60. Wrapping Mail.—All matter should be so wrapped that it will bear transmission without breaking or injuring mail bags, their contents, or the person handling them. Persons mailing liquids or other merchandise of a particular nature should secure a copy of the specified directions for wrapping prepared by the Post-Office Department.

Drawings, photographs, etc. should be backed by pieces of stout pasteboard; otherwise they are certain to be damaged. The pasteboard should be slightly larger than the drawing or photograph. In wrapping cuts, card plates, etc., place a piece of blotting paper or soft pasteboard over the faces before wrapping. Always use substantial paper and tie the packages with strong cord.

Second-, third-, and fourth-class matter must be so wrapped or enveloped that the contents may be examined easily by postal officials. When not so wrapped, or when bearing or containing writing not authorized by law, the matter will be treated as first-class.

61. Postage on drop letters is 1 cent for each ounce or fraction, but there is no drop-letter rate where the mail must be delivered by carrier.

62. Seeds, Bulbs, Roots, Etc.—By special legislation, seeds, bulbs, roots, scions, and plants are mailable at the rate of 1 cent for each 2 ounces or fraction, but are otherwise entitled to the privileges of fourth-class matter. Under this head are included samples of wheat and other grains in their natural condition.

Samples of flour, rolled oats, pearled barley, dried peas, and beans in which the germ is destroyed, cut flowers, dried plants, and botanical specimens, not susceptible of propagation, and nuts and seeds (such as the coffee bean) used exclusively as food, are subject to the regular fourth-class rate of 1 cent an ounce or fraction.

COPYING

PLAIN COPYING

1. Plain copying is the easiest of all grade subjects, as it requires only that a competitor should make an exact copy of an original. It appears, however, that either from carelessness or overconfidence it is the one subject in which many competitors fail to get a satisfactory mark.

The Civil Service Commissioners mark this subject very severely, as a deduction of 5 points is made for nearly every error, owing to the extreme simplicity of the work. Since any variation, however small, is marked as an error, great care must be taken to make an exact copy of the exercise.

2. The following rules should be carefully studied and thoroughly understood before the exercises are copied:

RULES FOR PLAIN COPYING

1. Punctuate, capitalize, spell, and paragraph exactly as in the exercise. Should there be an error in spelling in the exercise, make the same error in order that your copy shall be exact.

2. Figures should be copied exactly as they appear in the exercise. If Roman numerals are used they should appear as Roman numerals in your copy. If Arabic numerals are used they should appear as Arabic numerals in your copy.

3. A margin of about $\frac{1}{2}$ inch should be maintained on each side of the sheet. The margin on the left side should be kept as straight as if ruled off by a line. You may find

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it a help to rule light pencil lines, but be sure to erase them before sending us your work for correction.

4. The first word of the exercise and of each new paragraph (unless spaced otherwise in the exercise) should be indented; that is, the first word of the exercise and of each new paragraph should be written about ½ inch to the right of the point of beginning of the other lines.

5. If asterisks (*) occur in the exercise, make them in your copy as nearly as possible like those in the exercise and approximately in the same position. If more than one is used, be sure to use the exact number shown.

6. Quotation marks (" "), brackets [ ], parenthesis ( ), should be placed in exactly the same position as they appear in the exercise. Practice making punctuation marks, as you will be marked in penmanship on the manner in which they are formed.

7. The pen should not be raised between the letters of a word.

On page 2 is shown an exercise, and on page 3 an illustration of copying it that would receive a very low mark.

On page 4 is given a list of the corrections that such a copy of the exercise would receive, and on page 5 is given a copy of the exercise that would receive a perfect mark.

GOLD TUNNEL MINING CO.

A SUCCESSFUL COAST ENTERPRISE

I. The fact is that every eastern city of consequence has been, and is now, a financial base for the west, a point from which a steady stream of capitol has poured westward into oil, mining, oil refining and ore smelting enterprises.

* * * * * *

The Southwest, (published at Los Angeles, Cal.), contains the following:

"The surplus capital of the east is the power that is making knowable to the world the wonderful resources of the west."

(Report From Mining Journal)

Gold Tunnel Mining Co.
A Successful Coast Enterprise

(1) 1. The fact is that every eastern city of (2) consequence has been (3) and is now, a financial (4) (5) base for the west, a point from which (6) (7) (8) a streme of capital has foured west- (9) ward into the oil, mining, oil refining, (10) and ore smelting enterprises.

× (12) × × ×

(11) The (13) Southwest, [published at Los Angeles,] Cal., (14) (15) contained the following: (16) (17) The (18) surplus (19) Cap. of the east is the power (20) that is making knowable to the world the wonderful resources of the west." (21)

(Report From Mining Journal)

EXPLANATION OF RULES

3. In order that the student may become familiar with the practical application of the foregoing rules, we give above a copy of the printed exercise. This copy shows a large number of errors that are commonly made by competitors. On page 5 is shown how the exercise should be copied in order to receive a perfect mark in accordance with the rules for copying. Following this paragraph is given an explanation of the errors made in it, the rule that applies to each error, and the deduction made in each case. The errors in the copy are numbered from 1 to 21 and correspond to those in the explanation.

Error		Deduct	See Rule
1	Use of Arabic instead of Roman numeral	5	2
2	Pen rest	1	7
3	Punctuation mark omitted	5	1
4	Word misspelled	5	1
5	Word omitted	5	1
6	Word misspelled	5	1
7	Error in exercise corrected	5	1
8	Pen rest	1	7
9	Word inserted	5	1
10	Punctuation mark inserted	5	1
11	Irregular margin	5	3
12	Asterisks poorly made, not in same position, and not same in number	5	5
13	Failure to indent	5	4
14	Bracket used instead of parenthesis	5	6
15	Parenthesis in wrong place	5	6
16	Wrong word	5	1
17	Quotation mark omitted	5	1
18	Failure to paragraph as in exercise	5	4
19	Word abbreviated, which is written in full in exercise	5	
20	Blot	1	
21	Quotation mark made wrong	1	1

From these corrections it will readily be seen that the charge for errors is 89 and therefore this exercise would receive a mark of 11.

The following is a copy of the exercise that would receive a mark of 100:

Gold Tunnel Mining Co.
A Successful Coast Enterprise

I. The fact is that every eastern city of consequence has been, and is now, a financial base for the west, a point from which a steady stream of capitol has poured westward into oil, mining, oil refining and ore smelting enterprises.

* * * * * *

The Southwest, (published at Los Angeles, Cal.), contains the following:

"The surplus capital of the east is the power that is making knowable to the world the wonderful resources of the west."

(Report From Mining Journal)

The exercises on pages 6, 7, and 8 of this Instruction Paper are carefully selected to test your ability and give you a valuable training in this subject. Practice these exercises repeatedly until you are certain that you can copy them without an error, and then mail to us your last copy of each exercise and we will mark it and return it to you with such corrections as are required.

EXERCISE 1

DEPARTMENT OF GREENWICH

Asphalt Surfaces in London and in the United States.—In reference to the very interesting report on "Accidents to Horses on Carriageway Pavements," in London, By Mr. Heywood, engineer and surveyor to the commissioners of sewers of that city, (pp. 297–317), it should be understood that the asphalt surface referred to in this report is the natural rock asphalt, and facts derived from observations in connection therewith do not apply to the Trinidad asphalt surfaces, made of sand instead of the natural soft limestone, in use in the United States.—[NOTE BY THE DEPARTMENT.]

EXERCISE 2

Sec. 38. That no export duties shall be levied or collected on exports from Porto Rico; but taxes and assessments on property, and license fees for franchises, privileges and concessions may be imposed for the purposes of the insular and municipal government: Provided, however, that no public indebtedness shall be allowed in excess of 7 per-centum of the aggregate tax valuation of its property.

EXERCISE 3

AN ORDINANCE

To authorize the Department of Public Works to pay John Nolan for paving School lane.

I. *The Select and Common Councils of the City of do ordain,* That the DEPARTMENT OF PUBLIC WORKS be authorized to draw, and the City Controller is requested to countersign a warrant in the sum of thirteen hundred and Fifty-one (1351) dollars and ninety-six (96) cents, in favor of John Nolan, for paving School lane, from Morris street to Pulaski avenue, and charge the same to Item 2, in the annual appropriation to the Department of Public Works, Bureau of Highways, for the year 1895.

Approved the tenth day of January, A. D. 1895.

EXERCISE 4

The act of 1879 provides that promotions from the lower to the higher grades of letter carriers shall be made on the basis of "efficiency and faithfulness during the preceding year." Beyond this Congress has made no provision (except in the civil service act passed Jan. 16, 1883) for promoting the civil servants of the people by reason of their merits.

EXERCISE 5

PHILIPPINE SERVICE

Promotion.—It is the intent of the Civil Service Act to establish in these Islands a permanent civil service so administered that a person who enters one of the lower grades may by loyal and efficient service secure promotion to the highest offices in the civil service.

The Act expressly provides that vacancies occurring in the offices (enumerated in section 21) shall be filled by promotion without examination of persons in the service. It will be seen, therefore, that vacancies in most of the higher offices in the service are required to be filled by promotion.

EXERCISE 6

NEW YORK

The people of the State of New York, represented in senate and assembly, do enact as follows:

SECTION 1. The comptroller of the city of New York is authorized, upon the application of the board of education of said city and the approval of a majority of the board of estimate and apportionment of said city, to issue bonds in the name and on behalf of the mayor, aldermen, and commonalty of the city of New York for an amount not exceeding two hundred and fifty thousand dollars, par value, to be known as sanitary improvement schoolhouse bonds.

SEC. 2. This act shall take effect immediately.

(For balance of this exercise see following page.)

CONSTRUCTION OF BUILDING

Every building hereafter erected * * * or altered to be used * * * in whole or in any part as a school or place of instruction, the height of which exceeds thirty-five feet, except buildings for which specifications and plans have been heretofore submitted to and approved by the superintendent of buildings * * * shall be built fireproof. (Laws relating to the construction of buildings in the city of New York, 1892, ch. 275, sec. 484, p. 29.)

NOTE.—Students enrolled for second and third grade positions should not study or send in work on Clean Copying from Rough Draft.

CLEAN COPYING FROM ROUGH DRAFT

4. All applicants for positions requiring a first-grade examination are required to be able to make clean pen-written copies of interlined and corrected printed and written manuscripts, except applicants enrolled for the position of stenographer and typewriter, who will make the clean copies on typewriter.

Instead of following the copy exactly, as in the plain copying work, you are to correct all errors in grammar, spelling, punctuation, and capitalization, write in full all abbreviated words, and make such additions or transpositions as are included between enclosures and at the points indicated by the arrowheads leading from such enclosures. The purpose is to make the clean copy as nearly perfect as possible with the least number of changes. The use of capitals and all omissions and mistakes will be taken into consideration in marking this subject. Be especially careful not to substitute or omit any words that would make an entire change in meaning. The following corrected and interlined manuscript is an example similar, in style, to the rough draft given in the United States Civil Service Examinations. It is hardly possible to give any fixed rules for the making of a clean copy from a rough draft other than those given above.

On page 10 is a clean copy of the following example that would receive a mark of 100 in a United States Civil Service Examination:

The rural delivery system is yet in its infancy, and I desire to say that no law passed in many years by congress has been more in the interest of the masses of the toilers of this country than this ~~has been~~.

It is the best educator for the people in the rural districts that has ever been attempted by the government, and I must say that I am oposed to ~~commencing~~ beginning thus early to meddle with a system ~~which~~ that is doing so much good.

I only have a few routes in my district, yet wherever they are established the affect is soon apparent.

The first route established in ~~the~~ my district was a little less than two years ago, from my home town of Gadsen. The carrier, Mr Sutton, is a intelligent farmer, who is as proud of his route as an engineer, becomes of his engine, and has great pride in building it up.

At the aniversary of its establishment he had a little entertainment at his sons house and invited several friends to be present, and I was one of his honored guests, in the meantime I had secured two other routes from the same place. and the carriers were both their at the old mans reception, both of them men of reputation and inteligence, and the old man intertained us by detailing some facts concerning his route.

The rural delivery system is yet in its infancy, and I desire to say that no law passed by Congress in many years has been more in the interest of the masses of the toilers of this country than this. It is the best educator for the people in the rural districts that has ever been attempted by the Government, and I must say that I am opposed to beginning thus early to meddle with a system that is doing so much good.

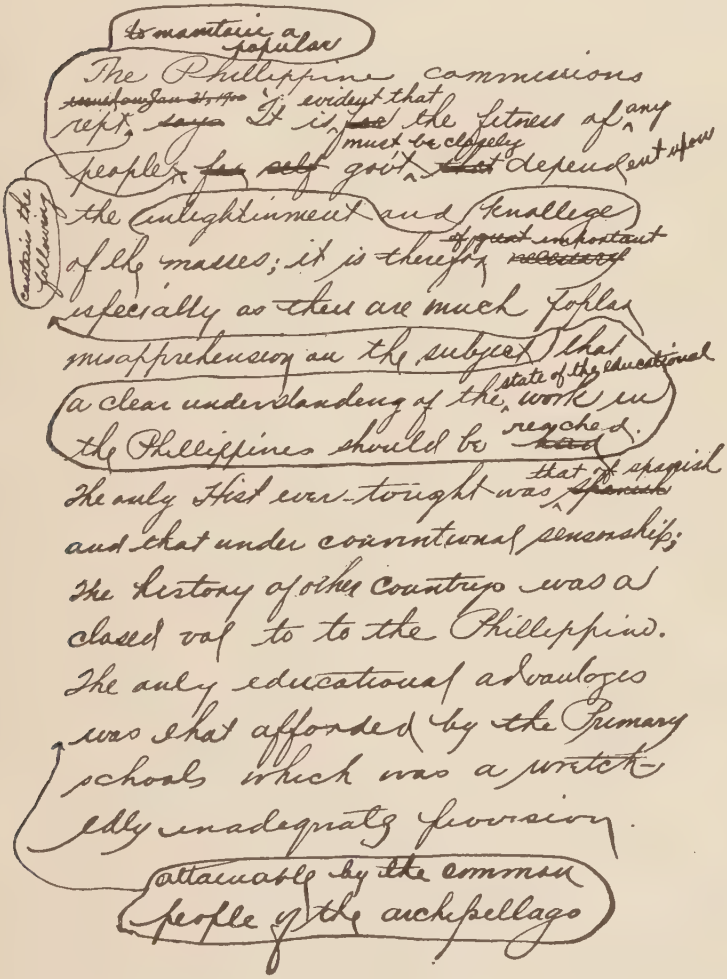
I have only a few routes in my district, yet wherever they are established the effect is soon apparent.

The first route established in my district was from my home town of Gadsen, a little less than two years ago. The carrier, Mr. Sutton, is an intelligent farmer, who is as proud of his route as an engineer becomes of his engine, and has great pride in building it up.

At the anniversary of its establishment he had a little entertainment at his son's house and invited friends, and I was one of his honored guests. In the meantime I had secured two other routes from the same place, and the carriers were both there at the old man's reception, both of them men of reputation and intelligence. The old man entertained me by detailing some facts concerning his route.

You should study all the corrections and changes made so that you thoroughly understand them. After studying the example make a clean copy of each of the following exercises, following the directions given at the beginning of this subject, and mail them to us for correction.

EXERCISE 1


to maintain a popular
The Philippine commissions
report says It is evident that
the future of any
people must be closely
govt dependent upon
the enlightenment and knowledge
of the masses; it is therefore of great important
especially as there are much popular
misapprehension on the subject that
a clear understanding of the state of the educational
work in
the Philippines should be reached.
The only Hist ever thought was that it stagnid
and that under conventional censorship;
The history of other countrys was a
closed vol to to the Philippins.
The only educational advantages
was that afforded by the Primary
schools which was a wretch-
edly inadequate provision.
attainable by the common
people of the archipelago

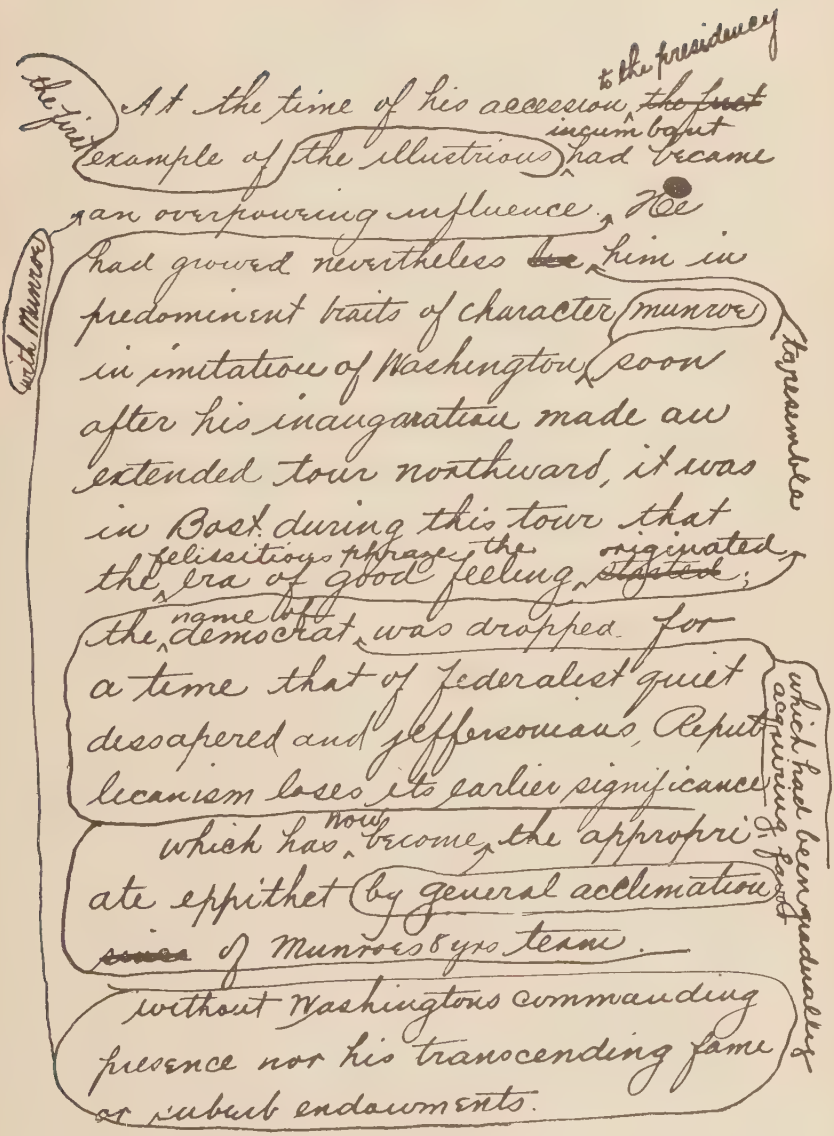
EXERCISE 2

The multiplication of books and its distribution through all conditions of Society ~~is~~ one of the very interesting features of our times is at a small expense a man can now posess himself of Eng. Literature Books by their castliness once confined to a few is now accessable to the multitude ~~to high and low~~ and in this way a change of habits are going on in society. For most all of their knowlege and objects of thought instead of depending on casual remark and careless conversation instead of ~~depending~~ forming their judgments in crowds and recieving their chief excitement from the voice of neighbors men are now learning to reflect alone and to study to follow out subjects continuously to determine for himself what shall engage his mind and to call to his aid the knowlege orginal views and reasonings of men of all ~~every~~ country and age must be the results. An independence of judgment and a thoroughness and extent of information in former times.

EXERCISE 3

At the time of his accession to the presidency ~~the first~~ the first example of the illustrious incumbent had became an overpowering influence. He had growed nevertheless ~~be~~ him in predominent traits of character to resemble munroe in imitation of Washington with Munroe soon after his inauguration made an extended tour northward, it was in Bost. during this tour that the felissitious phrase the era of good feeling originated ~~started~~; the name of democrat was dropped for a time that of federalist quiet dissapered and jeffersonians, Republicanism loses its earlier significance which has now become the appropriate eppithet by general acclimation ~~since~~ of Munroes 8 yrs term. without Washingtons commanding presence nor his transcending fame or suburb endowments.

which had been gradually acquiring fame



EXERCISE 4

Although laws may be enacted to prevent such distinctions

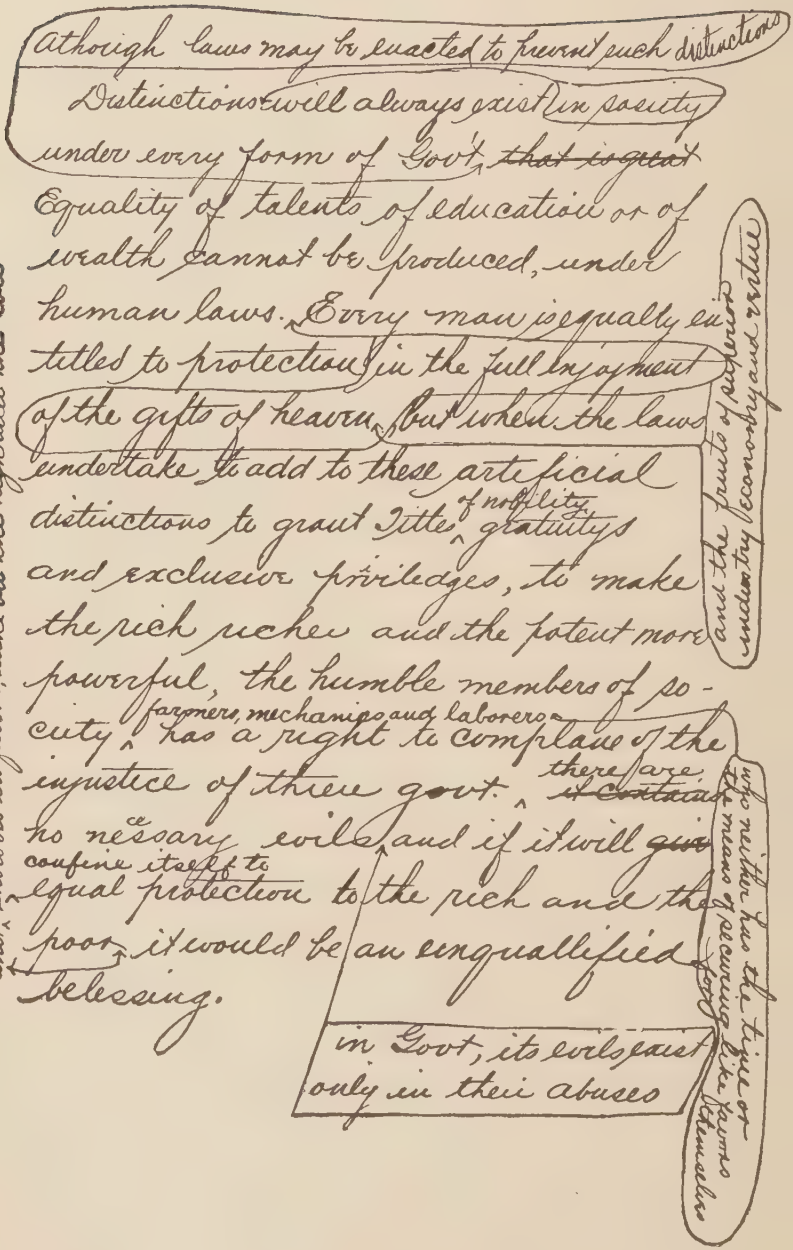
Distinctions will always exist in society under every form of Govt, ~~that is great~~ Equality of talents of education or of wealth cannot be produced, under human laws. Every man is equally entitled to protection in the full enjoyment of the gifts of heaven, but when the laws undertake to add to these artificial distinctions to grant Titles of nobility, gratuitys and exclusive priviledges, to make the rich richer and the potent more powerful, the humble members of society, farmers, mechanics and laborers, has a right to complain of the injustice of their govt. there are ~~it contains~~ no necessary evils, and if it will ~~give~~ confine itself to equal protection to the rich and the poor, it would be an unquallified belessing.

and the fruits of superior industry economy and virtue

who neither has the time or the means of securing like favors for themselves

in Govt, its evils exist only in their abuses

as Heaven does its rains and showers its favors, alike on the high and the low



EXERCISE 5

Each senator, Congressional District and territory -- also the D. C. and Porto Rico -- is entitled to have one cadet at the U. S. Miliatary Academy at West Point. There are also forty appointments conferred by the president of the U. S., at large. The number of ~~candidates~~ Students thus is ~~this~~ limited to 522.

Appointments are made usually one year in advance of date of admission by the Secretary of war, upon the nomonation of the Senator or Representative. These nominations may be either ~~bestowed~~ given after competitive examination or given direct at the option of the Representative. These two ~~The~~ alternates will receive from the War Department letters of appointment, and will be examined ~~together~~ at the same time and place with the regular Appointee, ~~and~~ the best qualified will be admitted to the Academy in the event of the failure of the Principal to pass the proscribed preliminary examination. Appointes to the military academy must ~~not be must~~ be between seventeen and twenty two years of age, and able to pass a carefull examination in reading, writing, spelling, English grammar, English composition, English literature, arithmatic, algebra, ~~through~~ up to and including quadratic equations, plane geometry, descriptive georgaphy, and the Elements of physical geography, especially the geography of the U. S., ~~history of the~~ United states history, the outlines ~~of~~ of General History, and the general principals of physiology and hygiene.

free from any deformity that may unfit them for military service

The Representative may nominate two legally qualified candidates to be designated as alternates

EXERCISE 6

Although spelling is no longer fashionable, and success in life may be obtained without it, as many of our millionaires have shown, writers cannot afford to neglect what it is to them a most important matter.

Errors in spelling in a MS. inevitebly prejudice readers and editors against it, since they give to it an appearance of illiteracy which must be much to its disadvantage of the author. The ability to spell correct is not a gift, as many poor spellers are only too willing to have us believe. Bad spelling is the result of Poor spelling is the results of careless, and inaccurate observation, and editors and Publishers has learned that no one can be a successfull writer who is a n inaccurate observer. Slovenly manuscript generally speaking is almost always inferior manuscript, and it is the common experience of Editors that the manuscript of good writers is generally well prepared, and that they may usually pass bye carelessly written manuscript without fear that they will have lost anything of merit.

Modern elementary education is unsatisfactory in many ways, but in no respect more so than in the matter of Orthography. It is widely know that a painfully small percentage of grammar and graduates can not spell even common words, and the teachers of English in the Colleges are constantly complaining because the students they have to learn show such ignorance.

A SERIES OF QUESTIONS

Relating to the Subjects Treated of in This Volume.

It will be noticed that the questions contained in the following pages are divided into sections corresponding to the sections of the text of the preceding pages, so that each section has a headline that is the same as the headline of the section to which the questions refer. No attempt should be made to answer any of the questions until the corresponding part of the text has been carefully studied.

ARITHMETIC.

(PART 1.)

EXAMINATION QUESTIONS.

(1) What is arithmetic?

(2) In what two ways may numbers be represented?

(3) What is a number? a unit?

(4) What is the difference between a concrete number and an abstract number?

(5) What is the reading of a number called?

(6) Express, in both Arabic notation and Roman notation, seven thousand five hundred three.

(7) For what purpose are ciphers used?

(8) Write each of the following numbers in words:
(*a*) 980; (*b*) 605; (*c*) 28,284; (*d*) 9,006,042; (*e*) 850,317,002; (*f*) 700,004.

(9) Express in Roman notation the following numbers: (*a*) 76; (*b*) 353; (*c*) 1,732; (*d*) 1,496; (*e*) 1,888.

(10) Represent in figures the following expressions:
(*a*) Seven thousand six hundred; (*b*) eighty-one thousand four hundred two; (*c*) five million four thousand seven; (*d*) one hundred eight million ten thousand one; (*e*) ten million six; (*f*) thirty thousand ten.

(11) Find the sum of 83,027 + 46,928 + 4,769 + 81,987 + 46,729 + 479,897 + 627 + 14,896 + 987,649.

Ans. 1,746,509.

(12) Multiply 29,800 by 390. Ans. 11,622,000.

(13) A man owes $1,000; he pays $329 the first year and $438 the second year. How much does he still owe? Ans. $233.

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(14) Find the difference between $23,896 + 4,982 + 96,875 + 59,674$ and $31,627 + 54,892 + 6,925 + 8,976$. Ans. 83,007.

(15) Multiply 8,765 by 987, and from the product subtract $4,695 \times 823$. Ans. 4,787,070.

(16) The greater of two numbers is 1,004, and their difference is 49. What is their sum? Ans. 1,959.

(17) A drover bought 36 oxen at \$24 each and 23 cows at \$96 each; how much did they all cost? Ans. \$3,072.

(18) Divide $2,937 \times 864$ by 923. Ans. $2,749\frac{2}{9}\frac{4}{2}\frac{1}{3}$.

(19) Multiply 8,976 by 4,298, and subtract 98,765 from the product. Ans. 38,480,083.

(20) An engine and boiler in a manufactory are worth \$3,246; the building is worth three times as much, plus \$1,200; and the tools are worth twice as much as the building, plus \$1,875. (*a*) What is the value of the building and tools? (*b*) What is the value of the whole plant?

Ans. { (*a*) \$34,689. (*b*) \$37,935.

(21) The divisor is 1,389, the quotient is 748, and the remainder is 1,263. What is the dividend? Ans. 1,040,235.

(22) The multiplicand is 4,896, and the product 3,862,944. Find the multiplier. Ans. 789.

(23) A man left \$25,375 to his widow and \$12,450 to each of his seven children. How much did they all receive?

Ans. \$112,525.

(24) If I sell my farm for \$375 an acre, I would get \$31,734 more for it than if it sold for \$246 an acre. How many acres in the farm? Ans. 246 acres.

(25) Solve the following by cancelation:

$$(a)\ \frac{231 \times 96 \times 192 \times 45}{11 \times 24 \times 48 \times 15 \times 7} = ?$$

$$(b)\ \frac{42 \times 64 \times 125 \times 98 \times 144}{14 \times 16 \times 25 \times 49 \times 36 \times 20} = ?$$

Ans. { (*a*) 144. (*b*) 24.

ARITHMETIC

(PART 2.)

EXAMINATION QUESTIONS.

(1) Find the sum of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{11}{12}$. Ans. $4\frac{1}{24}$.

(2) What is the value of $3\frac{3}{4} + 6\frac{7}{8} - 4\frac{5}{6}$? Ans. $5\frac{19}{24}$.

(3) A man bought $12\frac{3}{4}$ tons of coal. How much was left after he had burned $5\frac{4}{5}$ tons? Ans. $6\frac{19}{20}$ T.

(4) How many dresses can be made of $53\frac{1}{3}$ yards of gingham, if each dress contains $6\frac{2}{3}$ yards? Ans. 8 dresses.

(5) Find the product of $83\frac{2}{3} \times 67\frac{4}{5}$. Ans. $5{,}672\frac{3}{5}$.

(6) Divide $4\frac{1}{2} \times 6\frac{3}{4}$ by $1\frac{4}{5}$. Ans. $16\frac{7}{8}$.

(7) At $\$3\frac{7}{8}$ a yard, what must be paid for 24 yards of velvet? Ans. \$93.

(8) Simplify $\frac{4\frac{7}{8} \times 3\frac{2}{3} \times 3\frac{1}{5}}{1\frac{5}{8} \times 4\frac{2}{5}}$. Ans. 8.

(9) Multiply $8\frac{1}{20}$ by $\frac{1}{40}$. Ans. $\frac{161}{800}$.

(10) Subtract from 100 the sum of $8\frac{3}{4}$, $7\frac{5}{6}$, and $10\frac{1}{2}$. Ans. $72\frac{11}{12}$.

(11) A cask containing 40 gallons leaks at the rate of $3\frac{3}{4}$ gallons an hour. In how many hours will it be empty? Ans. $10\frac{2}{3}$ hr.

(12) Find the value of $\frac{65}{100} \times 4\frac{2}{3} \times 6\frac{1}{2}$. Ans. $19\frac{43}{60}$.

(13) The circumference of a wheel is $3\frac{55}{113}$ times its diameter. What is the circumference of a wheel whose diameter is $8\frac{2}{3}$ feet? Ans. $27\frac{77}{339}$ ft.

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(14) A company of soldiers consumes $165\frac{3}{4}$ pounds of meat, which is an allowance of $1\frac{5}{8}$ pounds each. How many soldiers are in the company? Ans. 102 soldiers.

(15) In a field of $4\frac{4}{5}$ acres there were raised 96 bushels of wheat. What was the average number of bushels per acre? Ans. 20 bu.

(16) Simplify $\left(\frac{2\frac{1}{2} - 1\frac{2}{3}}{\frac{2}{5} \times (\frac{3}{4} - \frac{1}{2})} - \frac{2}{3}\right) \times \frac{5}{2\,3}$. Ans. $1\frac{2}{3}$.

(17) Find the cost of $24\frac{3}{4}$ tons of hay at $\$23\frac{3}{4}$ a ton. Ans. $\$587\frac{1\,3}{1\,6}$.

(18) Three men in partnership gain \$3,696. In dividing the profits, one gets $\frac{1}{3}$ of the amount, the second gets $\frac{2}{5}$ of the remainder, and the third gets what still remains. Find the share of each. Ans. \$1,232; $\$985\frac{3}{5}$; $\$1,478\frac{2}{5}$.

(19) Simplify $\frac{1\frac{2}{3} \times (2\frac{1}{2} + \frac{2}{5} - 1\frac{2}{3} \div \frac{5}{6})}{(5 + 8\frac{1}{2} \times 1\frac{1}{3}) \div \frac{7}{9}}$. Ans. $\frac{1}{1\,4}$.

(20) The sum of two numbers multiplied by $18\frac{2}{3}$ is $296\frac{2}{3}$. One of the numbers is $12\frac{3}{4}$. Find the other number. Ans. $3\frac{1}{7}$.

(21) If $12\frac{7}{8}$ times the difference between two numbers is $41\frac{1}{6}$, and one of the numbers is 21, what is the other number? Ans. $24\frac{1}{6}$ or $17\frac{4}{5}$.

(22) What is the circumference of a wheel that goes 108 feet in turning $5\frac{3}{4}$ times? Ans. $18\frac{1\,8}{2\,3}$ ft.

(23) Simplify $\frac{2\,5}{1\,8} \times \frac{1\frac{1\,3}{1\,6}}{2\frac{1\,6}{2\,7}} \div \frac{4\frac{3}{5} \div \frac{7}{4}}{5\frac{3}{4} - \frac{1\,1}{3}}$. Ans. $\frac{5}{6}$.

(24) If a man travels $85\frac{5}{1\,2}$ miles in one day, $78\frac{9}{1\,5}$ miles the next day, and $125\frac{1\,7}{3\,5}$ miles the third day, how far did he travel in the three days? Ans. $289\frac{2\,1\,1}{4\,2\,0}$ mi.

(25) Bought $211\frac{1}{4}$ pounds of old lead for $1\frac{7}{8}$ cents per pound. Sold a part of it for $2\frac{1}{2}$ cents per pound, receiving for it the same amount as I paid for the whole. How many pounds did I have left? Ans. $52\frac{1\,3}{1\,6}$ lb.

ARITHMETIC.

(PART 3.)

EXAMINATION QUESTIONS.

(1) Change $2\frac{1}{2} \times 3\frac{1}{3} \div 8$ to a decimal. Ans. 1.0417—

(2) By the method of aliquot parts, multiply 448 by $62\frac{1}{2}$. Ans. 28,000.

(3) How many tons of coal at $\$2.87\frac{1}{2}$ a ton should be received in exchange for 128 cords of wood at $\$1.62\frac{1}{2}$ a cord and $436 in money? Ans. 224 T.

(4) A dealer bought peaches at $\$1.87\frac{1}{2}$ a basket, and sold them so as to lose $12.50 on 100 baskets. How much did he receive per basket? Ans. $1.75.

(5) Reduce to decimals and add the following fractions: $\frac{3}{4}$, $\frac{7}{8}$, $\frac{11}{16}$, $\frac{17}{32}$. Ans. 2.84375.

(6) If the diameter of the earth is 7,912 miles and its circumference is 3.1416 times as much, how many miles are in its circumference? Ans. 24,856.3392 mi.

(7) Find the value of $\frac{3}{5}$ of $\frac{2}{3}$ of .0168. Ans. .00672.

(8) The sum of .37 of a number and .23 of the same number is 33.6. Find the number. Ans. 56.

(9) If $\frac{2}{3}$ of a number be subtracted from .8 of the number, the remainder is 87.8. What is the number? Ans. 658.5.

(10) If $8.20 more is paid for $14\frac{5}{8}$ yards of cloth than for $9\frac{1}{2}$ yards, what is the price per yard? Ans. $1.60.

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(11) Change 4,620 feet to the decimal of a mile. One mile contains 5,280 feet. Ans. .875 mi.

(12) Find the cost of 6 barrels of flour at \$6.375 a barrel, 28 bushels of potatoes at \$.875 per bushel, and 120 pounds of sugar at \$.03$\frac{1}{8}$ a pound. Ans. \$66.50

(13) A lot cost \$3,300, and this was .165 of the cost of the house erected upon it. Find what was paid for the house. Ans. \$20,000.

(14) A man whose daily pay was \$1.87$\frac{1}{2}$ had his wages advanced to \$2.25. Counting 300 working days in a year, find the annual increase in his salary. Ans. \$112.50.

(15) A man paid .143 of his annual salary to his butcher, .347 of it to his grocer, .256 of it for clothing, and .154 for other expenses. How much did he save, if his butcher received \$327.47? Ans. \$229.

(16) A farm was bought at \$87.50 per acre and sold at \$112.50, by which there was a gain of \$3,670. How many acres were there? Ans. 146.8 A.

(17) If a cubic inch of water weighs .03617 pound, what will be the weight of 231 cubic inches? Ans. 8.355+ lb.

(18) Using the method of aliquot parts, divide (*a*) 475 by 12$\frac{1}{2}$; (*b*) 25 by 16$\frac{2}{3}$. Ans. (*a*) 38. (*b*) 1$\frac{1}{2}$.

(19) Express (approximately) .7292 as a common fraction whose denominator is 40. Ans. $\frac{29}{40}$.

(20) Find the value of the following expression when the result is carried to three decimal places:

$$\frac{74.26 \times 3.1416 \times 19.5 \times 19.5 \times 350}{33{,}000 \times 12 \times 4}$$

Ans. 19.601+.

(21) From 1 + .001 take .01 + .000001. Ans. .990999.

(22) Simplify $[(\frac{6}{12}\frac{7}{2} + 2\frac{1}{6} - 3\frac{13}{18}) \div (\frac{29}{8} - 1\frac{2}{9} + 5\frac{47}{72})] \times (.625 \times \frac{16}{25})$. Ans. .2.

(23) Solve the following: (*a*) $(\frac{7}{16} - .13) \times \sqrt{.625 + \frac{5}{8}}$; (*b*) $\frac{19}{32} \times .21 - .02 \times \frac{3}{16}$; (*c*) $(\frac{13}{4} + .013 - 2.17) \times \sqrt{13\frac{1}{4} - 7\frac{5}{16}}$.

Ans. { (*a*) .384375. (*b*) .1209375. (*c*) 6.4896875.

(24) Simplify and express the following, as a mixed number:

$$\frac{\frac{1.25 \times 20 \times 3}{87 + 11 \times 8}}{459 + 32}$$

Ans. $210\frac{3}{7}$.

(25) Simplify and express the following, as a decimal:

$$\frac{\frac{.875}{\frac{1}{2}} \times \frac{\frac{7}{8}}{.4}}{\frac{\frac{5}{16} \times .125}{.375 \times \frac{1}{4}}}$$

Ans. 9.1875.

ARITHMETIC.

(PART 4.)

EXAMINATION QUESTIONS.

(1) Change .67 of a mile to integers of lower denomination. Ans. 214 rd. 2 yd. 7.2 in.

(2) How many miles apart are two points on the earth's equator, one being at 23° 28′ west longitude, and the other at 5° 22′ east longitude. (Take a degree of longitude at the equator as 69.16 statute miles.) Ans. 1,994.11+ mi.

(3) How many yards of lining $\frac{7}{8}$ of a yard wide will be required to line a piece of goods 28 yards long and $\frac{3}{4}$ of a yard wide? (The area equals the product of the length and the width.) Ans. 24 yd.

(4) What must be paid for a rectangular block of granite, its dimensions being 3 ft. $\times 4\frac{1}{2}$ ft. $\times 6$ ft., at $\$1.37\frac{1}{2}$ per cubic foot? (The contents of the block equals the product of the length, breadth, and thickness.) Ans. $\$111.375$.

(5) Find to four decimal places the number of times that 2 ft. 8 in. must be repeated to be equal to the circumference of a flywheel 12 feet in diameter. (The circumference equals 3.1416 times the diameter.) Ans. 14.1372.

(6) Change $\frac{5}{8}$ of 365 da. 5 hr. 48 min. 49.7 sec. to integers of lower denomination.

Ans. 228 da. 6 hr. 38 min. 1.0625 sec.

(7) If 5 T. 875 lb. of coal cost $\$9.13\frac{1}{2}$, how much is it a ton? Use the long ton. Ans. $\$1.69+$.

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(8) How many 8-inch furrows, each 40 rods long, are equivalent to an acre? (See Example 3.) Ans. 99.

(9) Assuming that a degree of latitude is 69.16 miles, how far from the equator is a place whose latitude is 42° 20′ 30″? Ans. 2,928.3497— mi.

(10) A cubical cistern is 8 ft. 6 in. in each of its three dimensions. How many gallons will it hold? (See Example 4.) Ans. 4,594— gal.

(11) Find the exact time in years and days between Jan. 14, 1898, and July 23, 1900. Ans. 2 yr. 190 da.

(12) What decimal fraction of an entire circle is 60° 40′ 30″? Ans. .1685+.

(13) How many bushels will a bin contain whose dimensions are 6 ft. $\times 6\frac{1}{2}$ ft. $\times 8$ ft.? (See Example 4.) Ans. 250.7+ bu.

(14) A family used a quantity of sugar at the rate of $8\frac{3}{4}$ ounces per day and it lasted 80 days. How many pounds were there in all? Ans. $43\frac{3}{4}$ lb.

(15) Find the difference between 25 ft. square and 75 sq. ft. Ans. 550 sq. ft.

(16) A rectangular stick of lumber 36 ft. long and 10 in. thick contains 40 cu. ft. How wide is it? (See Example 4.) Ans. 16 in.

(17) A vessel having a capacity of 100 bushels will hold how many gallons? Ans. 930.92— gal.

(18) (*a*) How many square feet in a board 18.75 ft. long and 2 ft. 10 in. wide? (See Example 3.) (*b*) What will it cost at 8 cents a square foot? Ans. (*a*) 53.125 sq. ft. (*b*) $4.25.

(19) (*a*) What fractional part of 63 gallons is 18 gal. 3 qt. 1 pt.? (*b*) What decimal part? Ans. (*a*) $\frac{151}{504}$. (*b*) .2996+.

(20) A grocer bought 6 bushels of cranberries and sold them at 25 cents for 3 quarts. His gain was $4. What did they cost him per bushel? Ans. $2.

(21) A student spent 8 hours and 40 minutes daily in study for 4 years. How much time did he study in all, if school was in session 5 days each week for 40 weeks each year? Ans. $6,933\frac{1}{3}$ hr.

(22) From £21 5 s. 3 d. take £13 9 s. 6 d. Ans. £7 15 s. 9 d.

(23) If 16 square miles be equally divided into 62 farms, how much land will each farm contain?

Ans. 165 A. 25 sq. rd. 24 sq. yd. 3 sq. ft. 80+ sq. in.

(24) How many bushels of apples are contained in 9 barrels, if each barrel contains 2 bu. 3 pk. 6 qt.?

Ans. 26.4375 bu.

(25) Reduce 4,763,254 links to miles.

Ans. 595 mi. 32 ch. 2 rd. 4 li.

ARITHMETIC.

(PART 5.)

EXAMINATION QUESTIONS.

NOTE.—The answers obtained by the student to those examples which require a change of units from the metric system to English or vice versa may differ slightly from those here printed on account of the method used. If the student's work is correct, however, it will be accepted and full credit will be given.

(1) How many kilos of water will a tank contain that is 3 m. × 2 m. × 3 m.? Ans. 18,000 K.

(2) Find the cost of 1.5 tonnes of sugar at $2\frac{1}{2}$ cents a pound. Ans. $82.67+.

(3) How much will 25.875 kilograms of opium cost at 10 cents a grain? Ans. $39,930.30.

(4) At 25 cents a square yard, find how much must be paid to plaster a ceiling 12 meters long and 10 meters wide. Ans. $35.88.

(5) What would be the value of the 5-cent nickel coins that, placed side by side, would make a row 50 meters long, the diameter of the 5-cent nickel coin being 2 centimeters? Ans. $125.

(6) Find the cost, at 12 cents a quart, of 1,000 liters of cranberries. Ans. $108.94—.

(7) Change 56.8 meters to yards. Ans. 62.1+ yd.

(8) When $x = 10$, $y = 4$, $z = 6$, $a = 3$, $b = 5$, $c = 8$, what is the value of $\frac{5(xyz - abc)}{bc - ax}$? Ans. 60.

(9) How many cubic meters are equivalent to 93 cu. yd.? Ans. 71.1+ cu. m.

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(10) How many hektares of land in a field 40 rods square?
Ans. 4.0469+ Ha.

(11) Bought a farm of 200 hektares at $250 a hektare and sold it at $150 an acre; find the gain or loss.
Ans. $24,130 gain.

(12) A wall containing 30 m³. was built at $4.05 per cu. yd.; how much did it cost? Ans. $158.92.

(13) Find the sum of 2.75 K., 429.17 Dg., and 38,295 g., and express the result in hektograms. Ans. 453.367 Hg.

(14) What is the value of $(43\frac{1}{2})^2 + (4\frac{2}{3})^2$?
Ans. 1,993.879+.

(15) Find the number of liters in 100 gallons of water.
Ans. 378.546— l.

(16) A gold eagle of the United States weighs 258 grains Troy; what would be the value of a kilo of these coins?
Ans. $598.14—.

(17) If a cubic foot of marble weighs 168.75 lb., what will be the weight of a marble block containing 26.25 m³.?
Ans. 156,432 lb., nearly.

(18) What is the nearest whole number of city lots, each containing 2,500 sq. ft., that is equivalent to a hektare of land? Ans. 43 lots.

(19) A rectangular field 100 m. long contains 1 acre. How wide is it? Ans. 8.0488— rd.

SUGGESTION.—Divide the number of square rods in one acre by the number of rods in 100 meters.

(20) A fish weighing 50 Kg. was bought at 30 cents a Kg. and sold at 18 cents a pound; find the gain or loss.
Ans. $4.84+ gain.

(21) How many gallons will a cistern contain that holds 18 m³.? Ans. 4,755.06 gal.

(22) Mercury is 13.5 times as heavy as water; how many pounds of mercury will a vessel hold whose capacity is 35 cu. cm. Ans. 1.04+ lb.

(23) A cistern contains 13.5 m³.; what is the weight of water in pounds that it will hold? Ans. 29,762.1 lb.

(24) A liter is .26417 of a gallon; how many grams will a gill of water weigh? Ans. 118.295+ g.

(25) A rectangular piece of ground is 65.4 m. long and 39.8 m. wide. Find the cost of constructing a walk 1 m. and 5 dm. wide at \$.55 per yard of length: (*a*) when the walk is just outside the boundary; (*b*) when the walk is just inside the boundary. Ans. (*a*) \$130.13. (*b*) \$122.91+.

ARITHMETIC.

(PART 6.)

EXAMINATION QUESTIONS.

(1) A rectangle has a base of 35 rods and an area of $3\frac{1}{2}$ acres; find the altitude. Ans. 16 rd.

(2) The perimeter of a room is 140 feet, and its width is three-fourths of its length; what are its dimensions?
Ans. 30 ft. × 40 ft.

(3) The base of a right triangle is 60 inches, and its area is 18.75 square feet; what is its altitude? Ans. 90 in.

(4) The pressure on a circular piston is 60 pounds to the square inch; if its diameter is 20 inches, what is the entire pressure? Ans. 18,849.6 lb.

(5) How many bushels will a granary hold that is 12 feet long, 5 feet wide, and 4 feet deep? Ans. 192.9 bu.

(6) How many bricks in a wall 4 feet thick, 90 feet long, and 36 feet high, counting 574 bricks in a cubic yard of wall?
Ans. 275,520 bricks.

(7) Find the area of a circle whose circumference is 1,000 feet. Ans. 79,577.285+ sq. ft.

(8) Find the convex surface of a cylinder 24 inches long and 18 inches in diameter. Ans. 1,357.1712 sq. in.

(9) Find the cost of the following bill of lumber:
10 pieces hemlock $10' \times 3'' \times 4''$ at $16 a M.
25 pieces hemlock $12' \times 8'' \times 1''$ at $16 a M.
20 pieces pine $16' \times 8'' \times 2''$ at $30 a M.
Ans. $17.60.

(10) How many feet of 1-inch boards could be cut from a stick 40 feet long and 16 inches square, allowing $\frac{1}{4}$ inch for each saw-cut? Ans. $693\frac{1}{3}$ ft.

(11) Find the cost of 1 mile of iron water pipe 36 inches inside diameter, and 38 inches outside diameter, the price of the iron being \$40 per ton of 2,000 pounds. (Weight of 1 cubic foot of cast iron is 450 pounds.) Ans. \$38,358.94−.

(12) How much will it cost to enclose a rectangular lot 160 feet wide and 480 feet long with a post and board fence 6 feet high? Lumber costing \$30 per thousand feet and labor \$11.65 per thousand feet. Posts measuring 4 in. × 4 in. × 8 ft. are to be placed 8 feet apart, boards 1 inch thick. Ans. \$390.95+.

(13) An iron safe is everywhere $1\frac{1}{2}$ inches thick, and its external dimensions are 6 ft. × 4 ft. 6 in. × 3 ft. 6 in.; how much does it weigh? (Weight of 1 cubic foot of wrought iron is 480 pounds.) Ans. $7{,}237\frac{1}{2}$ lb.

(14) How many bricks, each $8\frac{3}{4}$ inches long, $4\frac{1}{4}$ inches wide, and $2\frac{3}{4}$ inches thick, will be required to build a wall 175 yards long, 12 feet high, and 1 foot $10\frac{1}{2}$ inches thick? (Allow for mortar; see Art. **85.**) Ans. 177,882 bricks.

(15) The perimeter of a field is 600 yards, and its length is 80 yards more than its width; how many acres in the field? Ans. 4.32−A.

(16) How many revolutions will a locomotive wheel 6 feet in diameter make in going from New York to Philadelphia, a distance of 90 miles? Ans. 25,210.08+ revolutions.

(17) A cylindrical tank 12 feet high and 40 inches in diameter is full of oil worth 8 cents a gallon; what is the value of the oil? Ans. \$62.67−.

(18) (*a*) How many perches of stone are required to build the walls of a church, 80 ft. × 40 ft. outside, the walls being 36 feet high and 3 feet thick, there being twelve windows, each 6 ft. × 12 ft., and three doors, each 7 ft. × 10 ft.? (*b*) What is the cost of laying the walls, at \$3.75 per perch?

Ans. { (*a*) 864.7 pch.
(*b*) \$3,927.27.

(19) A cubical cask 8 feet 6 inches each way is full of oil; a faucet capable of discharging an average of 5 gallons per minute is opened. In what time will the cask be emptied? Ans. 15 hr. 18.795— min.

(20) How many tons of Schuylkill coal will fill a car 40 feet long and $6\frac{1}{2}$ feet wide to a depth of $4\frac{1}{2}$ feet?
Ans. 33.4+ T.

(21) What must be paid, at $4\frac{3}{4}$ cents per square yard, for kalsomining the walls and ceilings of 4 rooms, each having a height of 9 feet 4 inches, the dimensions of the rooms being as follows: 12 ft. × 14 ft., 15 ft. × 15 ft., 16 ft. × 18 ft., and 15 ft. × 19 ft.? Ans. $17.31+.

(22) How many single rolls and how many double rolls of paper will be required to paper the walls of a room 16 ft. × 20 ft. 6 in., allowing for a baseboard 7 inches high, 5 doors 3 ft. × 7 ft., and 3 windows 3 ft. × 6 ft., the walls of the room being 11 feet high? Also, how many double rolls of border 18 inches wide are required?

Ans. { 7 double rolls.
17 single rolls.
2 double rolls border.

(23) At $1.25 a yard, what will be the cost of carpeting a room 15 ft. × 17 ft., with Brussels carpet, allowing 1 foot on each strip for waste and matching? Ans. $52.50.

(24) What would be the cost of erecting the walls of a building 145 ft. × 75 ft., and 30 feet high; the walls being 3 bricks (= 1 foot) thick, allowance to be made for 110 windows, each 7 ft. × 3 ft. 6 in.; for 4 doors, each 8 ft. × 10 ft.; and 2 doors, each 6 ft. × 8 ft.; the bricks to cost $5.60 per thousand, and the laying to cost $1.45 per thousand?

Ans. { $1,299.23+ for bricks.
$340.46— for laying.

(25) What would it cost to shingle a roof, each side measuring 30 ft. × 15 ft., shingles costing $5 per thousand?
Ans. $40.50.

ARITHMETIC.

(PART 7.)

EXAMINATION QUESTIONS.

(1) Express decimally: $\frac{1}{2}\%$, $\frac{3}{4}\%$, $1\frac{3}{8}\%$, $\frac{5}{8}$ of 1%, $\frac{2}{3}$ of $\frac{3}{4}\%$.

(2) A farmer planted $6\frac{1}{4}\%$ as many potatoes as his crop amounted to; this was 1,575 bushels more than he planted; how many bushels in his crop? Ans. 1,680 bu.

(3) I pay $37\frac{1}{2}\%$ of my month's salary to my grocer, $12\frac{1}{2}\%$ of it to my butcher, and $16\frac{2}{3}\%$ of it for other expenses, and then have $90 of it left; how much do I get per year?

Ans. $3,240.

(4) A merchant bought some goods at 20% below list price, and sold them so as to gain 30%; at what per cent. above list price did he sell them? Ans. 4%.

(5) Two horses were sold at $120 each; if by the transaction 25% was gained on one and 25% lost on the other, how much was gained or lost? Ans. $16 lost.

(6) A bought a horse and sold it to B at an advance of 20%; B sold it to C and gained 25%; what did each man pay for it if it cost C $60 more than it cost A?

Ans. { A, $120. B, $144. C, $180.

(7) What number increased by $12\frac{1}{2}\%$ of itself is equal to $3,690? Ans. $3,280.

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(8) A speculator, after losing 35% of his money and 10% of the remainder, had $17,550 left; how much did he have at first? Ans. $30,000.

(9) A man earned 20% more during the second of three years than he did the first, and 25% more during the third year than he earned the second; in the three years he earned in all $7,400. What sum did he earn each year?

Ans. { First year, $2,000. Second year, $2,400. Third year, $3,000.

(10) If I should sell my horse for $140, my gain would be 12%; for what must I sell him to gain 28%. Ans. $160.

(11) The sum of two numbers is 2,200, which is 275% of the smaller; what are the numbers? Ans. { Larger, 1,400. Smaller, 800.

(12) A commission merchant sold some goods for $8,407.96; his commission was 2%, for guaranteeing payment he charged $1\frac{1}{2}$%, and his other charges were $275. How much should he send his principal? Ans. $7,838.68.

(13) Find A's tax, knowing that the tax rate is $12\frac{1}{2}$ mills on 100 cents, that his assessment on real estate is $7,800, on personal property $5,640, and that he pays for three polls at $.75 each. Ans. $170.25.

(14) How much better on a bill amounting to $987.50 is a discount of 40% than discounts of 20% and 20%? Ans. $39.50.

(15) A boy buys oranges, and always receives for 3 of them as much as he pays for 5; what is his gain per cent.? Give reasons for your solution. Ans. $66\frac{2}{3}$%.

(16) If the premium for insuring a store is $267 when the rate is $1\frac{1}{2}$%, what is the amount of insurance? Ans. $17,800.

(17) When the rate is three-fourths of 1%, and the amount to be insured is $75,375, what will be the premium? Ans. $565.31+.

(18) After deducting $2\frac{1}{2}\%$ commission and $\frac{1}{4}\%$ for insurance, a commission merchant remitted \$6,613 as the proceeds from a sale of peaches; for how much were the peaches sold? Ans. \$6,800.

(19) A commission merchant received \$520.45 to buy peaches after deducting expenses; drayage cost \$18.75, storage, \$8.50, and his commission was $2\frac{3}{4}\%$. At \$.75 a crate, how many crates did he buy? Ans. 640.

(20) A merchant bought 200 barrels of flour at \$5.40 per barrel; he sold 65% of it at \$5.90 and the remainder at \$6.50. Find the gain per cent. Ans. $13\frac{4}{27}\%$.

(21) A merchant purchased a bill of goods, on which he received a serial discount of 20%, 15%, and 5%; if, instead, he had been given a discount of 20%, 10%, and 10%, his bill would have been \$25.48 more. Find the amount of the bill without discount. Ans. \$12,740.

(22) A life-insurance company took a risk at $\frac{7}{8}\%$ annual premium for \$25,000; the company reinsured 28% of their risk at $\frac{1}{2}\%$ in a second company, and 48% of it at $\frac{3}{4}\%$ in a third company. What was the annual gain of the first company by reinsuring? Ans. \$41.25.

(23) A discount of 20% on three-fourths of the amount of a bill, and 15% on the remainder is \$324.50 better than a discount of 16% on the whole bill; find the amount of the bill. Ans. \$11,800.

(24) A dealer receives \$33.90 more for a piano by allowing a serial discount of 20%, 10%, and 10% instead of one of 25%, 15%, and 5% from the list price; what is the list price? Ans. \$800.

(25) A merchant sold 20% of his stock at a gain of 20%, 30% of the remainder at 1.25 of its cost, and what still remained he sold at seven-eighths of its cost; did he gain or lose, and what per cent.? Ans. 3% gain.

ARITHMETIC.

(PART 8.)

EXAMINATION QUESTIONS

(1) By the method of Art. **12**, find the interest of \$5,628.40 at $3\frac{4}{5}\%$ for 5 yr. 7 mo. 29 da. Ans. \$1,211.39.

(2) By the six-per-cent. method, find the interest of \$5,670.80 for 1 yr. 11 mo. 27 da. at $4\frac{1}{2}\%$. Ans. \$508.25.

(3) By the 60-day method, find the interest of \$4,689.50 at 6% for 87 days. Ans. \$68.00.

(4) Find, in the shortest way, how much more the interest of \$6,600 will be for 3 yr. 8 mo. 15 da. at $5\frac{1}{2}\%$ than at $4\frac{2}{3}\%$. Ans. \$203.96.

(5) If, in finding the interest at 6% of any sum of money, the multiplier is .2365, what is the time?

Ans. 3 yr. 11 mo. 9 da.

(6) How much more at 6% is the exact interest of a debt of \$100,000 for February, 1896, than for the same month in 1897? Ans. \$15.14.

(7) In 3 yr. 5 mo. 18 da. at $4\frac{1}{2}\%$, what part of the principal equals the interest? Ans. .156.

(8) Find the exact interest at 7% of \$928.60 from April 8, 1899, to October 17, 1899. Ans. \$34.19.

(9) What is the compound interest of \$3,690 for 3 yr. 7 mo. 20 da. at 6%, interest being compounded semiannually?

Ans. \$886.05.

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(10) What is the difference between the exact and the ordinary interest of $3,600 at 6% for 270 days of a common year? Ans. $2.22.

(11) The amount of a certain principal at 5% for 2 yr. 3 mo. 18 da. is $535.20; what is the principal? Ans. $480.

(12) What is the present worth of $1,260 due in 6 mo. 24 da., money being worth 5% a year? Ans. $1,225.28.

(13) What is the face of a 90-day note that will yield $1,200 proceeds when discounted at 6%? Ans. $1,218.89.

(14) Find the bank discount of a note for $2,400, due in 60 days, the rate of discount being 6%. Ans. $25.20.

(15) Write, in proper form, a 90-day note which, when discounted at a bank at 5%, will yield $6,000 proceeds. Give the work for finding the face. Ans. Face, $6,078.51.

(16) A 90-day note for $1,200, dated July 1, 1900, and bearing interest at 8%, was discounted at 6% at a bank, September 1, 1900; find the proceeds. Ans. $1,218.47.

(17) A note for $2,920, bearing interest at 5%, is dated April 1, 1900; at what date will the exact interest amount to $58.40? Ans. Aug. 25, 1900.

(18) A demand note for $12,000 with interest at 6% is dated May 10, 1894, and bears the following indorsements: September 1, 1894, $200; January 9, 1895, $960; March 1, 1895, $150; May 1, 1895, $500; settled, September 1, 1895. Which is better for the lender, and by which will he receive the more interest—the merchants' rule or the United States rule, exact interest being computed?

Ans. U. S. rule, $28.02.

(19) At what rate per cent. will $1,280, in 2 yr. 5 mo. 24 da., give $148.31 interest? Ans. $4\frac{2}{3}$%, nearly.

(20) The bank discount on a 30-day note, bearing interest at 8%, is $66, the rate of discount being 6% per annum; find the face of the note, if the note was discounted on the day it was made. Ans. $11,912.64.

(21) I lend equal sums to each of two men, for which one pays me $5\frac{1}{2}\%$ and the other $4\frac{1}{3}\%$ per annum; at the end of 1 yr. 9 mo. 18 da., the interest due me from the first exceeds the interest due from the second by $42. What did I lend to each?

Ans. $2,000.

(22) If the interest of three-fourths of A's money added to that of two-thirds of B's in 4 years at $5\frac{1}{2}\%$ is $1,320, how much has each, if three-fourths of A's equals two-thirds of B's?

Ans. { A, $4,000. B, $4,500.

(23) What is the ad valorem duty at 22% on 18,600 yards of silk invoiced at 8 francs per yard? Ans. $6,317.96.

(24) Find the value, in English money, of the following United States invoice:

Goods.	United States Money.		English Money.		
	Dollars.	Cents.	£	s.	d.
12,000,000 pounds steel rails at $27.62 per ton					
4,304 steel plows at $7.18 each					
Total, in English money					

Ans. £36,754 15 s. 1 d.

(25) If the rate of duty is 28% ad valorem, what will be the duty on a lot of silk invoiced at £378 15 s.?

Ans. $516.04.

ARITHMETIC.

(PART 9.)

EXAMINATION QUESTIONS.

(1) How much must be invested in stocks paying $3\frac{1}{2}\%$ semiannual dividends to yield a yearly income of \$5,600, if they are bought at $108\frac{1}{2}$, brokerage being $\frac{1}{8}\%$?

Ans. \$86,900.

(2) A broker at a commission of $\frac{1}{4}\%$ receives \$50 for buying stock at $110\frac{1}{2}$. (*a*) How many shares does he buy? (*b*) What do they all cost without brokerage?

Ans. { (*a*) 200. (*b*) \$22,100.

(3) What will be the cost of 125 shares of bank stock bought at $114\frac{1}{8}$, brokerage being $\frac{1}{8}\%$? Ans. \$14,281.25.

(4) How much must be invested in stock paying a quarterly dividend of 2% to yield an annual income of \$4,800, their market price being $112\frac{3}{8}$ and brokerage $\frac{1}{8}\%$?

Ans. \$67,500.

(5) Which is better, and by how much per cent., to invest in stock at 110 that pays an annual dividend of 10%, or in stock at 90 that pays 8% a year?

Ans. The former, by $\frac{20}{99}\%$.

(6) What must be paid for a sight draft for \$5,600 on New Orleans when exchange is at $1\frac{1}{8}\%$ premium?

Ans. \$5,663.

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(7) What will be the cost of a draft for \$8,000 payable in 60 days after sight, exchange being at $1\frac{1}{2}\%$ premium, and money being worth 6% interest? Ans. \$8,036.

(8) Find the cost of a 90-day draft for \$6,800 when money is worth 5% interest and exchange is at $1\frac{1}{4}\%$ discount. Ans. \$6,627.17.

(9) Exchange being at $\frac{3}{4}\%$ premium, find the cost of a sight draft for \$2,800. Ans. \$2,821.

(10) Find the face of a 60-day draft costing \$12,000 when exchange is at $1\frac{1}{2}\%$ discount and money is worth 6% interest. Ans. \$12,314.01.

(11) What is the cost, in New York, of a draft on London for £400 15 s. when exchange is \$4.855? Ans. \$1,945.64.

(12) What must be paid in New York for a draft on Paris for 20,000 francs when \$1 is equal in exchange to 5.22 francs? Ans. \$3,831.42.

(13) In what time will the interest of \$1,000 be the same as the interest of \$800 for 30 days, \$500 for 20 days, and \$400 for 60 days, the rate in each case being the same? Ans. 58 da.

(14) A sold B a bill of goods as follows: silk for \$1,200 on 30 days' credit, cloth for \$3,600 on 60 days' credit, and cotton goods for \$1,200 on 90 days' credit; in what time can the three debts be equitably paid at once? Ans. 60 da.

(15) Find the equated time for paying in one payment the following debts: \$800 due September 1, \$900 due October 10, \$1,000 due October 30, and \$1,200 due December 1. Ans. Oct. 23.

(16) A bill was to be paid as follows: one-third in 20 days, one-fourth in 30 days, one-sixth in 60 days, and the remainder in 90 days; find the equated time for paying the whole bill in one payment. Ans. 47 da.

(17) When may the following account be settled by cash payment without loss of interest to either party?

HENRY STELL.

1899.					1899.			
June	8	Mdse., 10 days,	1,200		July	1	Cash,	1,000
July	10	" 60 "	2,000			25	Draft, 30 days,	500
Aug.	20	" 30 "	3,000		Sept.	30	Cash,	800
Sept.	25	" 90 "	2,400		Oct.	10	Real Estate,	6,000
Dec.	1	" 30 "	5,000		Nov.	20	Cash,	900

Ans. Jan. 14, 1900.

(18) A lends B at one time \$320 for 5 months, and at another time \$450 for 8 months; how long should B lend A \$500 to repay the favor? Ans. 10 mo. 12 da.

(19) Find the cost, in New York, of a draft for 10,000 reichmarks on Berlin through London and Amsterdam, exchange between New York and London being \$4.85, between London and Amsterdam 11.85 guilders to the £, and between Amsterdam and Berlin 1.7 reichmarks to the guilder, and brokerage at London and at Amsterdam being $\frac{1}{8}\%$. \$2,413.57—.

(20) On January 1, 1901, M. A. Davenport owed E. J. Collier \$1,708.95. Collier bought of Davenport 60 sheep weighing 4,380 pounds at 11 cents per pound; January 6, Davenport sold Collier 7,680 pounds of beef at $8\frac{1}{4}$ cents per pound; January 8, Collier accepted Davenport's note for \$620, less $\frac{3}{4}\%$ discount; January 10, Collier sold Davenport 600 bushels potatoes at $49\frac{1}{2}$ cents per bushel; January 12, Collier allowed Davenport a 12% rebate on potatoes that were damaged in transit; January 20, Davenport bought of Collier 12,810 bushels of corn at $35\frac{1}{4}$ cents per bushel; January 26, Davenport paid draft \$365.20 drawn on him by Collier; January 29, Davenport gave Collier note for \$195.62. Make an itemized statement of the above account as it should appear taken from the books of Collier; make a proper heading; close the account, and bring down the balance as it should appear February 1, 1901. Ans. \$4,194.27

(21) A man sells 160 shares of 4% stock at $87\frac{5}{8}$ and invests the proceeds of the sale in 6% stock at $124\frac{7}{8}$, brokerage in each case being $\frac{1}{8}\%$; find how much his annual income is increased. Ans. $32.

(22) Find the face of a 60-day draft on New York, purchased in Boston for $659.85, exchange being $1\frac{3}{8}\%$ premium and money being worth 7%. Ans. $658.86.

(23) A man pays 120 for stock that yields a quarterly dividend of 3%; what per cent. per annum does he receive on his investment? Ans. 10%.

(24) A speculator bought lots: 24 at $1,080 each, 36 at $2,160 each, and 48 at $3,240 each. He sold them all so as to average a gain of $100 a lot. Find the average price at which he sold them. Ans. $2,500.

(25) Wm. F. Smith, on May 1, 1902, opened an account with Frank J. Megargee, a stock broker, by depositing $10,000 for investment. May 3, Megargee bought for Smith 90 shares P. & R. stock at $138\frac{1}{4}$; May 5, Smith sold Megargee real estate valued at $4,270; May 7, Megargee bought for Smith 3,000 shares U. S. Steel Stock at $38\frac{7}{8}$; May 11, Megargee sold for Smith 79 shares P. & R. stock at $140\frac{1}{4}$; May 15, Megargee sold for Smith 2,864 shares U. S. Steel Stock at $37\frac{5}{8}$; May 26, Megargee paid draft drawn by Smith $2,910. Make an itemized statement of the above account as it should appear taken from the books of Wm. F. Smith; make a proper heading, close the account, and bring down the balance as it should appear June 1, 1902. Allow brokerage of $\frac{1}{8}\%$ for buying and selling stock. Ans. $376.12.

SPELLING

(PART 1, DIVISION 1)

INSTRUCTIONS FOR STUDYING

Before studying this lesson, read carefully Art. 9, page 4, in the pamphlet, "How to Proceed With Your Studies."

The Instruction Paper in Spelling, Part 1, is apportioned into two divisions. The First Division contains fifty exercises, and extends from page 1 to Exercise 51, page 12.

In studying the exercises in the Instruction Paper, look closely and carefully at each word, and form a correct mental picture of the word before attempting to write it. Cultivate the habit of training the eye to *see* the word forms correctly before *writing* them.

Write all the words in each exercise as many times as it is necessary until you can spell them correctly. If you can spell a word, it will not be necessary to write it; in case you cannot spell it, write it five, ten, or fifteen times, or until you *can* spell it. This may be done on paper or on a slate.

In order that the words in each exercise may be fully understood, we suggest that you consult a dictionary for the meaning of all words that are new.

INSTRUCTIONS FOR SENDING IN WORK

After you have written the words in the exercises in the First Division of the Instruction Paper, write correctly, without referring to the Instruction Paper, and send to us, the words in the following list. Begin each word with a capital letter.

1. pensel
2. claus
3. tonge
4. forhed
5. ant
6. sumer
7. glas
8. dauter
9. cosin
10. dredful
11. sadel
12. lach
13. pudden
14. venizen
15. scantie
16. hansome
17. nursry
18. libary
19. herth
20. nob
21. scholars
22. helth
23. wether
24. remidy
25. program
26. parrit
27. bobilink
28. glimse
29. atension
30. starecase
31. publick
32. vowel
33. untill
34. fireing
35. nauty
36. brom
37. teecup
38. kasters
39. truley
40. whissle
41. pigen
42. bronze
43. yong
44. strength
45. interrop
46. sauspan
47. quarel
48. merrie
49. speach
50. relation
51. bedsted
52. armchare
53. portrate
54. cushuns
55. mattres
56. napkin
57. amond
58. samon
59. plumer
60. sevral
61. tenniss
62. recieve
63. raindeer
64. icburgs
65. sedar
66. hickory
67. leaver
68. rench
69. ebony
70. hachet
71. molases
72. blossum
73. stomach
74. injur
75. coupel
76. dungon
77. quante
78. notise
79. neccesary
80. avenues
81. gaters
82. parisol
83. overalls
84. chessnut
85. centence
86. carrige
87. lettice
88. rubarb
89. pumkin
90. banana
91. skane
92. pilow
93. britle
94. weights
95. lerned
96. promt
97. generos
98. impolite
99. lasy
100. decetful

SPELLING

(PART 1, DIVISION 2)

INSTRUCTIONS FOR SENDING IN WORK

The Second Division of this Instruction Paper, Part 1, contains fifty exercises, and extends from Exercise 51, page 13, to the end of the Instruction Paper.

After you have written the words in the exercises in the Second Division of the Instruction Paper, write correctly, without referring to the Instruction Paper, and send to us, the following words and sentences. Begin each word with a capital letter.

1. Febuary
2. vessle
3. ciclone
4. huricane
5. bufalo
6. timothy
7. mullbery
8. squirel
9. feirce
10. gruge
11. pinapple
12. rassberry
13. tullips
14. larckspirs
15. marigold
16. holyhocks
17. troubel
18. losened
19. cheerfull
20. fredom
21. yearling
22. collum
23. tropick
24. circle
25. quosient
26. alspice
27. chocolate
28. vanila
29. cinamon
30. consert
31. jurney
32. timmid
33. mirthful
34. gabble
35. gizard
36. guinea
37. quikly
38. yuthfull
39. dumppling
40. custerd
41. poridge
42. missle
43. servis
44. coward
45. peirce
46. scurge
47. florrist
48. miliner
49. skateing
50. minister
51. leeping
52. nuting
53. sleighing
54. cilinder

55. celabrate
56. celestial
57. soler
58. nectar
59. cymble
60. thredbare
61. cronic
62. interger
63. numarator
64. prairie
65. torid
66. commet
67. dramma
68. fossle
69. verdent
70. ruller
71. entomed
72. mosy
73. breazy
74. nimph
75. citizen
76. insurrance
77. percentige
78. digit
79. squeek
80. compas
81. decemal
82. omilet
83. parralel
84. temparate
85. endeaver
86. atractive
87. benidiction
88. bachelor
89. cordiroy
90. seperate
91. brodcloth
92. pardner
93. dence
94. dictionery
95. calender
96. dolfin
97. sturgeon
98. securly
99. coriner
100. countrys

1. The (peel, peal) of thunder could be (heard, herd) (four, for, fore) a long time.

2. (Two, to, too) many (waist, waste) their time in (idol, idyl, idle) day dreaming.

3. The (heir, ere, e'er, air) (brake, break) did (not, knot) work, and the car went over the embankment.

4. After the box had been (scent, sent, cent) (two, to, too) him he (through, threw) it down (stairs, stares).

5. (Ewe, you, yew) could not tell that he was in (knead, need).

6. The (gate, gait) to the (mane, main, Maine) entrance was thrown open to (all, awl).

7. The (site, sight, cite) of the (great, grate) man brought (tiers, tears) to the eyes of the (fare, fair).

8. The (quire. choir) sang at (knight, night).

9. He (red, read) the (hole, whole) book, (leaf, lief) by (lief, leaf).

10. (Know, no) (won, one) (knew, new) the (bare, bear) facts (butt, but) himself.

SPELLING

(PART 2, DIVISION 1)

INSTRUCTIONS FOR STUDYING

Before studying this lesson, read carefully Art. 9, page 4, in the pamphlet, "How to Proceed With Your Studies."

The Instruction Paper in Spelling, Part 2, is apportioned into two divisions. The First Division contains fifty exercises, and extends from page 1 to Exercise 51, page 12.

In studying the exercises in the Instruction Paper, look closely and carefully at each word, and form a correct mental picture of the word before attempting to write it. Cultivate the habit of training the eye to *see* the word forms correctly before *writing* them.

Write all the words in each exercise as many times as it is necessary until you can spell them correctly. If you can spell a word, it will not be necessary to write it; in case you cannot spell it, write it five, ten, or fifteen times, or until you *can* spell it. This may be done on paper or on a slate.

In order that the words in each exercise may be fully understood, we suggest that you consult a dictionary for the meaning of all words that are new.

INSTRUCTIONS FOR SENDING IN WORK

After you have written the words in the exercises in the First Division of the Instruction Paper, write correctly, without referring to the Instruction Paper, and send to us, the words in the following list. Begin each word with a capital letter.

1. acquant
2. captave
3. yeilding
4. sucess
5. imigrate
6. decreace
7. icecle
8. delitfull
9. vipper
10. instruction
11. gipsum
12. scithe
13. ludecrus
14. mountians
15. niether
16. impire
17. dallia
18. nuckle
19. ocurred
20. profesion
21. inducment
22. acceptible
23. happyness
24. labirinth
25. charmming
26. picturesc
27. sensable
28. compells
29. surveyer
30. laboror
31. endustreous
32. capitel (chief town)
33. luccious
34. perfummed
35. hieght
36. busly
37. cemetary
38. seperate
39. cornise
40. zephir
41. turquoise
42. mussle (a small fish)
43. pedler
44. panteloon
45. spanial
46. navigater
47. fullfillment
48. cuticil
49. specticle
50. tragical
51. champain
52. victals
53. devisable
54. inumberable
55. comperison
56. companys
57. verified
58. intrest
59. absense
60. hazzard
61. kerasene
62. notiseable
63. courrage
64. marrageable
65. outrageous
66. leasure
67. herecy
68. jurnal
69. globbule
70. celary
71. retreive
72. reprieve
73. gravey
74. exorcize
75. marchendise
76. anxious
77. valueble
78. aggree
79. disimilar
80. disatisfy
81. legasy
82. jealoucy
83. acuracy
84. assistent
85. eloquence
86. fraudulance
87. misdemeaner
88. taxible
89. exclusivly
90. canidate
91. parigon
92. harbinger
93. clamering
94. croshey (to knit with a single needle)
95. Ecquador
96. De Moin
97. newter
98. berevement
99. tragidy
100. greivence

SPELLING

(PART 2, DIVISION 2)

INSTRUCTIONS FOR SENDING IN WORK

The Second Division of the Instruction Paper, Part 2, contains fifty-six exercises, and extends from Exercise 51, page 12, to the end of the Instruction Paper.

After you have written the words in the exercises in the Second Division of the Instruction Paper, write correctly, without referring to the Instruction Paper, and send to us, the following words and sentences. Begin each word in the following list with a capital letter.

1. monosylable
2. compeditor
3. conquorer
4. preparitory
5. arrangement
6. tyfoid
7. neumonia
8. independant
9. cartrige
10. pouder
11. patreot
12. tyrrany
13. napsack
14. goverment
15. amunition
16. Italain
17. Naraganset
18. tariff
19. transfered
20. busness
21. paralized
22. becomming
23. nucleus
24. corespondance
25. clericle
26. skillfull
27. antecedant
28. tortouse
29. Masachusets
30. Achison
31. battory
32. kernal (an officer)
33. strategem
34. assalent
35. supremecy
36. devestate
37. legicy
38. emagine
39. demonstrated
40. crystalize
41. shakey
42. enouf
43. conceed
44. boyency
45. pretensious
46. caricature
47. pamflet
48. referance
49. emmitate
50. vengance
51. brunette
52. capilary
53. surgon

54. catarr
55. discipate
56. seperate
57. chalise
58. garantee
59. convalessant
60. succede
61. bycicle
62. sheriff
63. stationery (paper, pens, etc.)
64. eddition (publication)
65. sentury (a sentinel)
66. presidant
67. pertition
68. surpluce (the remainder)
69. iminent (distinguished)
70. linament (liquid ointment)
71. emmunity
72. imbarras
73. milage
74. simultaneous
75. restarant
76. umbrela
77. telaphone
78. hesitency
79. valice
80. imparshal
81. dissarange
82. compeling
83. milenry
84. scoundel
85. majority
86. legable
87. tracable
88. unresonable
89. pavillion
90. consencus
91. apperant
92. eminate
93. guerrilla
94. mecanics
95. prosporous
96. incesant
97. expectency
98. releiving
99. patroling
100. statistecs

1. The (capital, capitol) of the syndicate was taken to (mete, meet, meat) (there, their) expenses

2. You could (hear, here) the (pare, pair, pear) talking under the (pare, pear, pair) tree.

3. The (pane, pain) in his arm made him (faint, feint).

4. The (martial, marshal) ordered the (core, corps) to march down the main (aisle, isle).

5. The (idle, idol, idyl) was read before the (principle, principal).

6. The (lightening, lightning) flashed through the room occupied by the (patience, patients).

7. The (sculpture, sculptor) looked down at his work in delight.

8. Can you (pear, pare, pair) a (pare, pair, pear) with a (pear, pare, pair) of scissors?

9. He lost his eighteen (caret, carrot, carat) gold watch.

10. He took the (bred, bread) to his starving children.

11. His (muscles, mussels) were poor because he was always (idyl, idol, idle).

UNITED STATES GEOGRAPHY

EXAMINATION QUESTIONS

(1) In what part of North America is the United States?

(2) (*a*) What ocean lies east of the United States? (*b*) What ocean lies west of the United States?

(3) (*a*) What country lies north of the United States? (*b*) What country and gulf lie south?

(4) What river forms part of the southern boundary of the United States?

(5) (*a*) What group of lakes is in the northeastern part of the United States? (*b*) Which of these lakes is wholly within the United States?

(6) What river, with its tributaries, drains the greater part of the United States?

(7) What river is the outlet of the Great Lakes?

(8) (*a*) Which is the largest state? (*b*) Which is the smallest?

(9) Which state is divided into two peninsulas?

(10) Which one of the Gulf States is a peninsula?

(11) How many states are there in the United States?

(12) (*a*) What is the capital of the United States? (*b*) Where is it situated?

(13) Locate and describe: (*a*) the Appalachian Mountain system; (*b*) the Rocky Mountain system?

(14) (*a*) Where is Long Island Sound? (*b*) Where is Chesapeake Bay?

(15) What sound is in the northwestern part of the United States?

(16) Mention: (*a*) two ranges of the Rocky Mountain system; (*b*) two ranges of the Appalachian system.

(17) Where is Yellowstone, or the National, Park?

(18) Locate Lake Champlain.

(19) (*a*) Locate the Adirondack Mountains. (*b*) Name a peak of these mountains.

(20) Locate Moosehead Lake.

(21) Where are the Black Hills?

(22) Name three large rivers that have their source in Colorado.

(23) On what waters would you sail in going from Chicago, Ill., to Annapolis, Md.?

(24) What lake is between Lake Huron and Lake Erie?

(25) Name one large river that flows into the Pacific Ocean.

(26) What river flows into the head of Chesapeake Bay?

(27) Which are the five principal bays on the Atlantic coast of the United States?

(28) Mention three tributaries of the Mississippi River from the west.

(29) Locate: (*a*) Cape Hatteras; (*b*) Cape Sable; (*c*) Penobscot Bay.

(30) Locate: (*a*) Great Salt Lake; (*b*) Lake Pontchartrain; (*c*) Humboldt Lake.

(31) (*a*) Where does the Susquehanna River rise? (*b*) Into what does it flow?

(32) (*a*) Where does the Missouri River rise? (*b*) Into what does it flow?

(33) (*a*) Where does the Mississippi River rise? (*b*) Into what does it flow?

(34) What is the principal tributary of the Mississippi River from the east?

(35) Where is the: (*a*) Grand Cañon? (*b*) Yosemite Valley? (*c*) Dead Valley?

(36) Locate Lake Okeechobee.

(37) What state is nearly in the center of the United States?

(38) Locate the following mountain peaks: (*a*) Pike's, (*b*) Marcy; (*c*) Washington.

(39) Name and describe the river that flows into the head of the Gulf of California.

(40) Locate: (*a*) Georgian Bay; (*b*) Straits of Mackinac.

(41) What is the most southerly city in the United States?

(42) Name the river or other body of water into which each of the following rivers flows: (*a*) Niagara; (*b*) Arkansas; (*c*) Snake.

(43) Mention, in order, the states that would be seen in sailing along the coast from the Bay of Fundy to the mouth of the Mississippi River.

(44) Name: (*a*) two states that border on both the Pacific Ocean and the Columbia River; (*b*) two that border on the Gulf of Mexico and the Mississippi River?

(45) (*a*) In what mountains does the Hudson River rise? (*b*) Into what does it flow?

(46) What river connects Lake Erie and Lake Ontario?

(47) Name ten states in which the largest city is the capital of the state.

(48) Name four rivers that flow into the Atlantic Ocean?

(49) Name four large rivers flowing into the Gulf of Mexico?

(50) Describe the Ohio River.

(51) Which states border on the Mississippi River?

(52) Between what states do the Bitter Root Mountains extend?

(53) Name: (*a*) two states that border on both the Ohio and the Mississippi Rivers; (*b*) two that border on both the Atlantic Ocean and the Potomac River.

(54) Locate each of the following cities, and name the river or other body of water on which each is situated: (*a*) Memphis; (*b*) Evansville; (*c*) Council Bluffs; (*d*) Toledo.

(55) What two states border Massachusetts on the south?

(56) Name three states that border Pennsylvania on the south.

(57) (*a*) What mountains form the boundary between Kentucky and Virginia? (*b*) To what system do these mountains belong?

(58) Between what states does the Delaware River flow?

(59) Name three rivers in Texas.

(60) What three rivers and gulf form part of the boundary of Texas?

(61) Name the lakes that border on Michigan.

(62) What large river rises in the northern part of Minnesota?

(63) Name the capital and largest city in each of the following states: (*a*) Vermont; (*b*) Indiana; (*c*) Illinois; (*d*) Oregon.

(64) In what state is each of the following cities situated: (*a*) Rockford; (*b*) Butte; (*c*) Utica; (*d*) Macon; (*e*) Ogden?

(65) (*a*) Name five states bordering on the Great Lakes. (*b*) Name one important lake port in each state.

(66) (*a*) Name five states that border on the Missouri River. (*b*) Name the capital of each state.

(67) Name and locate the largest city in each of the following states: (*a*) Connecticut; (*b*) West Virginia; (*c*) North Dakota; (*d*) Arkansas.

(68) Which of the New England States has no sea coast?

(69) What mountains are: (*a*) in New Hampshire? (*b*) in Vermont?

(70) What lakes and rivers form part of the boundary of New York state?

(71) Name and locate the capital of California.

(72) (*a*) What is the capital of New York state? (*b*) On what river is it situated?

(73) Name two bays on the coast of Texas.

(74) Name the states that border on Kentucky.

(75) What two sounds are east of North Carolina?

(76) Name the New England States.

(77) What two cities are situated near the junction of the Alleghany and Monongahela Rivers?

(78) Locate the following cities and name the river on which each city is situated: (*a*) Bismarck; (*b*) Des Moines; (*c*) Austin.

(79) Name and locate two important cities on Lake Michigan.

(80) Name the largest city in Vermont, and state on what lake it is situated.

(81) Name three important rivers of the United States that have the following cities situated near their mouths: (*a*) New Orleans; (*b*) St. Louis; (*c*) New York.

(82) Locate: (*a*) Boston; (*b*) Albany; (*c*) Richmond. State on what body of water each city is situated.

(83) On what waters could a cargo be shipped from New Orleans, La., to Pittsburg, Pa.?

(84) What river separates: (*a*) Louisiana from Texas? (*b*) Oregon from Washington?

(85) Bound Nebraska.

(86) Name two rivers that flow through both North Dakota and South Dakota.

(87) Name two rivers entirely within California.

(88) Describe an all-water route from New York to St. Louis.

(89) Locate the Big Horn Mountains.

(90) Name four states through which the Arkansas River flows.

(91) Bound Illinois.

(92) Name a river and a large body of water (not a river) that border on each of the following states: (*a*) New Hampshire; (*b*) South Carolina; (*c*) Vermont.

(93) Locate each of the following cities: (*a*) Sheboygan; (*b*) Fond du Lac; (*c*) Sandusky.

(94) What strait connects Lake Huron with Lake Michigan?

(95) (*a*) Name five states that border on Canada. (*b*) Name the capital of each.

(96) What five states are nearly enclosed by the Mississippi River, Ohio River, and the Great Lakes?

(97) What water boundaries has Oregon?

(98) What large river crosses New Mexico?

(99) (*a*) What river crosses Nebraska? (*b*) Name two cities situated on this river.

(100) (*a*) What city is opposite Council Bluffs? (*b*) What river separates these cities?

GAUGING AND ELEMENTARY PHYSICS

EXAMINATION QUESTIONS

(1) A lot of ten casks of the first variety measure 22 inches at the head, 2 feet 4 inches at the bung, and are 4 feet 2 inches long. A second lot of eighteen casks measure 19 inches at the head, 24 inches at the bung, and are 3 feet 9 inches long. How many gallons do the entire lot contain?

(2) Define the terms: (*a*) specific gravity; (*b*) capillary attraction; (*c*) distillation.

(3) What is the height of a cylindrical tub 9 feet in diameter that has the same capacity as a tub that is 12 feet in diameter at the bottom, 9 feet in diameter at the top, and 12 feet high?

(4) Make a sketch of and describe the following instruments: (*a*) hydrometer; (*b*) siphon; (*c*) pipette.

(5) Change into Fahrenheit temperatures: (*a*) 90° C.; (*b*) 10° C.; (*c*) — 30° C.

(6) The contents of a tub 10 feet in diameter at the bottom, 8 feet in diameter at the top, and 12 feet high is 80-per-cent. proof. It is mixed with spirits 110-per-cent. proof contained in twenty casks of the third variety that measure 23 inches at the head, 29 inches at the bung, and are 4 feet 4 inches long. (*a*) What is the percentage of proof of the mixture? (*b*) How many proof gallons are there?

(7) Define the term: (*a*) wine gallon; (*b*) beer gallon; (*c*) proof gallon; (*d*) cylindrical inch.

(8) A cask of spirits weighs 950 pounds; its tare (weight empty) is 84 pounds, the specific gravity of the spirit is .97549, which indicates 20.5-per-cent. alcohol. (*a*) How many gallons does the cask hold? (*b*) What is the proof? (*c*) How many proof gallons does it contain?

(9) Change into their corresponding centigrade temperatures: (*a*) 40° F.; (*b*) 18° F.; (*c*) — 30° F.

(10) (*a*) When is water at its greatest density? (*b*) At what temperature does water freeze? (*c*) At what temperature does water boil? (*d*) At what temperature does alcohol freeze? (*e*) At what temperature does alcohol boil? (*f*) At what temperature does mercury freeze? (*g*) At what temperature does mercury boil?

(11) What is the total tax on three casks of spirits, one of which contains 80 gallons of 70-per-cent. proof mixture; one, 40 gallons of 140-per-cent. proof mixture; and one, 75 gallons of 100-per-cent. proof mixture, the rate of tax being $1.10 per gallon?

(12) (*a*) What relation does the density of an alcoholic liquid bear to its weight and to its percentage of proof? (*b*) What is the relative action of the hydrometer in a high- and low-proof mixture? (*c*) What are the conditions favorable to evaporation? (*d*) What influence has temperature on liquids in general and on alcoholic liquid in particular?

(13) How would the boiling point of a liquid be affected: (*a*) by pressure? (*b*) by saline substances in solution? (*c*) by altitude?

(14) (*a*) If a tub of alcoholic mixture is set in the hot sun and allowed to evaporate, will the proof of mixture be increased or diminished? (*b*) Why?

(15) How can unsweetened spirits be recovered from spirits containing saccharine matter?

(16) (*a*) What is rectification? (*b*) In what way may it be conducted?

(17) Define the terms: mash; high wine; must; head; wine lees; malt; wort.

(18) (*a*) What is the difference between proof spirit and absolute alcohol? (*b*) Can spirits be drawn from an air-tight cask by a force pump?

(19) (*a*) Does immersing a body in a fluid affect the weight of the body? (*b*) If so, to what extent?

(20) (*a*) Explain the use of the hydrometer of constant weight. (*b*) How is the hydrometer, used in the internal-revenue service, graduated? (*c*) What effect has the introduction of ether into alcoholic spirits on the hydrometer?

IMMIGRATION LAW

EXAMINATION QUESTIONS

(1) How is the "immigrant fund" obtained?

(2) For what purpose is the "immigrant fund" used?

(3) Name the class of aliens that are excluded from entering the United States.

(4) What fine is imposed on the person or persons violating the provisions of the contract-labor law?

(5) What are the duties of any master or commanding officer of a vessel carrying aliens on entering a United States port?

(6) By whom is the head-tax paid?

(7) By whom are all physical and mental examinations of aliens entering the United States to be conducted?

(8) Are alien seamen landing in the United States, in the regular course of their pursuit and with the intention of departing as soon as practicable on some outward-bound vessel, to be held for examination?

(9) What disposition is made of aliens illegally brought into this country?

(10) What is the duty of an immigrant officer when an alien is one of the excluded classes and who is trying to land for the purpose of proceeding directly through the United States to a foreign country?

(11) How long may an alien who has entered the United States illegally reside here before he is exempt from deportation?

(12) What is the meaning of the words United States as used in the Immigration Law?

(13) In the case of illegal entry of an alien, by whom is the cost of deportation paid?

(14) What is the maximum fee an attorney can charge in the case of the detention of an alien?

(15) How can a larger fee than that fixed by the government be charged?

(16) By whom are the compensations fixed and the appointments made of the immigrant inspectors and other employes of the Bureau of Immigration?

(17) State the duties of the master, agent, or owner of vessels carrying aliens into a United States port before sailing from that port.

(18) What courts have jurisdiction in all suits arising from the enforcement of the Immigration Law?

(19) (*a*) What authority has the immigration officer to exercise? (*b*) Can he, of his own authority, accept a bond or guaranty that an alien will not become a public charge?

(20) In case of disagreement between immigration officers, as to the landing of an alien, how is the disagreement settled and who has the final decision?

CHINESE EXCLUSION LAWS

EXAMINATION QUESTIONS

(1) What do you understand by the privileged classes?

(2) Can a Chinese laborer, a resident of the Philippine Islands previous to April 29, 1902, immigrate to the mainland territory of the United States.

(3) Can a Chinese laborer, a resident of one of the Philippine Islands previous to April 29, 1902, immigrate to another of the islands included in this group?

(4) Can a Chinese laborer, a resident of one of the Philippine Islands previous to April 29, 1902, immigrate to any other island within the jurisdiction of the United States?

(5) May Chinese persons engaged under contract as mechanics, artisans, agents, etc. enter the United States?

(6) In the case of an arrest under the Chinese Exclusion Laws, by whom is the Government represented in securing the proper enforcement of these laws?

(7) By whom can orders be issued for the deportation of Chinese persons?

(8) May a Chinese person who has entered the United States as one of the exempted class become a laborer?

(9) What method is pursued to assure proper identification of Chinese persons arrested for improper entry into the United States?

(10) State the requirements under which a Chinese person may enter the United States as an exhibitor.

(11) State the duties of an officer in charge of the enforcement of the Chinese Exclusion Laws.

(12) Give the definition of a student with regard to the Chinese Exclusion Laws.

(13) Do the wife and minor children of a Chinese merchant have to present a certificate to be admitted to the United States?

(14) Do the Chinese Exclusion Laws apply to a Chinese person born in the United States?

(15) What are the duties of the master, agent, or owner of any vessel before sailing?

POSTAL INFORMATION

EXAMINATION QUESTIONS

(1) (*a*) When was the rural free delivery established? (*b*) What do you understand by the term United States as applied to domestic mail?

(2) (*a*) What is the rate for registering a letter or parcel? (*b*) Explain in detail how a letter should be prepared for registration.

(3) (*a*) How much would it cost to buy a postal money order for $747.32? (*b*) How many orders would be required? (*c*) For what amounts would they be issued?

(4) How would the following articles be classified by the United States postal authorities: (*a*) diplomas? (*b*) telegraph blanks? (*c*) electrotypes? (*d*) almanacs? (*e*) maps? (*f*) newspaper clippings? (*g*) magazines? (*h*) photographs? (*i*) architectural plans? (*j*) leather valentines?

(5) (*a*) Are there any limits to size or weight of parcels mailed Parcels Post? (*b*) If so, what are they? (*c*) What is the rate for sending a parcel to Bolivia by Parcels Post?

(6) What disposition is made of an undelivered registered letter or parcel?

(7) Can a pair of gloves or stockings be mailed to foreign countries as samples?

(8) Make out a registry receipt on the loose form enclosed for a letter addressed to Mortimer L. Johnson & Co., 116 E.

29th St., New York City, and indorsed on the back by the sender as follows:

Henry L. Cummings
Jenkintown
Montgomery Co.
R. F. D. No. 5 Pa.

(9) (*a*) Can a letter or postal addressed to a foreign country be registered? (*b*) If so, what is the rate of registration?

(10) What are the foreign postal rates on: (*a*) a letter? (*b*) double postal cards? (*c*) commercial papers? (*d*) samples?

(11) Are foreign letters or parcels inspected by customs officers?

(12) What do you understand by commercial papers?

(13) (*a*) What is the special-delivery rate? (*b*) During what hours are special-delivery letters delivered?

(14) What is the limit and distance for the delivery of a special-delivery letter from a post office not having free delivery?

(15) (*a*) Is a person indemnified in the evidence of a registered letter or postal being lost? (*b*) If so, what is the maximum amount that can be recovered?

(16) (*a*) Can a registered letter be delivered to any one other than the person to whom it is addressed? (*b*) If so, mention the conditions under which such registered letters can be delivered.

(17) Is it necessary that a person be identified who desires to have a money order cashed?

(18) Can a registered letter be forwarded in case of a written or telegraphic order? If so, under what conditions?

(19) How are articles classified under the Postal Union classification?

(20) Mention five articles that are prohibited from being carried in the mails.

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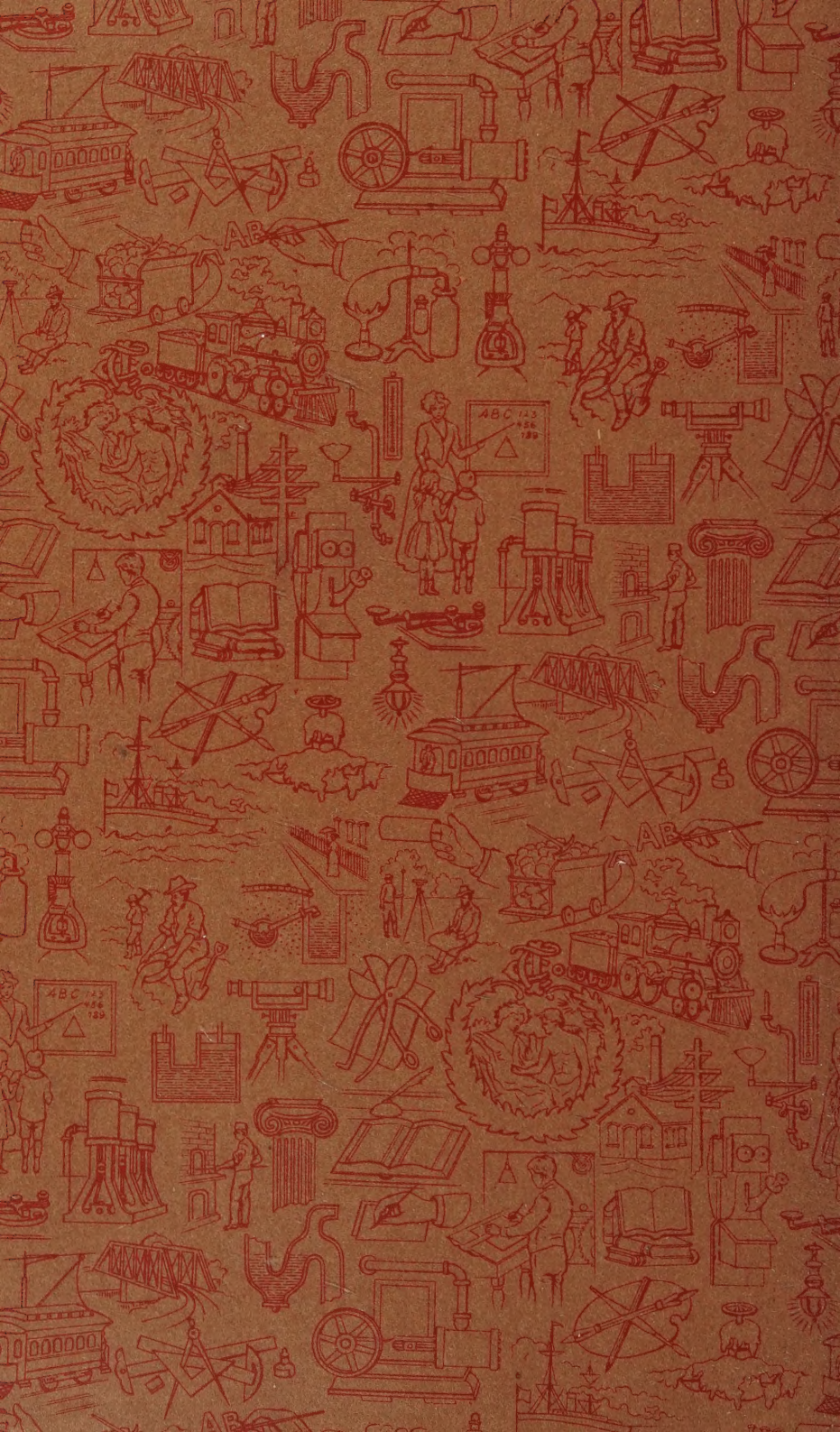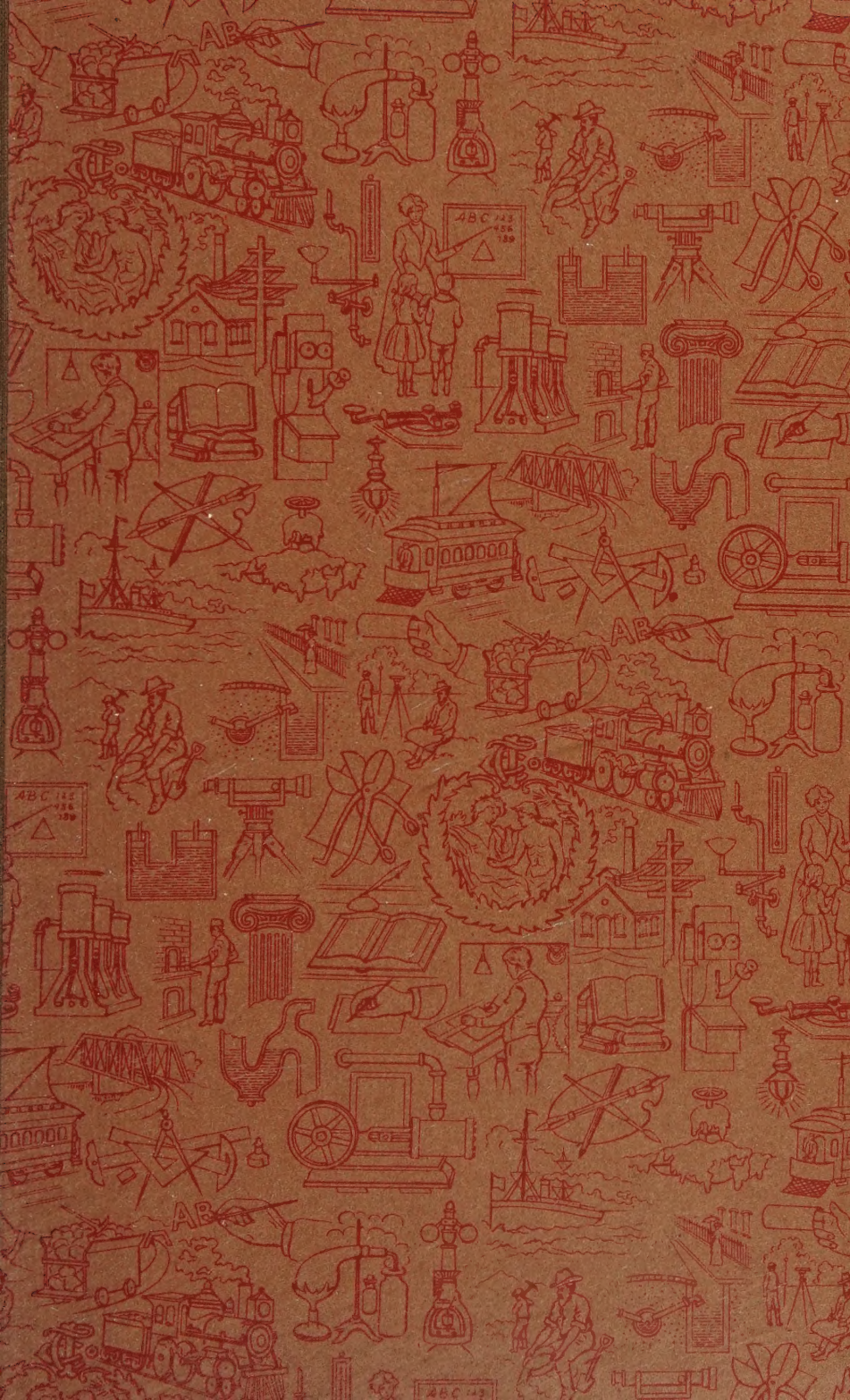
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